Supplemental Material: Phase Separation of Active Inertial Particles

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I. FINITE SIZE EFFECTS IN A SYSTEM WITH STATE OSCILLATION

In the main text, the AIP system is found to exhibit interesting state oscillation between the homogenous fluid state and the phase separated steady-state. To show that the state oscillation is not a simulation artifact due to finite system size, we use larger system sizes to perform the same AIP model simulation as the system shown in Fig. 7 of the main text. The packing fraction is 40%, $\chi = 3.1623$ and $F^A\sigma/k_BT = 5000$. All three simulations are run till 300τ without showing any sign of oscillation stopping. Clearly the steady-state oscillation observed in the main text is reproducible in simulations with larger system sizes. One intuitive explanation for the more pronounced oscillatory behavior as system size increases is that in a larger system the same cluster fraction corresponds to a larger cluster, which is harder to form collective motion and more resilient to collisions from the fluid phase particles.

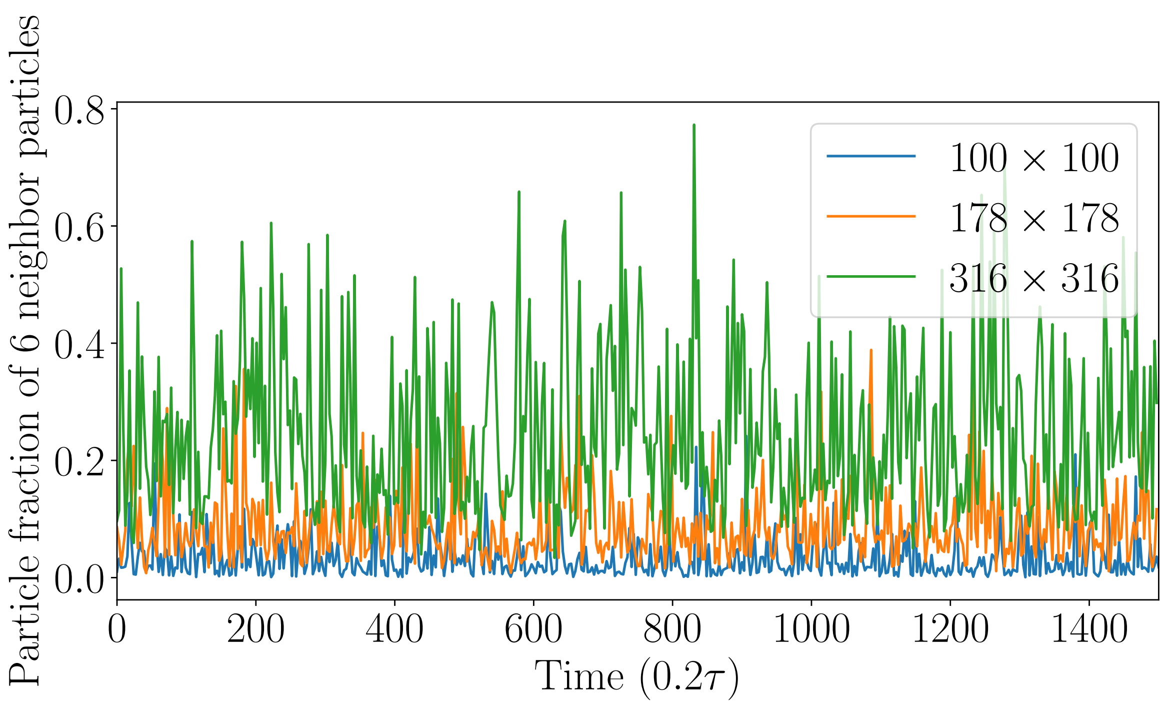

FIG. 1: Time series plot of the fraction of particle with 6 neighbors in contact in three identical simulations except different system size (100×100, 178×178, 316×316). The packing fraction is 40%, $\chi = 3.1623$ and $F^A\sigma/k_BT = 5000$.

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II. SUPPORTING PLOTS OF POWER SPECTRAL DENSITIES FOR THE SYSTEM SHOWN IN FIG.7 IN THE MAIN TEXT

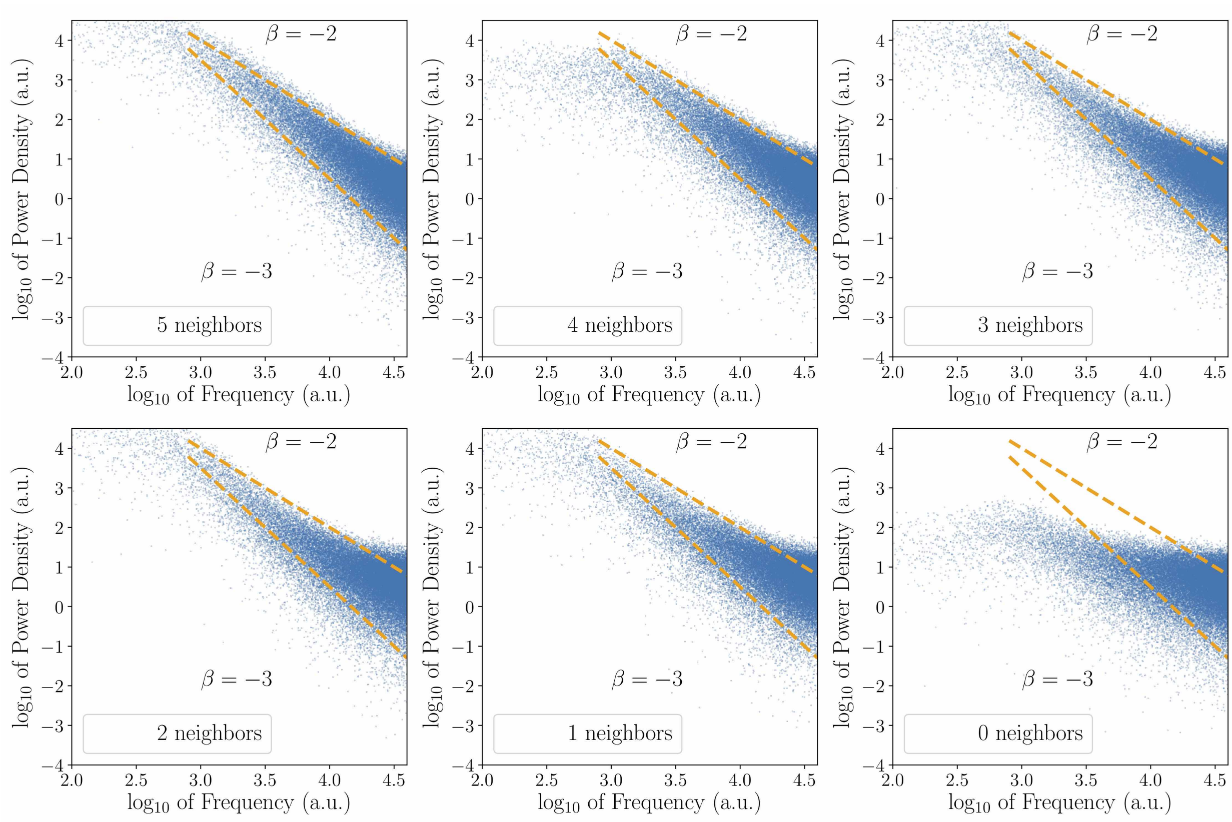


FIG. 2: The power spectral densities (PSDs) for the other time series of particle fractions shown in Fig.6 of the main text. The fraction of particles with 1-5 neighbors all qualitatively follow the power law distribution with exponent between -2 and -3, thus exhibiting fractional Brownian motion (fBM) like dynamics. The only exception is the fraction of particles with 0 neighbor, which are the free roaming particles in the active gas. It has a PSD that resembles more like a white noise (with PSD exponent 0). This might be due to their lack of coupling with other particles as free roaming particles have no direct neighbors to interact with.

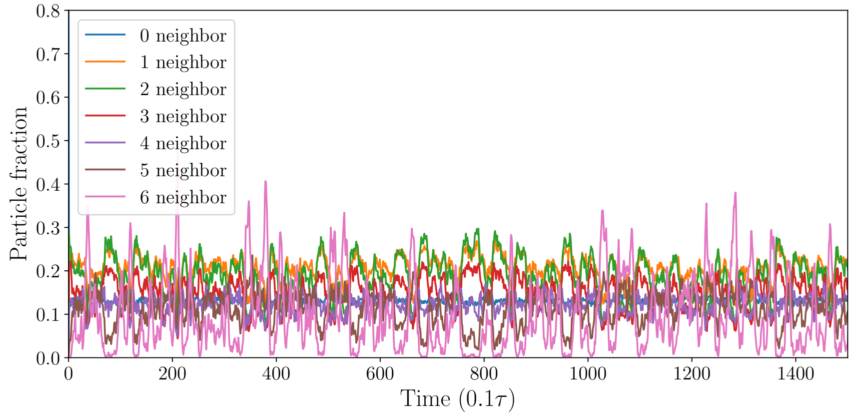
III. THE OSCILLATORY DYNAMICS OF SYSTEMS WITH DIFFERENT INERTIA

In Fig. 5 and Fig. 7 of the main text, a system with packing fraction at 40% , $\chi = 3.162$ and $F^A\sigma/k_BT = 5000$ was shown to exhibit the interesting state oscillation between homogeneous fluid state and phase separation state. Here two additional active inertial particle (AIP) systems that exhibit the same state oscillation behaviors are analyzed. These systems are at with very different parameters from the system in main text Fig. 7, and the simulation are done at lower time resolution due to data storage constraints. The figures below show that the power law distribution of power spectral densities, the exponent and the fBM-like oscillatory dynamics reported in the main text generalize well to other AIP systems within a wide range of particle inertia magnitudes.

IV. ON THE OSCILLATION TIME SERIES'S NON-GAUSSIAN PROPERTIES

To further study the oscillation time series, we focus on the empirical distribution of time increments of $\phi_6(t)$, i.e. the fraction of particles with 6 neighbors in contact. We use the same time series that is plotted in Fig. 7. The time increment is taken by calculating $\phi_6(t + \Delta t) - \phi_6(t)$ for each time frame t, where $\Delta t = 1/2500\tau$ is time period for taking snapshots from the simulation trajectory. To study ϕ_6 increment's distributional properties, we examine its quantile-quantile (Q-Q) plot, which is a probability plot commonly used in statistics to compare two probability distributions by plotting their quantiles (or in our case, the values corresponding to the same quantile) against each other. In Fig. 5, we plot the empirical distribution of ϕ_6 time series increments against a standard Gaussian distribution (zero mean and unit variance). Each data point's coordinate (x, y) can be understood by the following procedure: given an observed increment value y, find its quantile in the empirical distribution and then generate a value x such that in the standard Gaussian distribution x has the same quantile value. If the empirical distribution of the increments did follow a Gaussian distribution, the data would all fall on the red line. The Q-Q plot shows that the increments' distribution has fat tails, further suggesting that the oscillation time series may be generated by a non-Gaussian process.

(a)



(b)

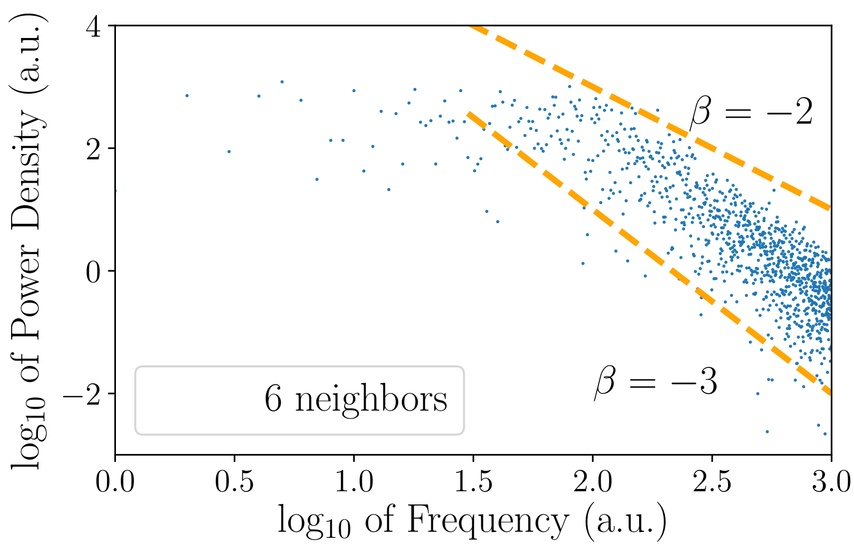
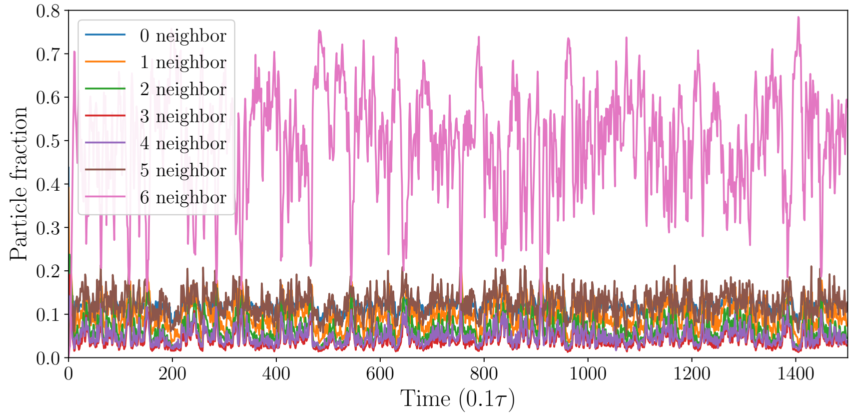


FIG. 3: (a) Time series plot of the particle fractions with different neighbors in contact. This AIP system is in oscillatory phase, with a packing fraction of 49%, $\chi = 10$ and $F^A\sigma/k_BT = 5000$. It shows the complete time evolution of particle fraction with 0-6 neighbors in contact. (b) The power spectral density (PSD) plot for the fraction of particles with 6 neighbors in (a), with both axes in log scale. The high frequency components of the phase oscillation exhibit a power law distribution with exponent between -2 and -3.

(a)



(b)

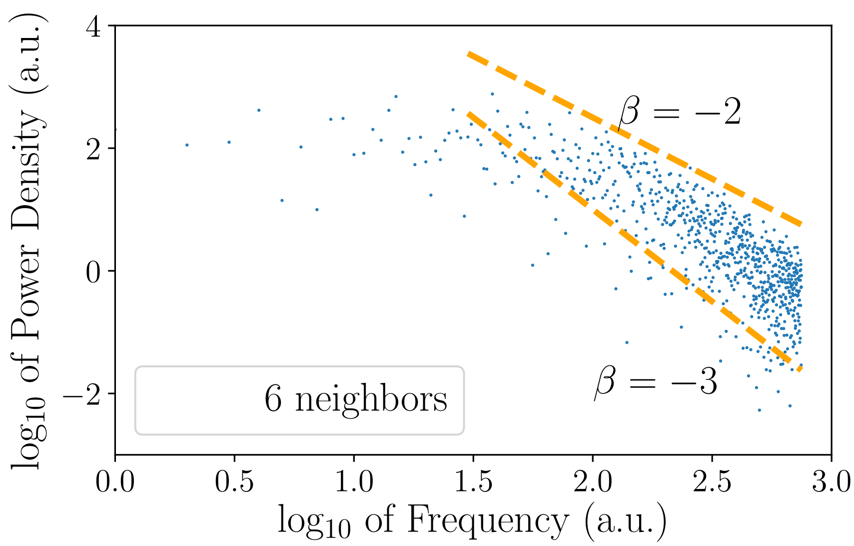


FIG. 4: (a) Time series plot of the particle fractions with different neighbors in contact. This AIP system is also in oscillatory phase, with a packing fraction of 39%, $\chi = 0.1$ and $F^A\sigma/k_BT = 5000$. It shows the complete time evolution of particle fraction with 0-6 neighbors in contact. Note that in this system, particles in the dense phase are much more than particles with fewer neighbors, however the state of the system still oscillates with no signs of stopping. (b) The power spectral density (PSD) plot for the same time series of fraction of particles with 6 neighbors in (a), with both axes in log scale. The high frequency components of the phase oscillation exhibit a power law distribution with exponent between -2 and -3. Although the fraction of particles with 6 neighbors are much higher in this system as compared to both the system shown in Fig. 7(a) of the main text, the PSD of the oscillatory dynamics still follows a similar power law distribution.

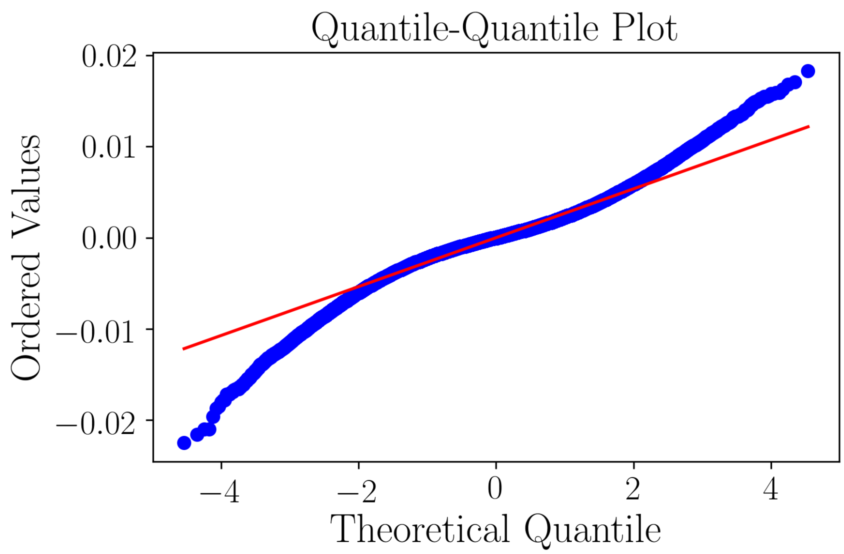


FIG. 5: Quantile-Quantile (Q-Q) plot for comparing the empirical distribution of ϕ_6 time series increments against a standard Gaussian distribution. Each data point's coordinate (x, y) can be understood by the following procedure: given an observed increment value y, find its quantile in the empirical distribution and then generate a value x such that in the standard Gaussian distribution x has the same quantile value. If the empirical distribution of the increments did follow a Gaussian distribution, the data would all fall on the red line. The Q-Q plot shows that the increments distribution has fat tails.