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Supplementary Information: Spin-layer Locked Gapless States in Gated Bilayer Graphene †

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We investigate how topological states interact with the vacancy state without spin polarization. The on-site Coulomb interaction is set to $U = 0$. We begin the study by considering two separate systems: (a) gated BLG with a SDW and (b) gated BLG with vacancies in the upper layer. For these two cases, the corresponding structural models and LDOS spectra around the cone valley are shown in Figs. 1 (a) and (b). In case (a), there are two spin-degenerate topologically protected states spanning the entire energy gap. We check that they remain spin degenerate even for $U \neq 0$.

In case (b), the vacancy state in the upper layer spans the entire reciprocal zone. Away from the cone, at $k = 0$ and $k = \pi$, the vacancy state is fixed at zero energy because this graphene layer has no gate. The LDOS structure of BLG with a SDW and vacancies is shown in Fig. 1 (c). The spectra described in panels (a) and (b) are mixed beyond simple superposition due to the interaction, mixing and anticrossing of the pair of gapless states with the vacancy state. Despite this strong interaction, one state connecting the valence and conduction bands is still present. Closer inspection reveals that the gapless state shown in Fig. 1 (c) has a mixed character of left and right topological states. When the on-site Coulomb interaction is included, all the states spin split, and we obtain Fig. 2 in the main article.

Fig. 2 shows the energy spectra for the three-state model Hamiltonian $H_{vac-SDW}$ defined in the main article. The spectra reveal that the interaction between the topological states and the vacancy state implies the survival of a topological state along the gap. A more detailed analysis confirms that the surviving gapless state is a mixture of the previous topological states and changes when asymptotically approaching the positive and negative energies away from the vacancy state. Furthermore, the effect of high voltages shows that for $V > \gamma_1$, the gapless state shifts to the right. This result aids understanding the results presented in Fig.3 of the main article.

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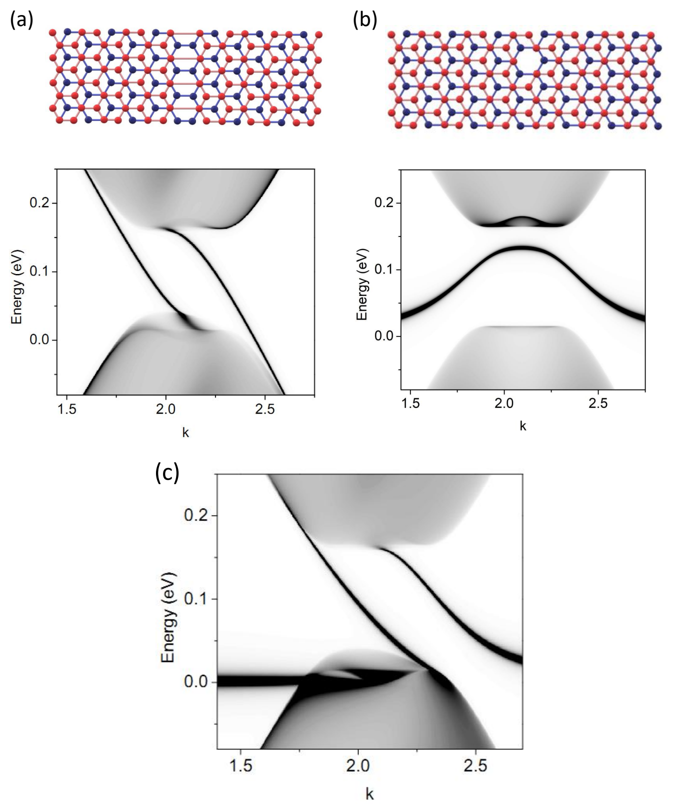

Fig. 1 Mixing gapless and vacancy states in gated BLG near the Fermi level. (a) BLG with a SDW. The LDOS shows a couple of spin-degenerate topological modes in the energy gap opened by a gate of V=0.18 in the bottom layer. (b) Gated BLG containing vacancies. The LDOS shows a vacancy band in the opened gap. (c) BLG including a SDW and vacancies. Note that the states described in (a) and (b) interact when calculated without the on-site Coulomb interaction.

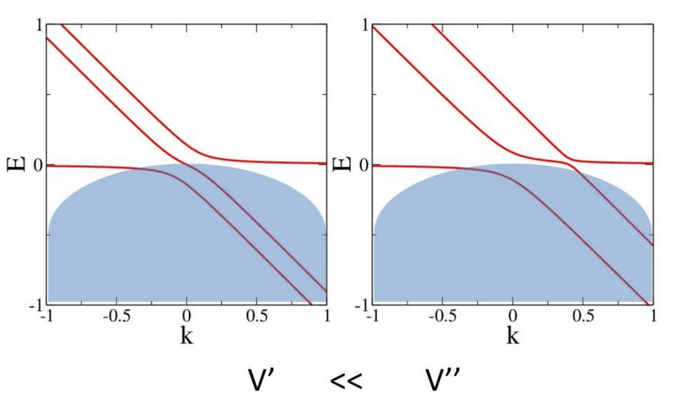

Fig. 2 Results of the three-state model for a vacancy state (at zero energy) interacting with the two topological states around a valley, as discussed in the text. It is noteworthy that a topological state survives following each of the topological states asymptotically moving away from the defect states for either positive or negative energies. The shaded area schematically marks the valence band continuum.