# **Deconvolution of Electroosmotic Flow in Hysteresis Ion Transport through Single Asymmetric Nanopipettes**

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#### Part I. Descriptions of simulation details and eperimental data treatments.

#### Simulation details

Finite element simulations were carried out using COMSOL Multiphysics (v. 4.3) by solving the coupled Poisson and Nernst-Planck (PNP) and the Navier-Stokes (NS) equations. The simulation structure in scheme S1 is a conical nanopore with radius, $r_p$ , of either 20 nm or 100 nm with AI as centerline for symmetry integration; half cone angle 10°; CH= 10 µm; HJ= 2 µm; IJ= 5 µm; DE= 1 µm; and FG=1 µm. Those parameters are representative for experimental devices and have been validated in previous simulation reports.<sup>1, 2</sup> The equations were coupled by using the three modules: 'Transport Of Diluted Species (TDS)'; 'Electrostatics' (ES); and Laminar Flow (LF). Boundary conditions are given in table S1. The three segements DE, EF and FG have defined SCD that affects transport. DE and FG use a gradient SCD<sup>3</sup> with exponential decay given by:

$$\sigma(z) = \sigma_0 e^{-z/\tau} + \sigma_b$$
$$\sigma(r) = \sigma_0 e^{-r/\tau} + \sigma_b$$

 $\sigma(z)$  or  $\sigma(r)$  is the position dependent SCD,  $\sigma_0$  is the maximal SCD (-70 mC/m<sup>2</sup>) and  $\sigma_b$  is the decay to bulk/flat surface value of -1 mC/m<sup>2</sup>.

Free triangular mesh was used. Finer mesh of between 0.6-3 nm was used on the charged surface and a larger mesh was used on the other areas to cut down on computing time. Mesh size was adjusted at higher concentrations to be finer. Total mesh elements consists of  $2.2 \times 10^5$ . A periodic triangular potential waveform was used at a scan rate of 100 V/s unless otherwise noted, and the current was sampled every 20 mV. Note that the scan rate in the simulation and experiments differ by 2-3 orders of magnitude. This is probably a result of the modelling of the ions as point charges as elaborated in our earlier report. <sup>1, 2</sup>



Scheme S1. Geometry of the simulation structure. The nanopore radius is r<sub>p</sub>, with AI as the centerline for symmetry integration.

Table S1. Boundary conditions for the coupled Poisson, Nernst-Planck and Navier-Stokes equations.

	TDS	ES	LF
AB	C=bulk	Ground (V=0)	Outlet p=0
CD	No flux	SCD=0	No slip
DE	No flux	SCD=-1 mC/m <sup>2</sup>	No slip
EF	No flux	SCD= Gradient (exponential decay)	No slip
FG	No flux	SCD= Gradient(exponential decay)	No slip
GH	No flux	SCD=0	No slip
ΗJ	No flux	SCD=0	No slip
JI	C=bulk	Working electrode (V=f(t))	Inlet p=0

## **Governing Equations**

Equations 1, 2 and 3 are the Nernst-Planck, Poisson and modified Navier-Stokes equations respectively. These governing equations are widely used in the simulations of nanotrasport and beyond. Equation 4 is the continuity equation which will be discussed in more detail below. The following descriptions of the governing equations for simulations in nanopores have been previously summarized<sup>4-6</sup> for the interested readers.

$$\boldsymbol{J}_{\boldsymbol{i}} = -\boldsymbol{D}_{\boldsymbol{i}} \nabla \boldsymbol{c}_{\boldsymbol{i}} - \frac{\boldsymbol{z}_{\cdot}^{F}}{\epsilon_{T}} \boldsymbol{D}_{\boldsymbol{i}} \, \boldsymbol{c}_{\boldsymbol{i}} \nabla \boldsymbol{\phi} + \boldsymbol{c}_{\boldsymbol{i}} \mathbf{u} \tag{1}$$

$$\nabla^2 \left( \varepsilon_0 \varepsilon_r \phi \right) = -F \sum z_i c_i \tag{2}$$

$$\frac{1}{\rho} \left( -\nabla p + \eta \nabla^2 \mathbf{u} - (F \sum z_i c_i) \nabla \phi \right) = 0$$
(3)

$$-\nabla \cdot \boldsymbol{J}_{\boldsymbol{i}} = \frac{\partial c_{\boldsymbol{i}}}{\partial \mathbf{t}} \tag{4}$$

The Nernst-Planck equation

The Nernst-Planck equation (1) is used for calculation of current and fluxes in the system. For species I,  $J_i$  is the ion flux;  $D_i$  is the diffusion coefficient;  $c_i$  is the concentration;  $z_i$  is the charge on the ion; F is Faraday's constant; R is the gas constant; T is temperature;  $\phi$  is the potential; **u** is the fluid velocity. The first term in equation 1 describes fluxes originating from a concentration gradient, i.e. diffusion. The second term calculates migration driven by potential gradient (obtained from Poisson equation) and the third from convection/flow (obtained from Navier-Stokes equation). All of these three gradients exist in nano-aperture systems leading to diffusional flux from concentration polarization, migration flux due to applied potential between the electrodes and fluid convection originating from electroossmotic flow (EOF). Equation 4 is the continuity equation with mass conservation.  $\nabla \cdot J_i = 0$  is the steady state condition often used in the literature. The simulations in this study are time-dependent simulations and the steady state approximation is not used. Equation 4 is known as the time-dependent Nernst-Planck equation. Equation 4 approaches the steady state condition at very low stimulus frequency or low scan rate.

## The Poisson equation

The Poisson equation (2) describes electrostatics which relates the ion concentrations (right hand side) to the potential (left hand side).  $\varepsilon_0$  is the relative permittivity in free space and  $\varepsilon_r$  is the relative permittivity of the solvent. Surface electric

field  $(-\varepsilon_0 \mathbf{n} \cdot \nabla \phi = \delta)$  is calculated from the surface charge density ( $\delta$ ) of the transport confining boundaries. When coupled to equation 1, the system of equations is known as the Nernst-Planck and Poisson equation system (PNP).

The modified Navier-Stokes equation

The Navier-Stokes equation (3) describes incompressible fluid flow in a system.  $\rho$  is the density of the fluid; p is the hydrodynamic pressure;  $\eta$  and is the viscosity of the fluid. Under the assumption of incompressible flow (constant fluid density) and mass conservation, the continuity equation is given by  $\nabla \cdot \mathbf{u} = 0$ . These assumptions result in equation 4 which gives the modified Navier-Stokes equation. The third term in equation 3 represents the external force on the solvent from the applied potential (or electric field). The third term in equation 3 generates EOF in the model which originates from counter-ion dominant charge distribution in solution, mostly in EDL region, dragging the solvent under an applied electric field. The resulting pressure and fluid velocity distributions can then be determined with appropriate boundary conditions. Coupling equation 4 with the PNP model (equations 1 and 2) gives the PNP-NS model.

# Calculating Debye Length using the Debye-Hückel approximation

A commonly used descriptor of the EDL is the Debye length<sup>4</sup> defined as (often using 1:1 electrolyte):

$$\lambda = \sqrt{\frac{\varepsilon_0 \varepsilon_r k_B T}{e^2 \sum_i c_i z_i^2}}$$

where is  $\varepsilon_0$  the relative permittivity of free space;  $\varepsilon_r$  is the relative permittivity of the solvent;  $k_B$  is Boltzmann's constant; T is the temperature; e is the fundamental charge;  $c_i \& z$  are the concentration and the charge of ion species *i* (1:1 electrolyte such as KCl). While Debye length is widely adopted to describe EDL in various research fields, it is well known that it falls short to account for many nano-transport phenomena quantitatively. The Debye length serves as a convenient parameter describing the experimental conditions under different bulk ionic strength.

## **Experimental data treatment**

Representative outcome from the data treatment steps are compared in Figure S13. The conductivity measurement results (current-potential data sampled at 1 mV per point/step) are first treated using the box car method where every 10 points are averaged. The differentiation function of Origin v 8E is used on the boxcar-ed i-V data (10 mV per point) to obtain  $G_{diff}$  (black points in panel A/F). A baseline (red curve in panel A/F) is created using the 'spectroscopy' section function. The percent relative standard deviation, 'noise level', of the  $G_{diff}$  is estimated by averaging the relatively constant values in a given potential range (~0.6-0.65 V in panel A). Those data points beyond 3X standard deviation (Stdev) from baseline are rejected. The resulting data is then smoothed using Origin v 8E via 'adjacent 10-point averaging'. The first 5 points and last 5 point of the resulting smoothed data are discarded accordingly. The differentiation function is then used to obtain the second derivative curve in panel D/I. Generally speaking, a baseline itself or outlier rejection makes it easier to discern the curve shape or trend from the experimental results. Common for most electrical measurements, the higher potentials have more 'noisy' response than lower potentials. Each data set should be analyzed based on the measurement conditions to minimize potential artifacts due to either data treatment or sampling.



Figure S1. Comparison of simulated conductance (G) and differential conductance ( $G_{diff}$ ). (A) and (C) PNP model at radius of 12 nm and 60 nm, respectively for a nanopore. (B) and (D) PNP-NS model at radius of 12 nm and 60 nm, respectively for a nanopore. At KCl concentration of 1 mM.

 $G_{diff}$  is defined as  $G_{diff}$  = di/dV and G as G=i/V. In general features are better resolved using  $G_{diff}$  compared to G and this is more significant when EOF is present as illustrated by the comparisons in panel B and D.



Figure S2. Comparison of simulated  $G_{diff}$  in other i-V segments from PNP and PNP-NS models. (A) LC backward scans (from cross point to -1.0V). (B) LC forward scans (from -1.0V to cross point). (C) HC backward scans (from 1.0V to cross point). Simulations for a nanopore of radius 12 nm in 1 mM KCl solution. Arrows show the direction of applied potential.

The potential switching or turning point at the end potential ( $\pm 1$ V herein ) affects different i-V scan segments as demonstrated in figure S2. It is more straightforward to use scan segments not immediately preceded by a turning point to analyze EOF, such as the LC backward scan in panel A and the HC forward scan used in the main text. The two scan segments immediately preceded by turning point (bottom panels) display additional curvatures from the threshold potential due to the continuous potential scan and switching.



Figure S3. Linear fitting of simulated  $G_{diff}$  from the 12 nm radius nanopore in different KCl concentrations. (A) 1 mM. (B) 5 mM. (C) 10 mM. (D) 100 mM. Data points from cross point to the divergent point between the two models were used in the fitting (as shown).



Figure S4. Simulated  $k_1$  at different concentrations from PNP and PNP-NS at different concentrations corresponding to the data in figure 2.



Figure S5. Second derivative ( $dG_{diff}/dV$ ) plots of the simulated conductivity for PNP and PNP-NS at different concentrations corresponding to the data in figure 2. (A) 1 mM. (B) 5 mM. (C) 10 mM. (D) 100 mM. Horizontal dashed (lines indicate k<sub>1</sub> obtained from the G<sub>diff</sub> fitting with PNP and PNP-NS models. The vertical dashed line indicates the divergent point ( $V_{div}$ ). Note the differences in k<sub>1</sub> between the two approaches will not affect the observed trends and related discussions.



Figure S6. Simulated peak k<sub>1</sub> vs fitted k<sub>1</sub> at different concentrations from (A) PNP and (B) PNP-NS.

The peak  $k_1$  vs fitted  $k_1$  has a slope of  $\sim 1$  which means the two methods for obtaining  $k_1$  are in reasonable agreement



Figure S7. Simulated scan rate dependence of (A) EOF-IF and (B) EOF-I from a 40 nm radius nanopore in 1 mM KCl. Solid symbols are PNP-NS results and open ones from PNP for reference.



Figure S8. Simulated  $G_{diff}$  in LC backward scans (from cross point to -1.0V) (A) 1 mM; (B) 5 mM KCl; and (C) 10 mM KCl in a 60 nm radius nanopore.



Figure S9. The potential with maximum EOF-IF,  $V_p$ , in different KCl concentrations for 12 nm (red) and 60 nm (black) radius nanopore from 1 mM to 100 mM.

Experimental results and analysis



Figure S10. Experimental EOF IF (A & C) and EOF-I (B &D) plots from two large nanopipette (with ca. 200 nm radius) in 1 mM KCl. The blue curves are from the nanopipette in figure 6 of radius ca. 200 nm (nanopipette #1) and the red curves are from another of ca. 200 nm radius (nanopipette #2)



Figure S11. Experimental I-V data (A) and corresponding analysis of  $G_{diff}$  (B), EOF IF (C) and EOF-I (D) from a large nanopipette (with ca. 60 nm radius) in different KCl concentrations. Some curves are multiplied by the listed arbitrary factors to fit in the same scale for direct visual comparison.



Figure S12. Experimental I-V data (A) and corresponding analysis of  $G_{diff}$  (B), EOF IF (C) and EOF-I (D) from a small nanopipette (with ca. 10 nm radius) in different KCl concentrations. Some curves are multiplied by the listed arbitrary factors to fit in the same scale for direct visual comparison. Because the level of fluctuation was so high, using a baseline outlier rejection procedure would have resulted in significant loss of data points. Therefore, outlier rejection was not used to remove potential outlier points.



Figure S13. The results of step-by-step data treatment on 2.5 mM (A-E) and 100 mM (F-J) KCl concentration with a 60 nm radius nanopipette. (A/F):  $G_{diff}$  and baseline after 10-point boxcar treatment of raw i/V data. (B/G):  $G_{diff}$  after outlier rejection (3x standard deviations from the average/threshold). (C/H): 10-pt-smoothing of  $G_{diff}$  after outlier rejection. (D/I): Derivative of the smoothed  $G_{diff}$  (second derivative of i/V measurement). (E/J): Comparison of the resulting EOF-IF from data treatment with (top) and without (bottom) outlier rejection (other steps are identical).



Figure S14. Experimental I-V data<sup>7</sup> (A) and corresponding analysis of  $G_{diff}$  (B), EOF IF (C) and EOF-I (D) from a small nanopipette (different from main text), with ca. 60 nm radius, in different KCl concentrations. Inset shows 1 mM conductivity response. The peak is highlighted by the dashed circle. The original raw data in panel A was previously reported in reference 4.



Figure S15. Experimental I-V data (A) and corresponding analysis of  $G_{diff}$  (B), EOF IF (C) and EOF-I (D) from a large nanopipette (nanopipette #3),with ca. 200 nm radius, in different KCl concentrations. Inset shows 1 mM conductivity response. and  $G_{diff}$  for 25 mM is multiplied by 0.2 to allow for comparison.



Figure S16. Additional time lapse images from a ca. 60 nm nanopipette after a +1.0V (reference inside) was applied at 0s. The 0 s image is the same as that in the main text in Figure 8. The solution inside the nanopipette contains 50 mM KCl and 10  $\mu$ M Rhodamine B, and the outside solution contains 50 mM KCl without dye.



Figure S17. Fluorescence imaging analysis of EOF at 0V (A & B) and -1V (C & D). Data from the same ca. 60 nm radius nanopi-pette in 50 mM KCl solution under +1V shown in Figure 8. (A) and (C) are the representative images and (B) and (D) are the corresponding analysis. The solution inside the nanopipette contains 50 mM KCl and 10  $\mu$ M Rhodamine B, and the outside solution contains 50 mM KCl without dye. The background, contrast and size of all time-lapse images are set consistent for direct comparison of grey scale intensity.

# References

- 1. D. C. Wang, J. Liu, M. Kvetny, Y. Li, W. Brown and G. L. Wang, *Chem Sci*, 2014, 5, 1827-1832.
- D. C. Wang, W. Brown, Y. Li, M. Kvetny, J. Liu and G. L. Wang, *Anal Chem*, 2017, 89, 11811-11817.
- 3. J. Liu, D. C. Wang, M. Kvetny, W. Brown, Y. Li and G. L. Wang, *Langmuir*, 2013, **29**, 8743-8752.
- 4. R. B. Schoch, J. Han and P. Renaud, *Rev. Mod. Phys.*, 2008, **80**, 839-883.
- 5. H. S. White and A. Bund, *Langmuir*, 2008, **24**, 2212-2218.
- 6. Y. Ai, M. Zhang, S. W. Joo, M. A. Cheney and S. Qian, J. Phys. Chem. C, 2010, 114, 3883-3890.
- 7. Y. Li, D. C. Wang, M. M. Kvetny, W. Brown, J. Liu and G. L. Wang, *Chem Sci*, 2015, **6**, 588-595.