# **Active Matter Invasion - ESI**

#### A. Role of reservoir and growth

In this section we examine the importance of the cell division and the presence of a reservoir on the dynamics reported in the main text. ESI Figure 1 shows simulation results for two systems that differ from the one considered in the main text, by the absence of a wider reservoir, and by the absence of cell division, respectively. We see that growth is an essential factor to create the phenomena reported in the main text as in the absence of growth the behaviour is qualitatively different. If the reservoir is absent, meaning that there is only a capillary of uniform width where growth takes place in the lower region, we in contrast observe the first crossover at the same values for the activity A (ESI Fig. 1A), together with the same change in the invasion speed (ESI Fig. 1B). The second crossover is however significantly shifted to higher values of A (see ESI Fig. 1C,D), indicating that the reservoir has long-range effects on the dynamics in the capillary. These, however, do not change the qualitative picture.

#### B. Universality for different channel widths

As described in the main text, the activity number A characterises the ratio of the channel width d to the intrinsic length scale of the active phase. In ESI Figure 2 the observables  $(h_{\text{max}} - h_{\text{min}})/d$  and  $N_c$  are plotted against A for varying channel width d. Both crossovers happen at the same values for A, independent of d. In addition, both observables are of the same order of magnitude and within regime II  $(h_{\text{max}} - h_{\text{min}})/d \sim 1$ . These results indicate that the activity number is the relevant dimensionless parameter in this setup and that  $(h_{\text{max}} - h_{\text{min}})/d$  is a meaningful observable.

## C. Description of Movies

All movies show the time-evolution of the velocity field on the left hand side and the orientation field on the right hand side, in the same way as panels B-D in Figure 3. We choose different values of the activity  $\zeta$  to display the qualitatively different phenomena. There are the following files:

- flows0020.mp4  $\zeta = 0.0020$ . System is in regime I, director homogeneous and parallel to the interface. Corresponds to Figure 2A, leftmost panel.
- flows0030.mp4  $\zeta = 0.0030$ . System is in regime I, but the director starts to show deviations from the perfectly homogeneous configuration, especially close to the reservoir. Corresponds to Figure 3B.
- flows0035.mp4  $\zeta = 0.0035$ . System is in regime 2, showing characteristic flow- and director-fields. At later times the periodic dynamics described in section 3.2.1 can be observed. Corresponds to Figures 2A (middle panel) and 3C.

- **flows0046.mp4**  $\zeta = 0.0046$ . System is the higher activity region of regime II. The video shows two examples of the switching behaviour described in section 3.2.2 and Figure 4B.
- flows0052.mp4  $\zeta = 0.0052$ . System is in regime III. This example shows the typical birth of small active clusters, corresponding to the situations shown in Figures 3D and 4C.
- flows0060.mp4  $\zeta = 0.0060$ . System is deep in regime III. The dynamics is more turbulent than in the video for  $\zeta = 0.0052$ ; there are more defects and clusters. Corresponds to the example shown in Figure 2A on the right panel.



### D. Supplementary figures

ESI Fig. 1: Comparison of different modifications of the setup. (A)  $(h_{\max} - h_{\min})/d$  as a function of A. The crossover from regime I to regime II takes place at the same values as in the unmodified setup for a system without reservoir but with growth (blue circles), but is absent for a system lacking growth (red crosses). Vertical lines mark the crossover points in the unmodified setup. (B) Total amount of active material in the system at the end of the simulation relative to the starting value. In the non-growing system, the amount of active material is trivially constant over time. The system without reservoir shows the same qualitative dynamics as the unmodified setup. Data in this figure has been taken from one sample per value for A only. (C) The number of clusters  $N_c$  as a function of A. The crossover from regime II to regime III takes place at higher values as in the unmodified setup for a system with reservoir but without growth, which also explains the longer linear range in subfigure B. Clusters are also present for a system lacking growth, but it takes longer for them to appear. (D) The number of +1/2 defects  $N_+$  as a function of A. The appearance of topological defects, for both setups, coincides with the appearance of additional clusters (compare subfigure C).



ESI Fig. 2: Universality for different channel widths.  $(h_{\text{max}} - h_{\text{min}})/d$  (panel A) and  $N_c$  (panel B) against A. Both crossovers happen at the same values for both channel widths.



ESI Fig. 3: Transition between regimes II and III. Comparison of number of clusters  $N_c$  (A) and number of +1/2 topological defects  $N_+$  in the system (B). Appearance of clusters and defects coincide around the second crossover.