Supplementary text for "Beating to rotational transition of a clamped active ribbon-like filament"

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FIG. SI-1. a) Binormal persistence length for various $\rho = \kappa_t / \kappa_b$ ratio. b) Ratio of amplitudes in the direction of beating plane and perpendicular to the beating plane.

The binormal persistence length (L_b) of the ribbon for various torsional strength ρ is displayed in the Fig. SI-1. It is estimated from the binormal-binormal correlation given as, $C_b(s) = \langle \mathbf{b}_{s+s_0} \cdot \mathbf{b}_{s_0} \rangle \sim \exp(-s/L_b)$. Interestingly, the persistence length varies linearly with ρ . The increase in the persistence length suggests the strengthening of torsional rigidity of the ribbon with ρ .

The ratio of mean of amplitudes of the periodic motion on the x-z plane defined as ϵ_z/ϵ_x (the direction along beating plane, and perpendicular to the beating plane) as shown in Fig. SI-1-b. If this ratio is small, it suggests amplitude in x-direction is much more than that in z-direction, and the trajectory is linear along x-axis. If this ratio is close to one, it suggests that amplitudes in x-direction and in z-direction are same, and the trajectory can be circular. For lower values of ρ , i.e. $\rho = 0, 0.1$, amplitudes is close to one, therefore shape of trajectory is nearly circular. Further, intermediate value of $\rho = 0.25$ gives elliptical trajectory. For $\rho = 0.25, \epsilon_z/\epsilon_x$ grows from a small value to one, and eventually reaches to the transi-



FIG. SI-2. a) Time-series of the azimuthal angle and variation of x component of the terminus end for Pe = 10 and $\rho = 0$. b) Frequency of oscillation ω obtained from the bending (\blacksquare) and torsion (•) energies as a function of Pe.

tion towards the butterfly to elliptical trajectory. For the higher torsion ratio, i.e., $\rho > 0.5$, ϵ_z/ϵ_x is very small suggesting motion is on the x-y plane only, this suggests end-monomer moves nearly along the x-axis. With Pe, ϵ_z/ϵ_x shifts progressively towards the higher values. The trajectory of the end monomer may become circular again in the large compressive force limit. This also confirms that the transition between beating to rotation is not sudden, it changes continuously with increasing compressive force. As Fig. SI-1-b suggests, transition from beating to rotation follows the path of butterfly to circular via elliptical trajectory.

We present here the discussion and the quantification of oscillation frequency through deflection of the ribbon in x-direction. The deflection is caused by bending of the filament, therefore measured frequency in this way quantifies only bending oscillations. Further, we analyse the oscillation frequency via the azimuthal angle of the ribbon's terminus end. This quantity can provide out-of plane oscillations at small ρ . In this limit, i.e., at $\rho = 0$ the oscillation of azimuthal angle coincides with the oscillation in x-deflection as illustrated in Fig. SI-2-a. When $\rho \simeq 0$, we observe local oscillations in the torsional parameter χ_i^t as given in Fig. 6-c of the main text. The frequency of oscillations in bending energy is computed by the Fourier transformation of the time evolution of $\chi^b(t)$ for different values of Pe. Similarly, we also obtain frequency from oscillation in torsional energy

 $\chi^t(t)$. Frequencies from torsional and azimuthal's angle coincides with the bending frequency Fig. SI-2-b.

We provide here two movie files showing the dynamics of the clamped active filament.

Movie SI-MOVIE-1: The movie shows the rotational motion of the filament at Pe = 4 for $\rho = 0$.

Movie SI-MOVIE-2: The movie shows the beating motion of the filament at Pe = 4 for $\rho = 1$.