Supplementary Online Materials

One should note that the carrier density of the modified TCO layer varies sharply away from the FO/TCO interface, so does the permittivity. Based on Drude model, the permittivity also depends on the wavelength, i.e. great dispersion exists for the permittivity. Therefore, one must take the spectral and spatial dispersion of the 2DEG into account when considering particular nonlinear optical processes, such as sumfrequency generation, difference-frequency generation, optical parametric oscillation, etc.

As shown in Fig. S1, the real part of the permittivity ε_m^r in the modified ZnO layer proximal to a Z-cut LT slab is shown in Fig. S1(a). The maximum is ε_m^r =4.0 shown in yellow. The minimum is ε_m^r =-131.60 shown in blue. Considering such as great range in ε_m^r , an orange node pointer is created at ε_m^r =0 to reveal more information about the epsilon near zero (ENZ) state since it is of significance in applications.

The imaginary part of the permittivity $\varepsilon_{\rm m}^{\rm i}$ in the modified ZnO layer proximal to an LT slab is shown in Fig. S1(b). The maximum is $\varepsilon_{\rm m}^{\rm i}$ =7.913 shown in yellow. The minimum is $\varepsilon_{\rm m}^{\rm i}$ =1.301×10⁻⁴ shown in blue. Then a pink node pointer is created at $\varepsilon_{\rm m}^{\rm i}$ =0.25, an orange node pointer is created at $\varepsilon_{\rm m}^{\rm i}$ =0.5 and a green node pointer is created at $\varepsilon_{\rm m}^{\rm i}$ =1.0 to reveal more information about the ENZ state for the reason stated above. The real part of refractive index *n* in the modified ZnO layer for the LT slab is

The real part of refractive index n in the modified ZnO layer for the LT slab is shown in Fig. S1(c). Spring colormap is used to depict the variation of the refractive index near the vicinity of the LT/ZnO interface. The maximum (n=1.998) and minimum (n=0.032) are set for pink and yellow, respectively.

The imaginary part of the refractive index κ in the modified ZnO layer next to the LT slab is shown in Fig. S1(d). Spring colormap is used to depict the variation of the refractive index near the vicinity of interface. The maximum ($\kappa = 11.477$) and minimum ($\kappa = 3.256 \times 10^{-5}$) are set for pink and yellow, respectively.

Similarly, the permittivity ε_m^r in the modified Al:ZnO layer next to the LT slab is shown in Fig. S1(e). The maximum is $\varepsilon_m^r = 3.9$ shown in yellow. In fact, the maximum value always equals to the unmodified ITO permittivity. The minimum is $\varepsilon_m^r = -136.87$ shown in blue. Then an orange node pointer and a pink node pointer are created at $\varepsilon_m^r = 0$ and $\varepsilon_m^r = -10$ to reveal more information about the ENZ state.

The permittivity ε_m^i in the modified Al:ZnO layer proximal to a LT slab is shown in Fig. S1(f). The maximum is $\varepsilon_m^i = 8.2204$ shown in yellow. The minimum is $\varepsilon_m^i = 0.0046$ shown in blue. Then a pink node pointer is created at $\varepsilon_m^i = 0.25$, an orange node pointer is created at $\varepsilon_m^i = 0.5$ and a green node pointer is created at $\varepsilon_m^i = 1.0$ to reveal more information about the ENZ state.

The real part of the refractive index n in the modified Al:ZnO layer for the LT sample is shown in Fig. S1(g). Spring colormap is used to depict the variation of refractive index near the vicinity of the LT/Al:ZnO interface. The maximum (n= 1.9268) and minimum (n= 0.0318) are set for pink and yellow, respectively.

The imaginary part of refractive index κ in the modified Al:ZnO layer for the LT slab is shown in Fig. S1(h). Spring colormap is used to depict the variation of refractive index near the vicinity of the LT/ZnO interface. The maximum (κ =11.704) and the minimum (κ =0.0012) are set for pink and yellow, respectively.



Fig. S1 Spectral dependences of real part ε_m^r (a) and imaginary part ε_m^i (b) of dielectric function, real part *n* (c) and imaginary part κ (extinction coefficient) (d) of refractive index in the modified ZnO layer proximal to the LT slab at varying distance to the ZnO/LT interface; Spectral dependences of real part ε_m^r (e) and imaginary part ε_m^i (f) of dielectric function, real part *n* (g) and imaginary part κ (extinction coefficient) (h) of refractive index in the modified Al:ZnO layer proximal to the LT slab at varying distance to the Al:ZnO/LT interface. To show these figures clearly, the yellow ([R G B]=[255 255 0]), green ([R G B]=[0 255 0]), orange ([R G B]=[217 84 26]), pink([R G B]=[255 0 255]) and blue ([R G B]=[0 0 255]) node pointers are used in the customized colorbars.

As shown in Fig. S2(a-d), ZnO thin film is coated onto a 1.0 a.t. % La doped PLZT [111] film. The permittivity ε_m^r in the modified ZnO layer proximal to the PLZT film is shown in Fig. S2(a). The maximum is ε_m^r =4.0 shown in yellow. The minimum is ε_m^r =-60.464 shown in blue. Considering the large variation of ε_m^r , an orange node pointer is created at ε_m^r =0 to reveal more information about the ENZ state.

The imaginary part of the permittivity ε_m^i in the modified ZnO layer proximal to a PLZT film is shown in Fig. S2(b). The maximum is ε_m^i =3.762 shown in yellow. The minimum is ε_m^i =1.179×10⁻⁴ shown in blue. Then a pink node pointer is created at ε_m^i =0.25, an orange node pointer is created at ε_m^i =0.5 and a green node pointer is created at ε_m^i =1.0 to reveal more information about the ENZ state.

The real part of refractive index n in the modified ZnO layer for the PLZT film is shown in Fig. S2(c). Spring colormap is used to depict the variation of the refractive index near the vicinity of the PLZT/ZnO interface. The maximum (n= 1.998) and minimum (n= 0.038) are set for pink and yellow, respectively.

The imaginary part of the refractive index κ in the modified ZnO layer for the PLZT film is shown in Fig. S2(d). Spring colormap is used to depict the variation of the refractive index near the vicinity of the PLZT/ZnO interface. The maximum ($\kappa = 7.78$) and minimum ($\kappa = 2.949 \times 10^{-5}$) are set for pink and yellow, respectively.

Similarly, for a case that a ZnO thin film is coated onto a 0.5 a.t. % La doped PLZT [001] film, as shown in Fig. S2(e-h).

The permittivity ε_m^r in the modified ZnO layer proximal to the PLZT film is shown in Fig. S2(e). The maximum is ε_m^r =4.0 shown in yellow. The minimum is ε_m^r =-319.69 shown in blue. Considering the large variation of ε_m^r , an orange node pointer is created at $\varepsilon_m^r = 0$ to reveal more information about the ENZ state.

The permittivity ε_{m}^{i} in the modified ZnO layer proximal to a PLZT film is shown in Fig. S2(f). The maximum is ε_{m}^{i} =18.89 shown in yellow. The minimum is ε_{m}^{i} =1.371×10⁻⁴ shown in blue. Then a pink node pointer is created at ε_{m}^{i} =0.25, an orange node pointer is created at ε_{m}^{i} =0.5 and a green node pointer is created at ε_{m}^{i} =1.0 to reveal more information about the ENZ state.

The real part of refractive index n in the modified ZnO layer for the PLZT film is shown in Fig. S2(g). Spring colormap is used to depict the variation of the refractive index near the vicinity of the PLZT/ZnO interface. The maximum (n= 1.998) and minimum (n= 0.032) are set for pink and yellow, respectively.

The imaginary part of the refractive index κ in the modified ZnO layer for the PLZT film is shown in Fig. S2(h). Spring colormap is used to depict the variation of the refractive index near the vicinity of PLZT/ZnO interface. The maximum ($\kappa = 17.89$) and minimum ($\kappa = 3.431 \times 10^{-5}$) are set for pink and yellow, respectively.



Fig. S2 Spectral dependences of real part ε_m^r (a) and imaginary part ε_m^i (b) of dielectric function, real part *n* (c) and imaginary part κ (extinction coefficient) (d) of refractive index in the modified ZnO layer proximal to the PLZT film at varying distance to the ZnO/PLZT interface in a 1.0 a.t. % La doped [111] PLZT film; Spectral dependences of real part ε_m^r (e) and imaginary part ε_m^i (f) of dielectric function, real part *n* (g) and imaginary part κ (extinction coefficient) (h) of refractive index in the modified ZnO layer proximal to the negative PLZT film at varying distance to the ZnO/PLZT film. To show these figures clearly, the yellow ([R G B]=[255 255 0]), green ([R G B]=[0 255 0]), orange ([R G B]=[217 84 26]), pink([R G B]=[255 0 255]) and blue ([R G B]=[0 0 255]) node pointers are used in the customized colorbars.