Electronic Supplementary Material (ESI) for Soft Matter.

Supplementary Information for
The effect of size-scale on the kinematics of elastic energy release

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BENDING AND RECOIL OF A CANTILEVERED BEAM

Here we consider a long, thin beam with length, L, width, w and thickness, h, which is fixed at one end and undergoes a deflection, x, at its free end in response to an applied force, F. For the case of $L >> w >> h$, with the beam being comprised of a uniform, linear elastic material with modulus, E, and density, ρ, the force is related to the displacement by [1]

$$F = \frac{Ewh^3}{4L^3}x, \tag{1}$$

in the limit of small displacements. For the case where the largest stress component occurs along the longitudinal axis of the beam, the maximum stress in the beam, $\sigma_{\max}$, is given by

$$\sigma_{\max} = \frac{6FL}{wh^2}. \tag{2}$$

Calculating the energy per unit volume for the cantilevered beam,

$$\frac{U}{Lwh} = \frac{Fx}{2Lwh} = \frac{\sigma_{\max}^2}{18E}, \tag{3}$$

gives an energy per unit volume that is 9 times smaller than that for uniaxial extension ($\sigma^2_{\max}/2E$), the geometry used in the main text.

For the recoil of a long cantilevered beam driving a heavy mass, m, such that $m >> \rho Lwh$, from an initial maximum strain s_{in}, the maximum velocity of the mass is given by [2]

$$v_{\max} = \sqrt{\frac{2U}{m + \rho Lwh/3}} = \frac{(c s_{\text{in}})}{3\sqrt{\frac{m}{\rho Lwh} + \frac{1}{3}}}, \tag{4}$$

where we have used $c = \sqrt{E/\rho}$ and $\sigma_{\max} = E s_{\text{in}}$. Since the recoil dynamics of the mass are described by a quarter period of a harmonic motion, the duration of recoil Δt is given by

$$\Delta t = \frac{\pi}{2}\sqrt{\frac{m + \rho Lwh/3}{k}} = \pi \left(\frac{L}{c}\right) \frac{L}{h} \sqrt{\frac{m}{\rho Lwh} + \frac{1}{3}}, \tag{5}$$

where the beam stiffness $k = \frac{Ewh^3}{4L^3}$. Finally, the maximum acceleration is given by

$$a_{\max} = \frac{\pi v_{\max}}{2\Delta t} = \frac{1}{6}\left(\frac{c^2 s_{\text{in}}}{L}\right) \frac{h}{L} \frac{1}{\frac{m}{\rho Lwh} + \frac{1}{3}}. \tag{6}$$

Since the dimensionless ratios of h/L and $m/\rho Lwh$ are constant under an isometric change of size, the overall size-scalings for the recoil of a cantilever driving a heavy mass (the terms in round brackets in Eqs. (4)-(6)) are the same as uniaxial recoil described in the main text.

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[1] S. P. Timoshenko and J. N. Goodier, *Theory of Elasticity*, Vol. 3 (McGraw-Hill, New York London, 1970).

[2] M. Ilton, M. S. Bhamla, X. Ma, S. M. Cox, L. L. Fitchett, Y. Kim, J.-S. Koh, D. Krishnamurthy, C.-Y. Kuo, F. Z. Temel, A. J. Crosby, M. Prakash, G. P. Sutton, R. J. Wood, E. Azizi, S. Bergbreiter, and S. N. Patek, Science **360**, eaa01082 (2018).