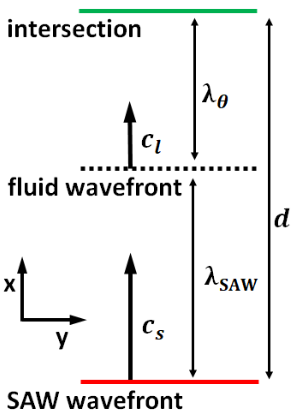
Note S1: Equation 1

Equation 1 from the manuscript is the answer to a simple question: if a fast moving wavefront is catching up with a slower moving one, how long will it take them to intersect? This intersection is the point at which these wavefronts will constructively interfere. Because the Huygens-Fresnel wavelet travels at a velocity of c_l (~1500m/s, water), which is less than that of the SAW-coupled fluid wavefront travelling at c_s (~4000m/s, lithium niobate), this intersection will occur when the SAW-coupled fluid wavefront overtakes the Huygens-Fresnel wavelet. We term this distance λ_θ, or the distance between the effective source of a fluid wavelet (a channel/fluid interface, for example) and the point at which a SAW-coupled fluid wavefront interferes with it.

S1



Looking at this in a simplified case where both wavefronts are travelling in the same direction, we can solve for λ_θ with the knowledge that they intercept at time t from the initiation of the fluid wavelet.

$$t = \frac{\lambda_\theta}{c_l} = \frac{d}{c_s} \qquad \text{Eqn. S1}$$

Since $d = \lambda_\theta + \lambda_{\text{SAW}}$ when both waves are travelling in the same direction

$$\frac{\lambda_\theta}{c_l} = \frac{\lambda_\theta + \lambda_{\text{SAW}}}{c_s} = \frac{\lambda_\theta}{c_s} + \frac{\lambda_{\text{SAW}}}{c_s}, \qquad \text{Eqn. S2}$$

and grouping all λ_θ terms

$$\lambda_\theta \left(\frac{1}{c_l} - \frac{1}{c_s}\right) = \frac{\lambda_{\text{SAW}}}{c_s}, \qquad \text{Eqn. S3}$$

then solving for λ_θ

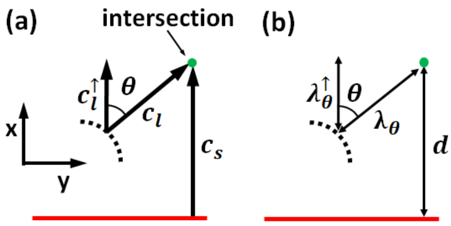
$$\lambda_\theta = \frac{\frac{1}{c_s}\lambda_{\text{SAW}}}{\frac{1}{c_l} - \frac{1}{c_s}} = \frac{\frac{c_l}{c_s}\lambda_{\text{SAW}}}{1 - \frac{c_l}{c_s}}, \qquad \text{Eqn. S4}$$

we have an expression for λ_θ in terms of the known quantities c_s, c_l and λ_{SAW}. Since the fluid wavelength is given by $\lambda_{\text{l}} = \frac{c_l}{c_s}\lambda_{\text{SAW}}$, this expression becomes

$$\lambda_\theta = \frac{\lambda_{\text{l}}}{1 - \frac{c_l}{c_s}}, \qquad \text{Eqn. S5}$$

thus recovering the result from Devendran et al (2017)[1], where a channel wall was placed in the path of a SAW with wavefronts parallel to said wall. In this case, the θ in λ_θ is 0° because the SAW wavefront and Huygens-Fresnel wavelet are propagating in the same direction. The present work, however, seeks to generalize this model for any orientation of the SAW wavefronts with respect to the source of the Huygens-Fresnel wavelets (such as a channel wall). At the limit where the radius of curvature approaches zero, as in Rayleigh scattering, the wavelets take the form of expanding circular wavefronts. Calculating the distance between the wavelet source and its intersection with a SAW-coupled wavefront for a given value of θ must then take into account that the velocity component of the fluid wavefronts in the +x direction ($c_l^{\uparrow}$), which will be decrease with increasing θ.

S2



The above diagram shows this scenario expressed in terms of either (a) velocity or (b) distance. For a time period equal to $t = \frac{d}{c_s} = \frac{\lambda_\theta}{c_l} = \frac{\lambda_\theta^{\uparrow}}{c_l^{\uparrow}}$, , the length of the (a) velocity vectors and (b) distances are equal. The value of $c_l^{\uparrow}$ is given by

$$c_l^{\uparrow} = \cos(\theta)\, c_l. \qquad \text{Eqn. S6}$$

Substituting this value into Equation S4, we arrive at an expression for the vertical (+x direction) component of λ_θ

$$\lambda_\theta^{\uparrow} = \frac{\frac{c_l}{c_s}\cos(\theta)\,\lambda_{\text{SAW}}}{1 - \frac{c_l}{c_s}\cos(\theta)} = \frac{\cos(\theta)\,\lambda_{\text{l}}}{1 - \frac{c_l}{c_s}\cos(\theta)}, \qquad \text{Eqn. S7}$$

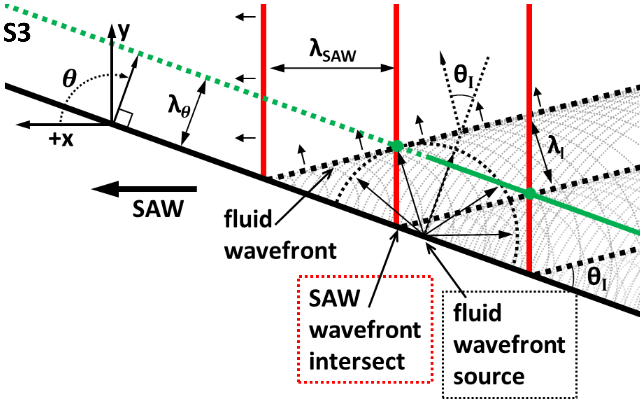
and noting that

$$\lambda_\theta = \frac{\lambda_\theta^{\uparrow}}{\cos(\theta)}, \qquad \text{Eqn. S8}$$

we arrive at Equation 1 from the text, with

$$\lambda_\theta = \frac{\lambda_l}{1 - \frac{c_l}{c_s}\cos(\theta)}. \qquad \text{Eqn. S9}$$

This expression is valid for the case where the second SAW wavefront intersects with the first fluid wavelet at the same time the third SAW wavefront arrives at the origin of the first fluid wavelet. This expression is valid when the effective radius of curvature for a channel wall approaches zero ($R < \lambda$), as in the case of a pillar or post smaller than the acoustic wavelength. In the case of a flat channel wall, however, we see that this is not the case in examining Figure 2b from the text in detail below: the intersection of the SAW wavefront along the channel wall is displaced from the source of the spherical wavefronts that ultimately intersected with that wavefront.



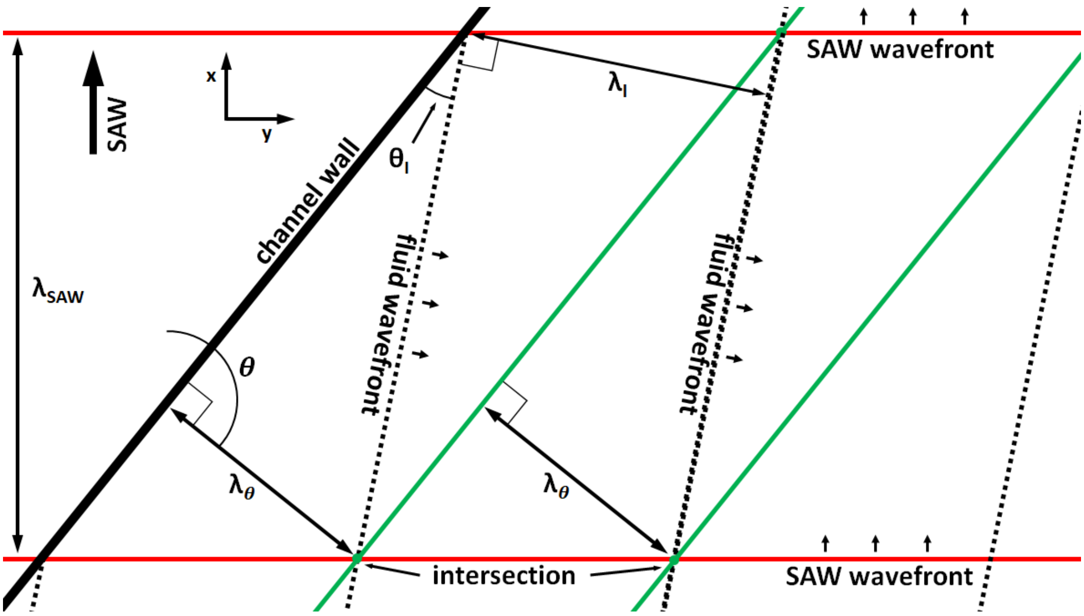
Because of this displacement, the time to fluid and SAW wavefront intersection (as described previously in Figure S1 and Equation S1) will change, and requires the consideration of a separate model to determine the value of λ_θ for flat channel walls.

<u>Note S2: Equation 2</u>

A travelling SAW produces a fluid wavefront that propagates at c_s when viewed in the plane of the transducer, whilst the intersection of this wavefront with a channel feature (in this case a flat wall) generates Huygens-Fresnel wavelets which give rise to a wavefront that that propagates at an angle θ_I to the normal vector of the wall. The interference of these wavefronts produces a combined field with intersection spacing λ_θ, as illustrated in S4.

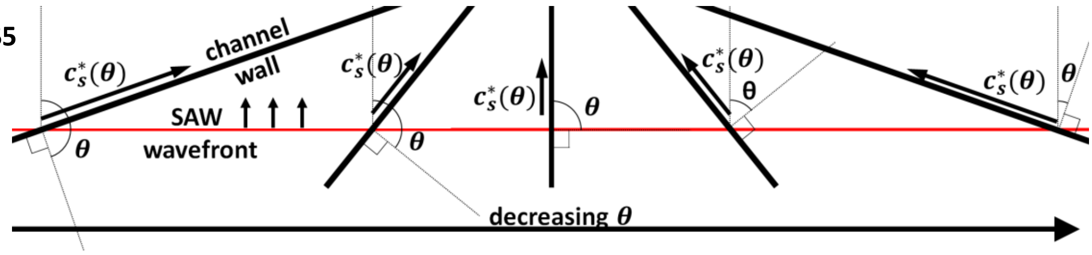
Important parameters here include the distance between SAW wavefronts (λ_{SAW}), the wavelength in the fluid (λ_l) and the angle of the channel wall relative to the SAW propagation direction θ.

S4



The value of θ_I is a function of the angle at which the SAW wavefronts intersect the channel wall; the velocity at which the wavefront travels along the axis of the wall is minimized (and equal to c_s) when $\theta = \pi/2$ and approaches infinity for θ values of 0 and π, and is given by $c_s^*(\theta) = \frac{c_s}{\sin(\theta)}$. This change in effective $c_s^*(\theta)$ as a function of θ is illustrated in S5, where a SAW wavefront has a higher velocity along the channel wall for more oblique angles.

S5

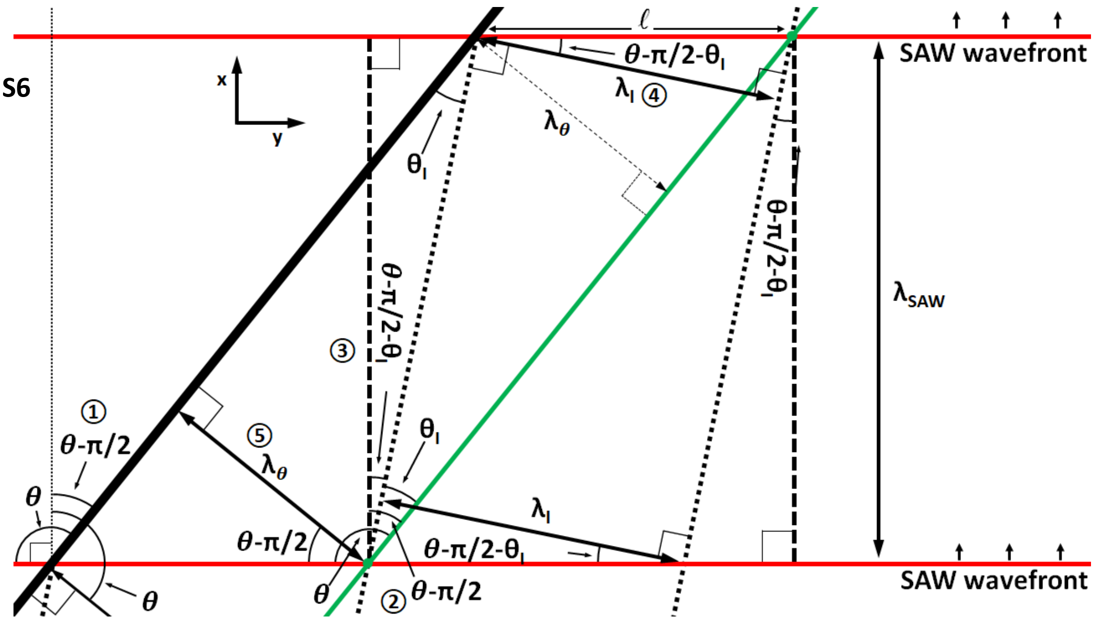


Noting that this effective c_s is a function of θ, the intersection angle as a function of the channel wall angle is given by

$$\theta_\mathrm{I}(\theta) = \sin^{-1}\left(\frac{c_l}{c_s}\sin\theta\right). \qquad \text{Eqn. S10}$$

Looking at the diagram in Figure S4, our challenge is to determine the value of λ_θ from the geometries in this system. To do so we find the value for one of the lengths of the triangle bounded by the SAW wavefront, intersection line and the line marked λ_θ above. This line is marked ℓ in Figure S6. The paragraph following details the geometric considerations involved.

S6



To find ℓ, we populate our diagram with angles defined in terms of the known quantities θ and θ_I. ① First we note that the angle between the channel wall and the x-axis is equal to $\theta - \pi/2$. ② Translating this known quantity to the right, we can use this angle and θ_I to ③ find the angle between the x-axis and the dotted line representing the fluid wavefront, given by $\theta - \pi/2 - \theta_\mathrm{I}$. Noting that the combination of the lines denoting λ_l and the fluid wavefronts constitute a rotated rectangle within a rectangle comprised by the dashed lines and (red) SAW wavefronts, ④ the angle shown adjoining the line ℓ is also given by $\theta - \pi/2 - \theta_\mathrm{I}$. Since the fluid wavelength is a known quantity, the value of ℓ is simply given by

$$\ell = \frac{\lambda_l}{cos(\theta - \pi/2 - \theta_\mathrm{I})}. \qquad \text{Eqn. S11}$$

⑤ We can then determine λ_θ using

$$\lambda_\theta = \ell \cos(\theta - \pi/2). \qquad \text{Eqn. S12}$$

From here, the expression for λ_θ in terms of know quantities can be determined, with

$$\lambda_\theta = \frac{\lambda_l}{\cos(\theta - \pi/2 - \theta_\mathrm{I})} \cos(\theta - \pi/2). \qquad \text{Eqn. S13}$$

Given $\cos(\theta - \pi/2) = \sin(\theta)$, this is equivalent to

$$\lambda_\theta = \lambda_l \sin(\theta) \csc(\theta - \theta_\mathrm{I}). \qquad \text{Eqn. S14}$$

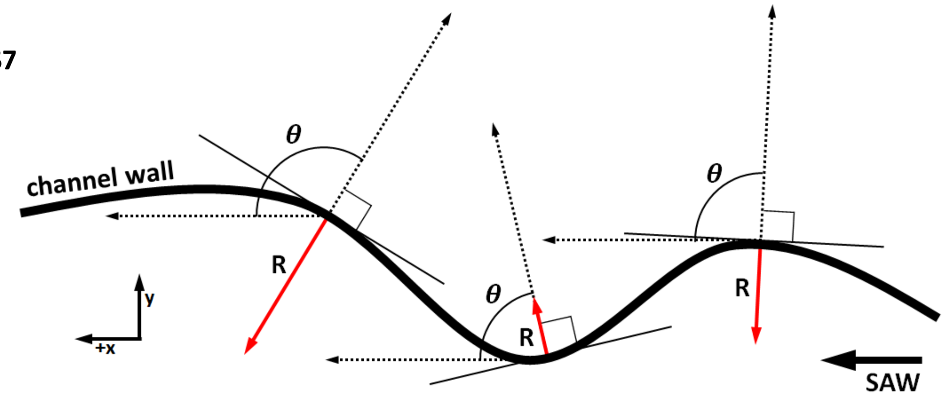
Finally, substituting Eq. S10 for θ_I we arrive at the expression for acoustic force periodicity in terms of θ and the fluid and substrate properties, with

$$\lambda_\theta = \lambda_l \sin(\theta) \csc\left(\theta - \sin^{-1}\left(\frac{c_l}{c_s} sin\,\theta\right)\right). \quad \text{Eqn. S15}$$

Note S3: Theta Definition For Arbitrary channel wall orientations

The periodicity for an arbitrary radius of curvature (between the $R \to 0$ and $R \to \infty$ cases represented by Equations 1 and 2) is discussed in the text. Figure 7 shows the transition between these two cases for finite R values as a function of the ratio of sound speeds in the fluid and substrate. For channel walls with such a curvature, the value of θ is still defined as the angle between the direction of acoustic propagation and a line extending orthogonally from the channel wall.

S7



References

[1] C. Devendran, D.J. Collins, Y. Ai, A. Neild, Huygens-Fresnel acoustic interference and the development of robust time-averaged patterns from traveling surface acoustic waves, Phys. Rev. Lett. 118 (2017) 154501.