Electronic Supplementary Information for

Controlling Shear Jamming in Dense Suspensions via Particle Aspect Ratio

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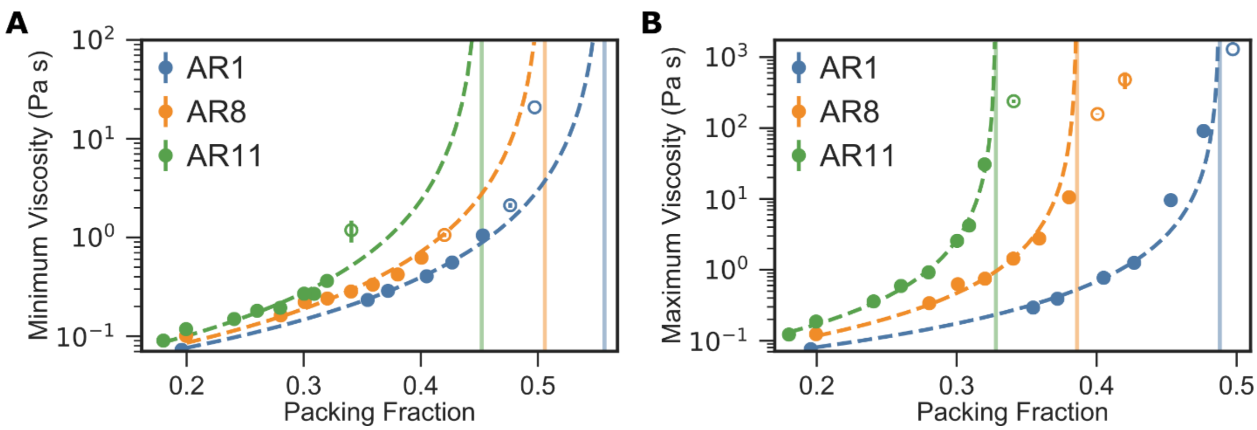

Figure S1. A. The divergence of the minimum, Newtonian viscosity before shear thickening, as a function of packing fraction. Open circles indicate data that was excluded from fitting due to the presence of shear thinning which may obscure the baseline, Newtonian suspension viscosity. Dashed lines indicate the fit determined by Equation 1, using $\beta = 2$ for all aspect ratio systems. The fitted value of ϕ_0 is shown by the vertical lines: $\phi_0(\mathrm{AR1}) = 0.557$, $\phi_0(\mathrm{AR8}) = 0.506$, and $\phi_0(\mathrm{AR11}) = 0.452$. B. The divergence of the maximum viscosity at the peak of shear thickening, as a function of packing fraction. Open circles indicate data that was excluded from fitting due to the possibility of the suspension being driven into the shear jammed state during rheology, resulting in an inaccurate measured viscosity. Dotted lines indicate the fit determined by Equation 1, using $\beta = 1.8$ for all aspect ratio systems. The fitted value of ϕ_m is shown by the vertical lines: $\phi_m(\mathrm{AR1}) = 0.488$, $\phi_m(\mathrm{AR8}) = 0.386$, and $\phi_m(\mathrm{AR11}) = 0.328$.

Generation of the State Diagram from the Wyart-Cates Model

The Wyart-Cates (WC) model[1] predicts a suspension with packing fraction $\phi < \phi_m$ will exhibit two Newtonian flow branches: a low-shear branch with viscosity dependent on the frictionless packing fraction ϕ_0, and a high-shear branch with viscosity dependent on the frictional packing fraction ϕ_m. Both these viscosities can be described using a Krieger-Doughtery relation[2]:

$$\eta = \eta_0 \left(1 - \frac{\phi}{\phi_J}\right)^{\beta}$$

Where η is the suspension viscosity, η_0 is the suspending solvent viscosity, ϕ_J is the relevant packing fraction (ϕ_0 at low shear and ϕ_m at high shear), and β is a fitting parameter.

The WC model posits that the system transitions between the low-shear and high-shear viscosities in accordance with an effective jamming point, ϕ_{eff}. ϕ_{eff} is a weighted average of ϕ_0 and ϕ_m, using the fraction of particles in frictional contact, $f(\tau) = 1 - e^{-\tau/\tau^*}$, where τ^* describes the breakdown of lubrication and the initiation of friction-dominated flows.

ϕ_{eff} is thus given by $\phi_{\text{eff}} = \phi_m \left(1 - e^{-\frac{\tau}{\tau^*}}\right) + \phi_0 \left(e^{-\frac{\tau}{\tau^*}}\right)$.

By substituting ϕ_{eff} into the Kriger-Dougherty description, we find:

$$\eta = \eta_0 \left(1 - \frac{\phi}{\phi_m + e^{-\frac{\tau}{\tau^*}}(\phi_0 - \phi_m)}\right)^{\beta}$$

Thus, armed with the previously-determined values of ϕ_0 and ϕ_J, we can fit the experimental flow curve, $\eta(\tau)$, to this equation and find τ^*. In all cases, β is a free parameter.

Armed with ϕ_0, ϕ_m, τ^*, and β, it is possible to calculate the DST and SJ boundaries and generate a state diagram. This is most easily done as a function of shear stress and shear rate; thus we reformulate the above expression for $\eta(\phi_{\text{eff}})$:

$$\dot{\gamma} = \frac{\tau}{\eta_0} \left(1 + \frac{\phi}{\phi_m + (\phi_0 - \phi_m)e^{-\frac{\tau}{\tau^*}}}\right)^{\beta}$$

The SJ boundary can be found by solving this equation for $\dot{\gamma} = 0$, which will generate two families of answers, including one at $\tau = 0$ which is the trivial solution at rest. The relevant solution is for nonzero τ.

The DST boundary occurs when there is a discontinuous increase in the viscosity or shear stress at a given shear rate. Thus, the DST condition is that $\frac{d\log(\eta)}{d\log(\dot{\gamma})} = \frac{d\log(\tau)}{d\log(\dot{\gamma})} = \infty$. Thus can be inverted to give $\frac{d\log(\dot{\gamma})}{d\log(\tau)} = 0$, which can be more easily solved. This equation yields two solutions, which form the edges

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[1] Wyart, M. & Cates, M. E. Discontinuous Shear Thickening without Inertia in Dense Non-Brownian Suspensions. *Phys. Rev. Lett.* **112**, 098302 (2014).

[2] Krieger, I. M. & Dougherty, T. J. A Mechanism for Non-Newtonian Flow in Suspensions of Rigid Spheres. *Trans. Soc. Rheol.* **3**, 137–152 (1959).

of the “nose” of the DST regime.

When these boundaries are plotted as $\tau(\phi)$, they yield the DST and SJ state diagram regions shown in Figure 4. Above $\phi = \phi_0$, the system exhibits isotropic jamming indicated in gray.