DISCRETE ELEMENT SIMULATIONS

Simulations employ the discrete element method (DEM) based on a soft-sphere approach. The basic principle of DEM simulations is to treat each particle as a sphere submitted to the gravity and contact forces with both other particles and boundaries. These contact forces are described by the well-known linear spring-dashpot force model in both normal (spring stiffness k, damping γ_n) and tangential directions (spring stiffness k_t, damping γ_t). In the tangential direction, the spring elongation is truncated to satisfy Coulomb law. The values of γ_n and γ_t are adjusted to obtain the desired value of the normal and tangential restitution coefficient, e and e_t. In all simulations, $k_t = 2k/7$ and $\gamma_t = 0$, so that the coefficient of tangential collisional restitution is unity. See Table 1 for a summary of the properties of the particles in the simulations employed in the present work. The acronyms in the Table refer to: Volume- or Pressure-imposed Simple Shearing (VSS and PSS, respectively); Pressure-imposed Shear Flows (PSF); Chute Flows (CF); and Inclined, Free surface Flows between frictional sidewalls (IFF).

In the IFF case, the simulated chute is a rectangular cuboid which can be inclined relatively to the horizontal by an angle θ. Its size in the flow direction (x-axis) is set to 25.3 diameters and periodic boundary conditions are employed along x. In the z-direction (*i.e.* perpendicular to the free surface), the size of the cell is considered as infinite. Frictional sidewalls are located at positions $y = -W/2$ and $y = W/2$, being W, therefore, the channel width. They are treated as sphere of infinite radius and mass.

Initially, we set the angle of inclination to a large value, $\theta \approx 70°$, and pour grains in the system from an ordered, slightly dilute, hexagonal compact packing. Each components of the grains' velocity is initially randomly and uniformly assigned between $-\sqrt{gd}$ and $+\sqrt{gd}$. The angle was then decreased instantaneously to the desired value and the system slowly relaxes to a steady and fully developed state that has no sign of the initial ordered structure. Note that this protocol leads to relatively loose erodible beds and other protocols (*e.g.* initially building a dense static packing and triggering the flow by suddenly increasing the inclination) lead to much denser erodible base and thus to potentially different results. All measurements presented in the text for the IFF case have been averaged along the y-direction, perpendicular to the sidewalls. The contacts between grains and sidewalls are note taken into account to calculate the coordination number.

Table I. Particle properties in the discrete element simulations employed in the present work. Here, $t_c = (\rho_p d^3/k)^{1/2}$ is the contact duration for a particle-particle interaction in the linear contact model, while $t_{ff} = (d/g)^{1/2}$ is the time associated with free fall in a gravitational field. In the VSS and PSS configurations, $t_c/t_{ff} = 0$ because gravity is absent. For comparison, for 1 mm silica grains subjected to Earth's gravitational field, $t_c/t_{ff} \approx 10^{-5}$.

Configuration	Source	e (-)	μ (-)	t_c/t_{ff} (-)
VSS	[1]	0.7	0.5	0
VSS	Present work	0.88	0.5	0
PSS, PSF, CF	[2]	0.1	0.4	0, 0.0001, 0.0017
IFF	Present work	0.88	0.5	0.0006

[1] Chialvo S, Sun J, Sundaresan S. Bridging the rheology of granular flows in three regimes. Physical Review E. 2012;85(2):021305. Available from: http://link.aps.org/doi/10.1103/PhysRevE.85.021305.

[2] Zhang Q, Kamrin K. Microscopic Description of the Granular Fluidity Field in Nonlocal Flow Modeling. Physical Review Letters. 2017;118:058001.