Electronic Supplementary Material (ESI) for Soft Matter.

Supplementary information

May 7, 2019

Supplementary Video 1

Side view of the motion of two granular rafts at an oil-water interface. Both rafts are formed by the aggregation of respectively 50 and 30 particles ZrO beads (density ρ_s = 3,800 kg.m^{-3}, radius R_{part} = 0.45 mm). The movie is played at real speed.

Supplementary Video 2

Top view of the motion of two granular rafts at an oil-water interface. Both rafts are formed by the aggregation of respectively 50 and 30 particles ZrO beads (density ρ_s = 3,800 kg.m^{-3}, radius R_{part} = 0.45 mm). The movie is played at real speed.

Theoretical description of the raft deformation

Here, we propose a minimal model to bring a theoretical foundation to the experimental result described in the previous section. To account for the vertical force the raft is imposing on the interface, we distribute F_{vert} along the perimeter of the raft, through the introduction of the pressure P at the point $\overrightarrow{r}$

$$P(\overrightarrow{r}) = \frac{F_{vert}}{2\pi R_{raft}} \delta(\|\overrightarrow{r}\| - R_{raft}) \tag{1}$$

where δ is a Dirac function. We then calculate the pressure jump at the oil-water interface, for $\|\overrightarrow{r}\| \geq R_{raft}$, in the limit of small deformations. The classic differential equation that describes the height of the interface h is modified by the presence of P:

$$\left(\nabla^2 - \frac{1}{\ell_c^2}\right) h = \frac{P(\overrightarrow{r})}{\gamma}. \tag{2}$$

The solution of equation (2) is given by the convolution of P and the green function $G(\overrightarrow{r}, \overrightarrow{s}) = -\frac{1}{2\pi} K_0(\frac{\|\overrightarrow{r} - \overrightarrow{s}\|}{\ell_c})$, associated with the linear operator $(\nabla^2 - 1/\ell_c^2)$. K_0 is the modified Bessel function of the second kind of order zero.

$$h(\overrightarrow{r}) = \int_{\Re^2} G(\overrightarrow{r}, \overrightarrow{s}) \frac{P(\overrightarrow{s})}{\gamma} s ds d\theta \tag{3}$$

$$h(\overrightarrow{r}) = -\frac{F_{vert}}{4\pi^2\gamma} \int_0^{2\pi} K_0 \left(\frac{\sqrt{\|\overrightarrow{r}\|^2 + R_{raft}^2 - 2R_{raft}\|\overrightarrow{r}\| \cos(\theta)}}{\ell_c} \right) d\theta. \tag{4}$$

Because we are only interested in the depth of the interface at the edge of the raft, we apply the previous equation for $\|\overrightarrow{r}\| = R_{raft}$:

$$h_{raft} = \frac{F_{vert}}{\pi^2\gamma} \int_0^{\pi/2} K_0 \left(\frac{2R_{raft}}{\ell_c} \sin(\theta) \right) d\theta. \tag{5}$$

In the limit $R_{raft} \ll \ell_c$, we can go further with the calculation, using $K_0(x) \underset{x \to 0}{\sim} -\ln(x)$:

$$h_{raft} \sim -\frac{F_{vert}}{\pi^2\gamma} \int_0^{\pi/2} \ln \left(\frac{2R_{raft}}{\ell_c} \sin(\theta) \right) d\theta \tag{6}$$

$$h_{raft} \sim -\frac{F_{vert}}{2\pi\gamma} \ln \left(\frac{R_{raft}}{\ell_c} \right). \tag{7}$$

Our minimal model thus gives the following result (in the limit of small deformations and small granular rafts):

$$-\frac{h_{raft}/\ell_c}{\ln(R_{raft}/\ell_c)} \sim \frac{F_{vert}}{2\pi\gamma\ell_c} \propto n, \tag{8}$$

which corresponds to the previous experimental and numerical results observed in the same limits. All these results validate that:

$$F_{cap\ B \to\ A} = n_A n_B F_{cap\ 1 \to\ 1}. \tag{9}$$