Supplementary text

Actin shells control buckling and wrinkling of biomembranes

Linear stability analysis

We discuss in this section the stability of a spherical actin shell around a vesicle at the scaling level. We use the notations of the main text, the liposome has a radius R_0 and the lipid membrane a bending modulus κ. The actin shell, which we consider incompressible for simplicity, has a thickness h_0 and a Young modulus E. The deflation of the membrane creates a relative decrease of the vesicle volume $-\delta v$ and induces a negative tension $-\gamma$ of the actin layer with $\gamma = \frac{2Eh_0\delta v}{3}$.

We first ignore the curvature of the surface and discuss the stability of a flat membrane covered with an elastic layer of thickness h_0 under a negative tension $-\gamma$. We consider an undulation of the membrane $u \cos qx$. This deformation creates an elastic stress in the elastic layer and costs elastic energy. In the limit where the wave vector is small, $qh_0 \ll 1$ the elastic energy per projected area corresponds to the bending of a thin elastic sheet $E_{el} = \frac{1}{18}Eh_0^3q^4u^2$. In the opposite limit where the elastic layer is thick $qh_0 \gg 1$, the elastic deformation only penetrates over a length $1/q$ inside the elastic layer and the elastic energy per projected area is $E_{el} = \frac{Equ^2}{3}$. As the relevant length scales are well separated, it is sufficient for our purpose to extrapolate the elastic energy per projected area between these two limits

$$E_{el} = \frac{1}{18}\frac{Eh_0^3q^4u^2}{1+(qh_0)^3/6} \tag{1}$$

The total energy F per projected area is the sum of the bending energy of the membrane, the elastic energy per projected area E_{el}, and the negative tension (-γ). The stability of the layer is best discussed in terms of its effective tension. We define the effective tension by writing the total energy as $F = \frac{1}{2}\Gamma(q)q^2u^2$. The surface of the layer becomes unstable with respect to undulations when the effective tension vanishes. The effective tension is given by

$$\Gamma(q) = \kappa q^2 + \frac{1}{9}\frac{Eh_0^3q^2}{1+(qh_0)^3/6} - \gamma \tag{2}$$

If the thickness of the elastic layer h_0 is larger than λ , the tension is a non monotonic function of the wave vector q with two minima, a minimum at zero wave vector corresponding to buckling and a minimum at finite wavevector corresponding to wrinkling. The wrinkling wave vector is

$$q_w = \frac{2\pi}{\lambda} = \left(\frac{E}{3\kappa}\right)^{1/3} \tag{3}$$

If the thickness h_0 of the elastic layer is smaller than the wave length λ the wrinkling minimum disappears and the only instability is buckling. At the scaling level, for a spherical vesicle of finite radius, one can still study the instability from the tension Γ given by Eq. 2. However, the smallest wave vector where buckling can occur is $q = 2\pi/R_0$.

To fully describe the transition between buckling and wrinkling, we build a diagram of states of the liposomes looking at the instability with respect to both of buckling and wrinkling for which $\Gamma = 0$. We obtain the two lines of Fig. S3 that limit the domains for buckling and wrinkling: using the expression of γ, we find that the vesicle is unstable with respect to buckling if $h_0/R_0 \leq (\delta v)^{1/2}$ and it is unstable with respect to wrinkling if $\lambda/h_0 \leq \delta v$. The relative volume change δv is fixed and we describe the state of the vesicle in the plane of the two dimensionless variables $(\lambda/h_0, h_0/R_0)$ (Fig. S3).

The transition between buckling and wrinkling occurs when the effective tension Γ is equal in the buckled and wrinkled states. This transition is independent of δv and occurs if

$$\lambda = \frac{(2\pi h_0)^3}{9R_0^2}$$

We compare this equation to the experimental results in the main text.

Supplementary Figures

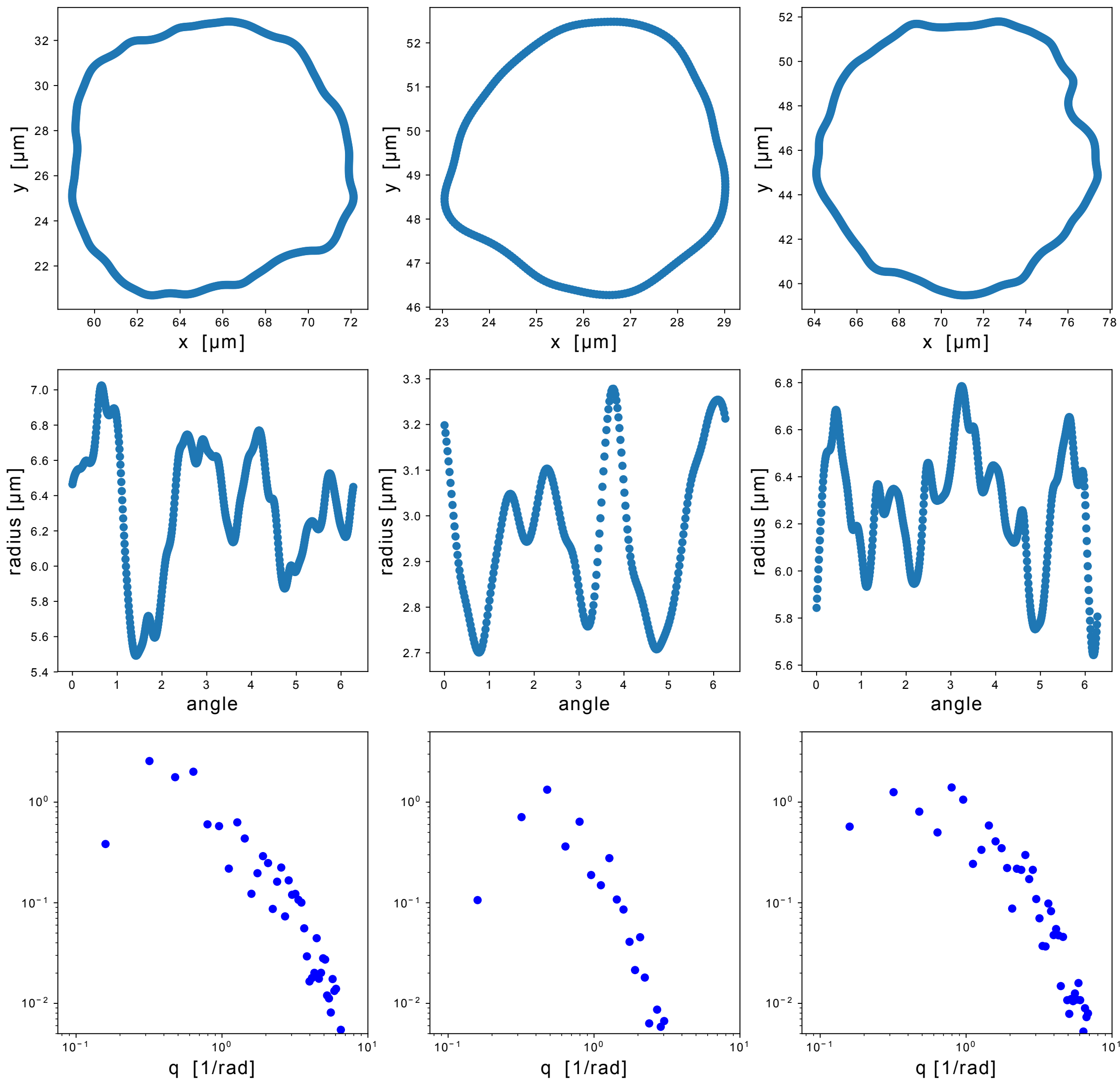

Figure S1 Three examples of wrinkling (top row) with their corresponding radius as function of the angle (middle row) and their Fourier spectrum (bottom row). The maximal value of the Fourier spectrum is selected as characteristic wavelength.

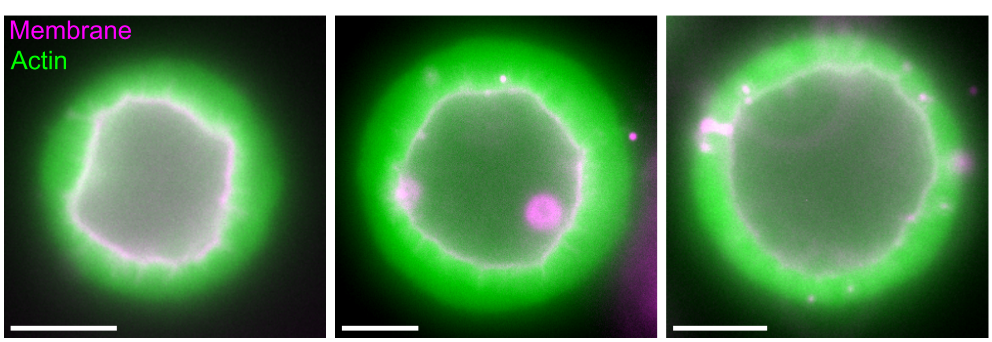

Figure S2 Three examples of membrane (magenta) and actin (green) in the case of wrinkled shapes. Note that the outer contour of the actin network is spherical and un-deformed whereas the membrane wrinkles. Scale bars: $5\ \mu\mathrm{m}$.

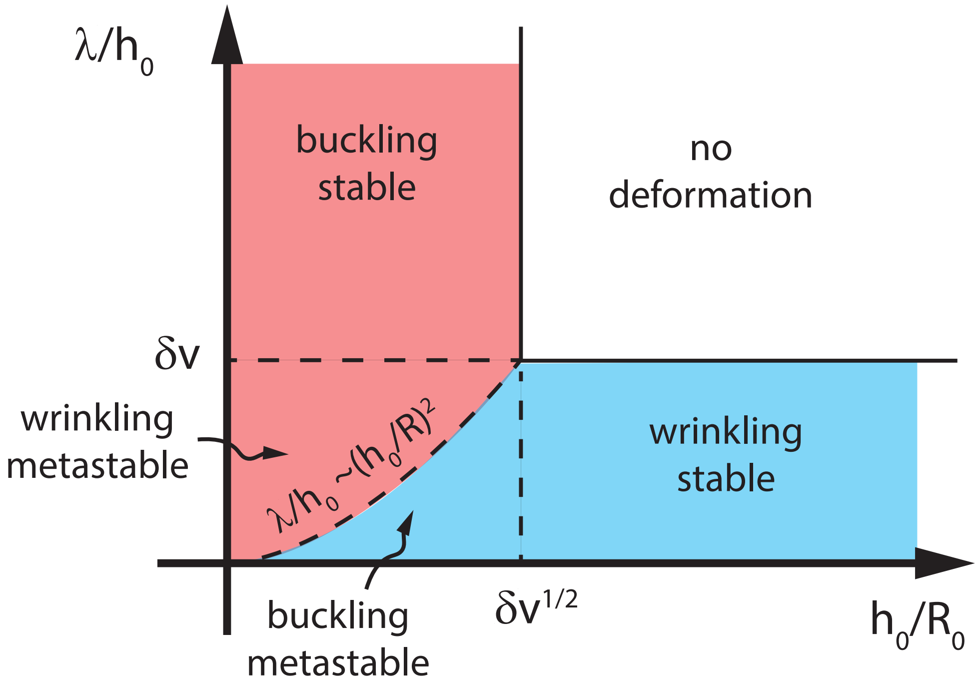

Figure S3 Stability diagram of a spherical vesicle with an actin shell undergoing both wrinkling and buckling. The dotted line represents the equality of tension γ for both buckling and wrinkling.

Supplementary Movies

VideoS1. Naked liposome when preparation evaporates over time. Equatorial plane observed by epifluorescence as a function of time. Bar, 5 μm.

VideoS2. Naked liposome under osmotic shock. Equatorial plane observed by epifluorescence as a function of time. Time indicated is the time after the osmotic shock. Bar, 5 μm.