Passive droplet generation in aqueous two-phase systems with a variable-width microchannel

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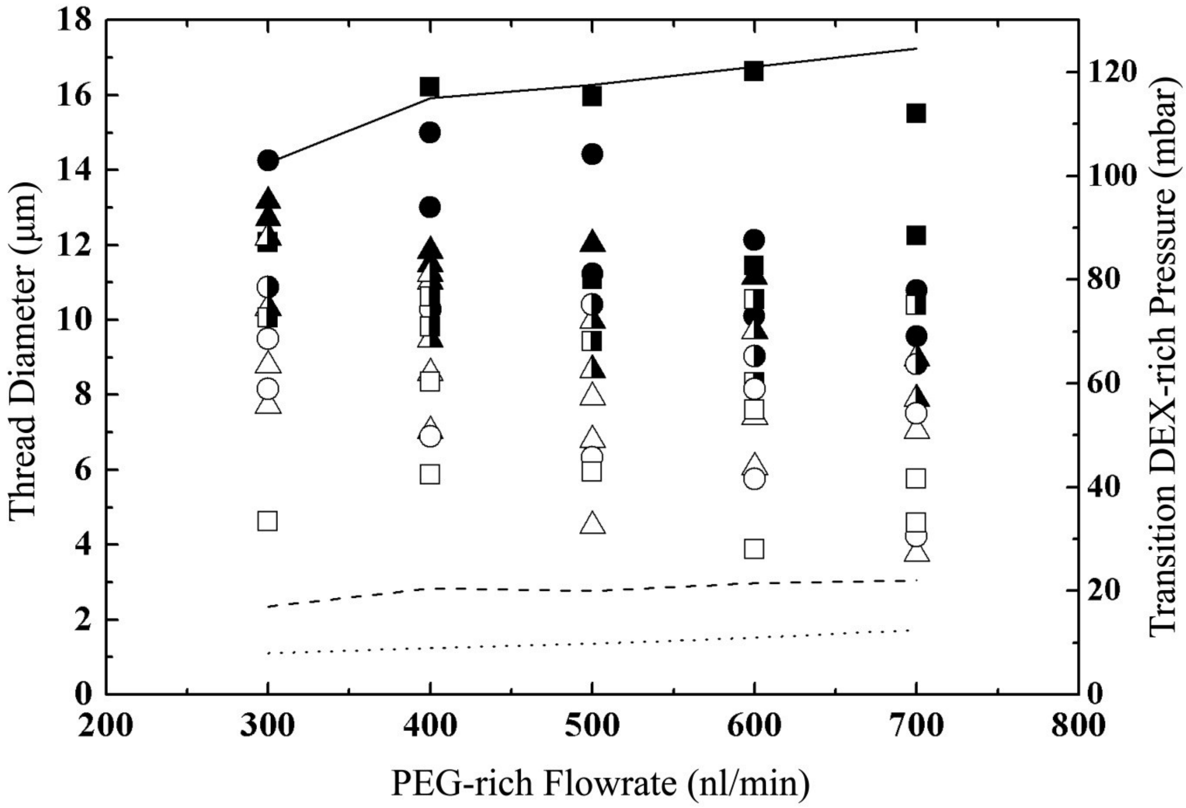
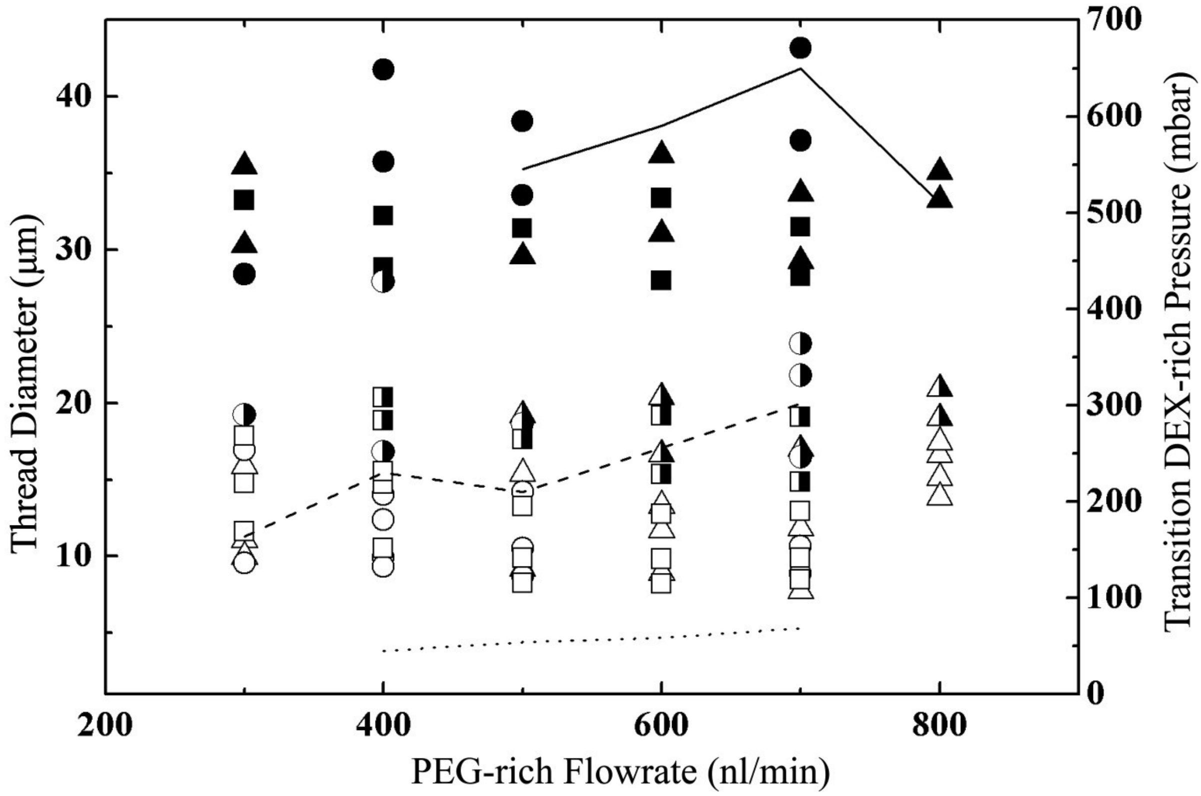
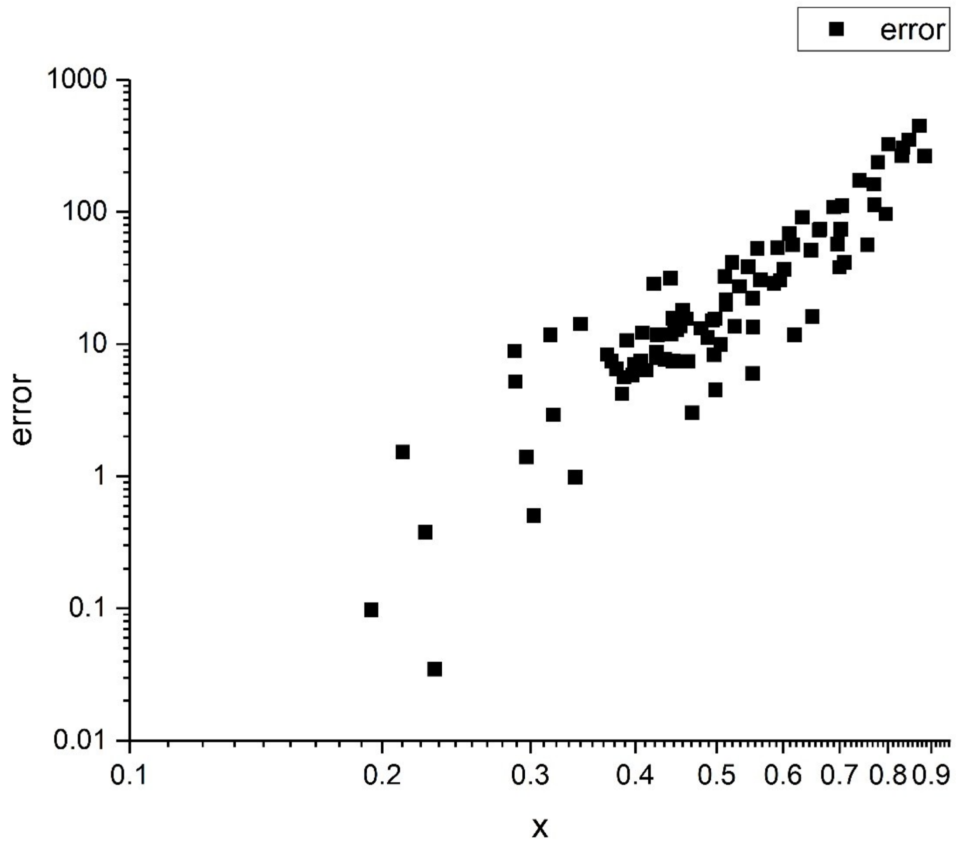


[Fig.SI-1] The concentration case of DEX/PEG= 6.4/5 (% w/v). The triangle symbol stands for DEX-inlet width of 2.89 ± 0.08, the circular symbol for 4.74 ± 0.03, and the rectangular symbol for 10.36 ± 0.09 μm. The open symbol stands for the droplet-generation regime, the half-filled symbol for the transition regime, the filled symbol for the stable jet regime. The solid line stands for the width of 2.89 ± 0.08, the dashed line for 4.74 ± 0.03, and the dotted line for 10.36 ± 0.09 μm.



[Fig.SI-2] The concentration case of DEX/PEG= 16/10 (% w/v). The triangle symbol stands for DEX-inlet width of 2.89 ± 0.08, the circular symbol for 4.74 ± 0.03, and the rectangular symbol for $10.36 \pm 0.09\ \mu m$. The open symbol stands for the droplet-generation regime, the half-filled symbol for the transition regime, the filled symbol for the stable jet regime. The solid line stands for the width of 2.89 ± 0.08, the dashed line for 4.74 ± 0.03, and the dotted line for 10.36 ± 0.09 μm.



[Fig.SI-3] The breakup time error(%) of theoretical prediction (Eq.(3)) from the measured data. X is the ratio of the thread diameter and channel height.

[Derivation of Eq. (1)]

An incompressible, laminar flow in a cylindrical pipe is considered that is fully-developed, unidirectional, and with no applied body force. The Navier-Stokes equation then gives

$$0 = -\frac{dP}{dx} + \mu\frac{1}{r}\frac{d}{dr}\left(r\frac{dv}{dr}\right) \quad \text{(SI-1)}$$

We first consider the inlet channel of the DEX-rich phase the radius of which is R_D. The velocity profile is then obtained as

$$v_D^i = -\frac{1}{4\mu}\frac{dP}{dx}\left({R_D}^2 - r^2\right)$$

and $Q_D = \int_0^{R_D} 2\pi r dr v_D^i$. So,

$$Q_D = -\frac{1}{8\mu}\frac{dP}{dx}{R_D}^4 \quad \text{(SI-2)}$$

Here we put the pressure gradient as

$$\frac{dP}{dx} = \frac{P_j - P_D}{l_D}$$

And get

$$Q_D = -\frac{\pi}{8\mu}\frac{P_j - P_D}{l_D}{R_D}^4 \quad \text{(SI-3)}$$

Where the subscript j stands for the location where the DEX and PEG flows joins. We then consider the flow in the main channel where the DEX-rich phase occupies the inner region in the pipe, $0 \le r < r_D$, and the PEG-rich phase occupies the outer region, $r_D < r \le R$.
After integrating Eq. (SI-1), the boundary condition determine that,
at $r = r_D$,

$$\frac{1}{4\mu_D}\frac{dP}{dx}r_D^2 + B_D = \frac{1}{4\mu_p}\frac{dP}{dx}r_D^2 + B_P$$

and, at $r = R$,

$$0 = \frac{1}{4\mu_p}\frac{dP}{dx}R^2 + B_P$$

So that we get the velocity profiles as

$$v_P = \frac{1}{4\mu_p}\frac{dP}{dx}r^2 - \frac{1}{4\mu_p}\frac{dP}{dx}R^2$$

and

$$v_D = \frac{1}{4\mu_D}\frac{dP}{dx}r^2 + \frac{1}{4}\left(\frac{1}{\mu_p} - \frac{1}{\mu_D}\right)\frac{dP}{dx}r_D^2 - \frac{1}{4\mu_p}\frac{dP}{dx}R^2$$

By integration, we get the volume flow rates,

$$Q_P = \int_{r_D}^{R} 2\pi r dr v_P$$

$$Q_P = -\frac{\pi}{8\mu_p}\frac{dP}{dx}\left(r_D^2 - R^2\right)^2 \qquad \text{(SI-4)}$$

$$Q_D = \int_{0}^{r_D} 2\pi r dr v_D$$

$$\therefore Q_D = -\frac{\pi}{8\mu_D}\frac{dP}{dx}r_D^4 - \frac{\pi}{4\mu_D}\frac{dP}{dx}r_D^2\left(R^2 - r_D^2\right) \quad ..(3)$$

Applying $\frac{dP}{dx} = \frac{P_{out} - P_j}{l_D}$ to Eq. (SI-4), we get

$$Q_P = -\frac{\pi}{8\mu_p}\frac{P_{out} - P_j}{l_D}\left(r_D^2 - R^2\right)^2$$

And, therefore,

$$P_j = Q_P\frac{8\mu_p}{\pi}\frac{l_D}{\left(r_D^2 - R^2\right)^2} + P_{out} \qquad \text{(SI-5)}$$

Putting Eq.(SI-5) into Eq.(SI-3), we get Eq.(1):

$$Q_D = \frac{\pi}{8\mu_D}\frac{R_D^4}{l_D}\left(P_D - P_{out}\right) - Q_P\frac{\mu_P}{\mu_D}\left(\frac{l}{l_D}\right)\frac{R_D^4}{\left(r_D^2 - R^2\right)^2} \qquad \text{(SI-6)}$$

We can also start out with a geometry with rectangular channel with a high aspect ratio such as width, $2w$, height, $2h$, and $w \gg h$. Following similar steps in the previous derivation, we can obtain:

$$Q_D = \frac{4}{3}\left(P_D - P_{out}\right)/\mu_D\left(\frac{l}{wh^3} + \frac{l_D}{w_D h_D^3}\right) - Q_P\frac{\mu_P}{\mu_D}\frac{1}{1 + \frac{l_D}{l}\frac{wh^3}{w_D h_D^3}} \qquad \text{(SI-7)}$$