Supplementary information

Micro-mechanical theory of shear yield stress for strongly flocculated colloidal gel

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THEORY

The displacement vector in the rotated coordinate frame is given as,

$$\mathbf{u}' = \frac{D\epsilon_o}{4}\left(n'_2\mathbf{e}'_2 - n'_3\mathbf{e}'_3\right) \tag{1}$$

where $\mathbf{n}' = sin\theta cos\phi \mathbf{e}'_1 + sin\theta sin\phi \mathbf{e}'_2 + cos\theta \mathbf{e}'_3$. Then the expression $n'_2\mathbf{e}'_2 - n'_3\mathbf{e}'_3$ simplifies to,

$$n'_2\mathbf{e}'_2 - n'_3\mathbf{e}'_3 = \mathbf{e}'_r\left(sin^2\theta sin^2\phi - cos^2\theta\right) + \mathbf{e}'_\theta sin\theta cos\theta\left(1 + sin^2\phi\right) + \mathbf{e}'_\phi sin\theta sin\phi cos\phi \tag{2}$$

The total contact force $F'_i(\mathbf{n}')$ exerted by a neighboring sphere at a contact point consists of two parts, namely, normal components due to an externally imposed strain, P', and a tangential component, T'_i, due to the external strain,

$$F'_i = -P'n'_i + T'_i. \tag{3}$$

Further, since T'_i is perpendicular to n'_i, we have

$$T'_i n'_i = 0. \tag{4}$$

Since the displacement at the particle level is due to the externally imposed strain, the force components that originate from the external strain is related to the components of the displacement. For small displacements, the magnitude of the normal component of contact force is assumed to vary linearly with the normal displacement of the contact point,

$$P' = k_n\delta'_n \tag{5}$$

where k_n is the normal stiffness coefficient. Similarly the magnitude of the tangential component of the force is related to the tangential displacement,

$$T' = k_t s' \tag{6}$$

when T' is less than the critical value required for the sphere to slide, $T'_r = \mu(P' + F_0)$; the latter being the expression for Coulomb friction. The inter-particle attractive force is represented by F_0 with the assumption that this force exists only when the particles are in contact. Here, k_t is the tangential stiffness coefficient and μ is the friction coefficient for sliding.

Now, at the onset of sliding, the critical tangential displacement of the contact point is given by,

$$\begin{aligned} s'^{E}_{\theta,\phi} &= \frac{T'_r}{k_t} = \frac{\mu\left(P' + F'_0\right)}{k_t} = \frac{\mu\left(k_n\delta'_n + F_0\right)}{k_t} \\ &= \bar{\mu}\delta'_n + \frac{\mu F_0}{k_t} \end{aligned} \tag{7}$$

where $\bar{\mu} \equiv \mu k_n / k_t$.

Now there will be two conditions for no slip, one is obtained for θ component of tangential displacement and another one is for ϕ component.

For θ component, the condition becomes,

$$s'^{E}_{\theta} \leq \overline{\mu}\delta'_n + \frac{\mu F_o}{k_t}, \quad \overline{\mu} \equiv \frac{k_n \mu}{k_t} \tag{8}$$

Now, from Eqn. 1 and 2, s'^{E}_{θ} and δ'_n is given as,

$$s'^{E}_{\theta} = \frac{D\epsilon_0}{4} sin\theta cos\theta\left(1 + sin^2\phi\right) \tag{9}$$

$$\delta'_n = \frac{D\epsilon_0}{4}\left(sin^2\theta sin^2\phi - cos^2\theta\right) \tag{10}$$

Then from Eqn. 8 and 10 we get,

$$\overline{\epsilon_0} F_1\left(\theta, \phi\right) \leq 1 \tag{11}$$

where $F_1(\theta,\phi) = \left\{(sin\theta cos\theta\left(1 + sin^2\phi\right) - \overline{\mu}\left(sin^2\theta sin^2\phi - cos^2\theta\right)\right\}$ and $\overline{\epsilon_0} = \frac{D\epsilon_0 k_t}{4\mu F_0}$

Similarly s'^{E}_{ϕ} is given as,

$$s'^{E}_{\phi} = \frac{D\epsilon_0}{4} sin\theta sin\phi cos\phi \tag{12}$$

Then similarly the second condition for ϕ component is given as,

$$\overline{\epsilon_o} F_2(\theta,\phi) \leq 1 \tag{13}$$

whereas

$F_2(\theta,\phi) \equiv \left\{sin\theta sin\phi cos\phi - \overline{\mu}\left(sin^2\theta sin^2\phi - cos^2\theta\right)\right\}$.