

Supplementary Information for

**Topological nodal lines and nodal points in the
antiferromagnetic material $\beta\text{-Fe}_2\text{PO}_5$**

Tingli He, Xiaoming Zhang,* Weizhen Meng, Lei Jin, Xuefang Dai, and Guodong Liu*

School of Materials Science and Engineering, Hebei University of Technology, Tianjin 300130, China.

E-mail: zhangxiaoming87@hebut.edu.cn; gdliu1978@126.com

I. Electronic band structure at other magnetic configurations

The electronic band structures of β -Fe₂PO₅ under the NM, AFM2 and AFM3 states are shown in Fig. S1 (a)-(c), and that for the FM state is shown in Fig. S2 (a)-(d). We find β -Fe₂PO₅ exhibits quite interesting topological band structure under the FM state. As shown in Fig. S2 (a)-(d), band crossings in R1, R2 and R3 form Weyl nodal line, type-II Weyl point, and Dirac nodal line, respectively.

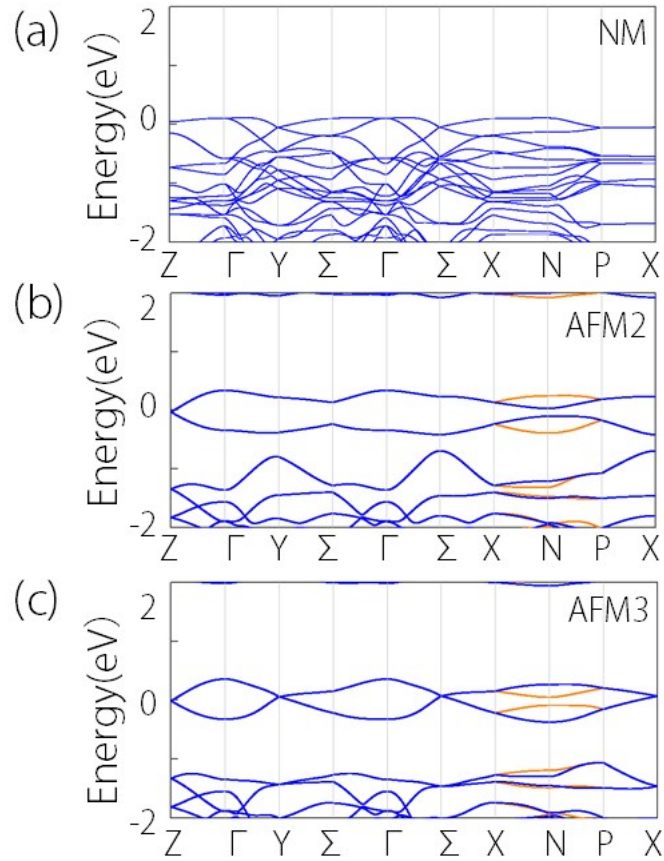


Fig. S1 Electronic band structure of β -Fe₂PO₅ in the (a) NM (b) AFM2 and (c) AFM3 states. In (b) and (c), bands in the spin-up and spin-down channels are shown by the blue and orange lines, respectively.

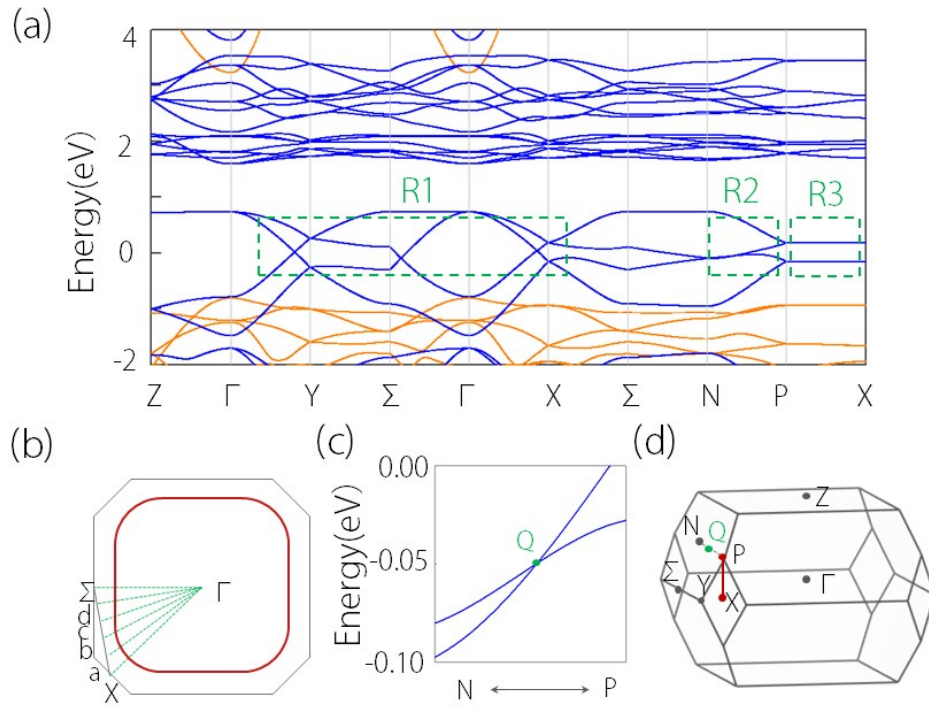


Fig. S2 (a) Electronic band structure of $\beta\text{-Fe}_2\text{PO}_5$ in the FM state. Bands in the spin-up and spin-down channels are shown by the blue and orange lines. R1, R2 and R3 highlight the band crossings along the k -paths Γ -Y, X - Γ - Σ , N -P, and P -X respectively. (b) The profile of the Weyl nodal line in R1. (c) Enlarged band structure for R2. Q is a type-II Weyl point. (d) The bulk Brillouin zone and the positions for the type-II Weyl point and Dirac nodal line along P -X.

II. Electronic band structures at different U values

Our calculations show the band crossings near the Fermi level in $\beta\text{-Fe}_2\text{PO}_5$ will not change when U shifts in the range from 0 eV to 10 eV.

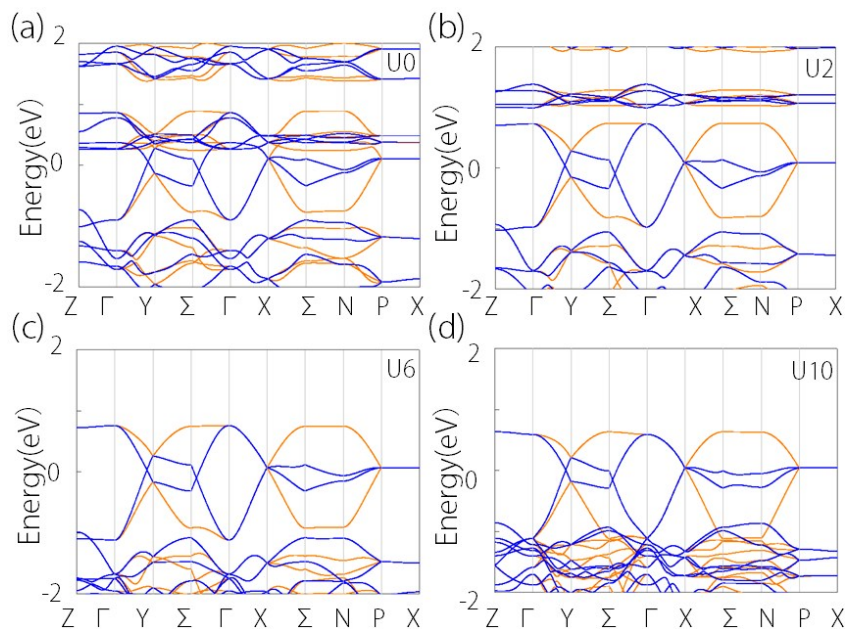


Fig. S3 Band structure of $\beta\text{-Fe}_2\text{PO}_5$ calculated by the GGA+U method with different U values: (a) U = 0 eV, (b) U = 2 eV, (c) U = 6 eV, (d) U = 10 eV. Bands in the spin-up and spin-down channels are shown by the blue and orange lines.

III. Electronic band structures under MBJLDA functional

The MBJLDA calculation shows a similar band structure with GGA that all the linear band crossings near the Fermi level retain in $\beta\text{-Fe}_2\text{PO}_5$.

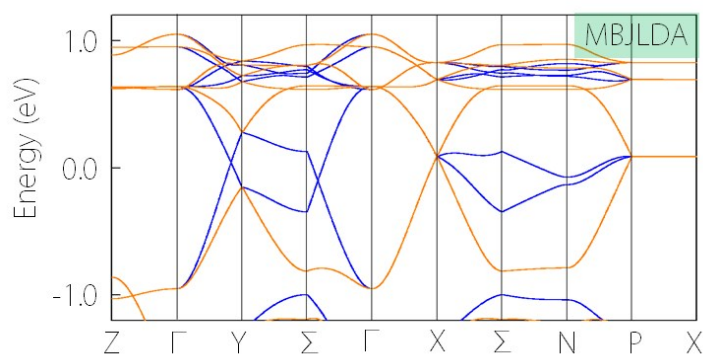


Fig. S4 The electronic band structure of $\beta\text{-Fe}_2\text{PO}_5$ under the MBJLDA functional.

IV. Electronic band structures under different lattice distortions

We find the Weyl nodal line in $\beta\text{-Fe}_2\text{PO}_5$ is very robust upon lattice distortion (which usually happens during the preparation of materials). Our calculations show the band crossings at the Γ -Y and Γ - Σ paths can retain under 10% tensile and compressive lattice distortions.

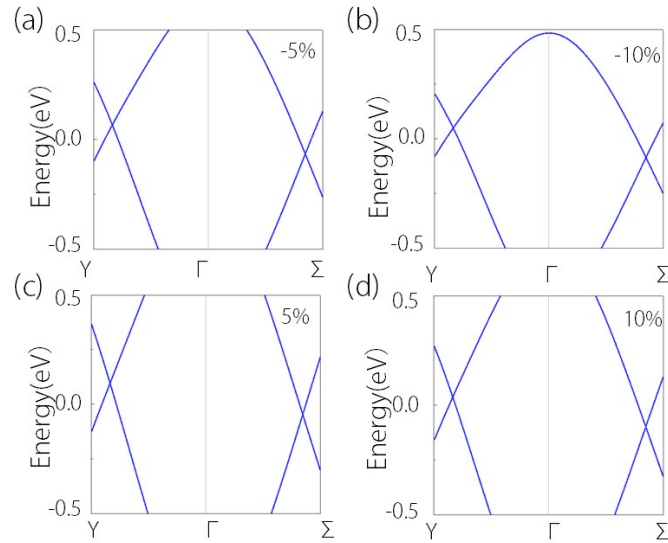


Fig. S5 Enlarged band structure of $\beta\text{-Fe}_2\text{PO}_5$ along the Y- Γ - Σ path under (a) 5% compressive, (b) 10% compressive, (c) 5% tensile, and (d) 10% tensile distortions.

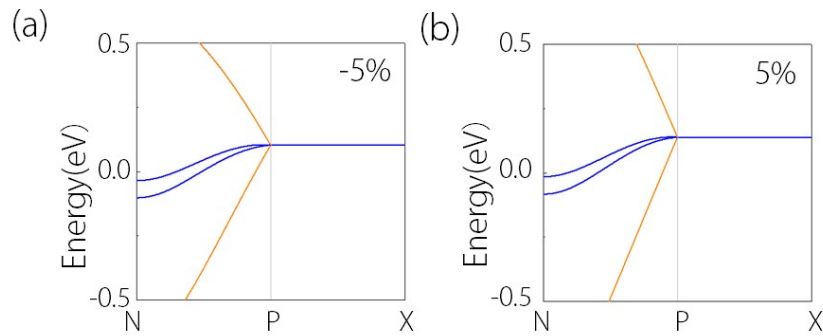


Fig. S6 Enlarged band structure of $\beta\text{-Fe}_2\text{PO}_5$ along the N-P-X path under (a) 5% compressive, and (b) 5% tensile distortions.

V. Surface states of $\beta\text{-Fe}_2\text{PO}_5$

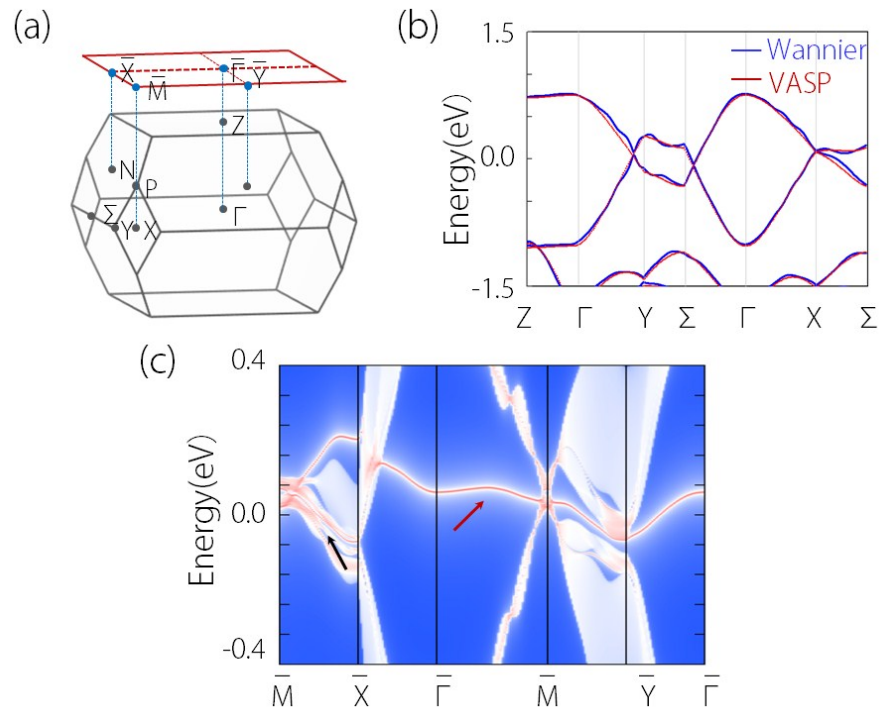


Fig. S7 (a) The bulk Brillouin zone and its projection onto the (001) surface projection of $\beta\text{-Fe}_2\text{PO}_5$. (b) Comparison of band structure from DFT and the Wannier. (c) The (001) surface band structure of $\beta\text{-Fe}_2\text{PO}_5$. In (c), the surface bands for the Weyl nodal line and Dirac point are pointed by the red and black arrows, respectively.