Electronic Supplementary Information (ESI)

Lateral migration of viscoelastic droplets in a viscoelastic confined flow: role of discrete phase viscoelasticity

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S.1 Properties of the fluids

Table S1: Measured properties of the fluids use in our experiments.

Liquids	Density (gm/cm^3)	Viscosity (mPa-s)	Viscosity ratio w.r.t PDMS (k)	IFT w.r.t PDMS (mN/m)	IFT w.r.t castor oil (mN/m)	Relaxation time(s)	Elasticity ratio w.r.t PDMS (ξ)
PVP 3%	1.0047	12.725	52.32	17.83-20.02	17.20-18.48	0.5[Go et al.[1]]	0.002
PVP 5%	1.0087	35.966	18.52			2.2×10^{-3} [Liu et al. [2]], 1.6×10^{-3} [Naillon et al.[3]]	0.45
PVP 6%	1.0107	57.154	12.0	16.03-17.31	20	$\sim 9.4\times10^{-4}$ [Yang et al.[4]]	1.06
PVP 9%	1.017	177.965	3.74			$\sim 3.0\times10^{-3}$ [Romeo et al.[5]]	0.33
PVP 10%	1.0187	257.695	2.58	20.3	17.80	6×10^{-3} [Naillon et al.[3]]	0.16
PVP 13%	1.0239	654.67	1.01	16.05-19.71			
PEG 15%	1.0293	57.655	12	21-25	16-20	~ 0	
PDMS	1.0051	666				$\sim 10^{-3}$	
Castor oil	0.960	650				~ 0	

S.2 Rheometry data

Fig S1 summarizes rheometry results. Viscosities of different concentrations of PVP remain constant w.r.t strain rate (Fig S1a). The same holds true for PDMS base and PDMS 1.5:1 mixture (Fig S1b). Oscillatory shear test of PDMS 1.5:1 reveals very high value of storage modulus, which indicates strong elastic property of cross-linked PDMS (Fig S1c).

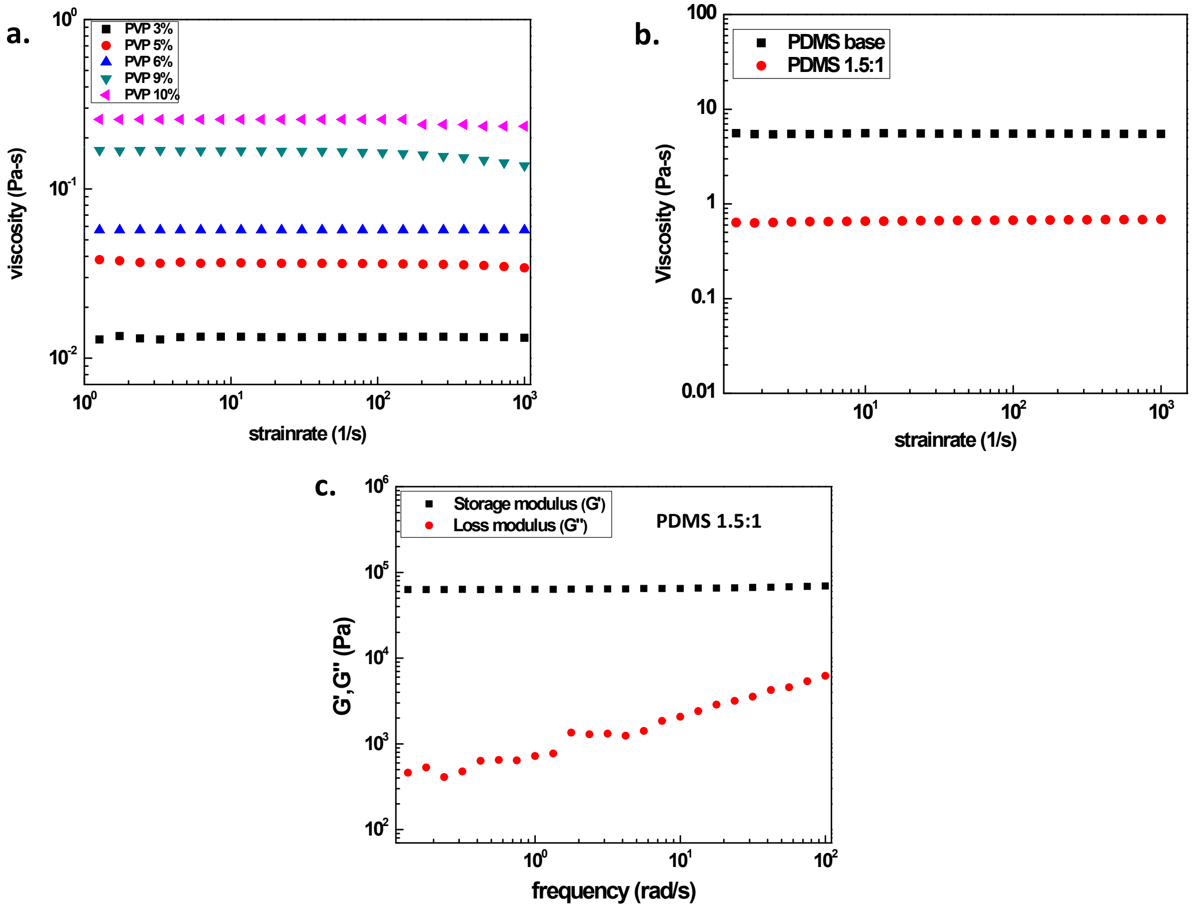

Fig. S1: (a) viscosity vs. strain rate plot of PVP 3%, 5%, 6%, 9% and 10%, (b) viscosity vs strain rate plot of PDMS base and 1.5:1 PDMS, (c) variation of storage modulus ($G^{'}$) and loss modulus ($G^{''}$) of PDMS 1.5:1 with frequency.

S.3 Empirical modeling and analytical scaling of F_{vd}

F_{VD} originates from discrete phase viscoelasticity. Hence, it must be a strong function of relaxation time of the droplet phase, λ_D. From experiment it is evident that F_{VD} is a function of viscosity ratio ($k = {}^{\mu_D}/_{\mu_M}$) as PDMS droplets are reversing the direction of migration for a range of k. Moreover, F_{VD} is a function of λ_M too as experiments revealed that reversal won't be observed in Newtonian continuous phase for the same viscosity ratio. F_{VD} has a complicated dependency on droplet diameter (D) which in turn is a function of continuous phase flowrate (strain rate, $\dot{\gamma}$). Fig S2 elaborates the effect of droplet size on lateral position in PVP 6%. For example, as we decreased the PVP 6 w/w % flowrate keeping PDMS flowrate constant, increasingly bigger PDMS droplets could be generated and their lateral equilibrium position shifted more towards the wall (Fig S2). Hence, for $\dot{\gamma}_1 > \dot{\gamma}_2 > \dot{\gamma}_3$ we get $r_1 < r_2 < r_3$ and $\Delta_1 > \Delta_2 > \Delta_3$, where, Δ_i is the film thickness separating PDMS droplet surface and the wall for corresponding droplet size r_i generated at $\dot{\gamma}_i$. From the above discussion F_{VD} takes the following functional form:

$$F_{VD} = f(\mu_D, \mu_M, \lambda_D, \lambda_M, D, \dot{\gamma}, h) \quad \text{(S1)}$$

Instead of Δ_i we choose wall to droplet center distance, h to follow the convention in literature.

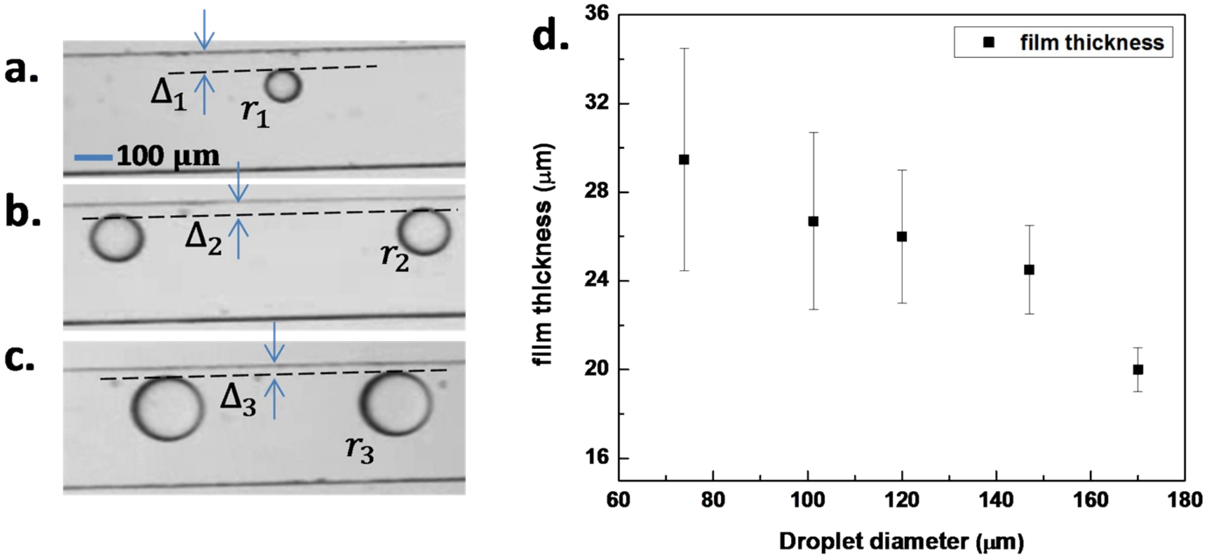

Fig. S2 (a-c) Decrease in film thickness with increase in PDMS droplet diameter in PVP 6%. (d) plot of film thickness vs. PDMS droplet diameter.

Using Buckingham's Pi theorem, we get five non dimensional numbers and Eq. (S1) can be expressed as

$$f\{(\mu_D/\mu_M), (D/h), (\lambda_D\dot{\gamma}), (F_{VD}/\mu_D D^2\dot{\gamma}), (\lambda_D/\lambda_M)\} = 0 \quad \text{(S2)}$$

To determine the interrelation among these five non-dimensional numbers we studied droplet dynamics in 300, 500 and 800 μm channel thereby changing the strain rate for same PVP flowrate. For each channel, flowrate was varied from 4-16 $\mu l/min$ with a step of 4 $\mu l/min$ and for each case same-sized droplets of both PDMS and castor oil were generated. For the same size, we propose that F_{VD} should be the difference of drag between PDMS and castor oil while they migrate laterally at different rates. Drag for flow past a viscous drop is given by [6]

$$F_{Drag} = 3\pi\mu_M D v \left(\frac{1 + {}^{2\mu_M}/_{3\mu_D}}{1 + {}^{\mu_M}/_{\mu_D}} \right) \quad \text{(S3)}$$

The viscoelastic force due to discrete phase viscoelasticity, F_{VD} in Equation S2 can be calculated using Equation S3. Hence,

for PVP 3 and 10 w/w %
$$F_{VD}^{+} = F_{Drag,PDMS} - F_{Drag,castor} = 3\pi\mu_M D(v_{pdms} - v_{castor}) \left(\frac{1 + {}^{2\mu_M}/_{3\mu_D}}{1 + {}^{\mu_M}/_{\mu_D}} \right) > 0 \quad \text{(S4)}$$

and for PVP 6 w/w %
$$F_{VD}^{-} = F_{drag,PDMS} - F_{drag,castor} = 3\pi\mu_M D(0 - v_{castor}) \left(\frac{1 + {}^{2\mu_M}/_{3\mu_D}}{1 + {}^{\mu_M}/_{\mu_D}} \right) < 0 \quad \text{(S5)}$$

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