

Supplementary Material:

Negative and near-zero Poisson's ratio of interlayer space in 2D

Graphene/MoS₂ and Graphene/h-BN heterostructures

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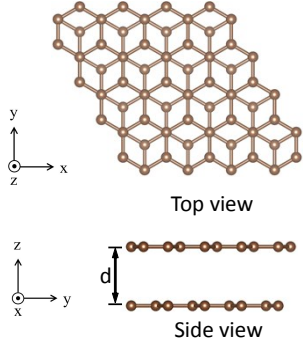
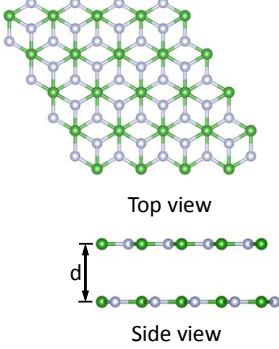
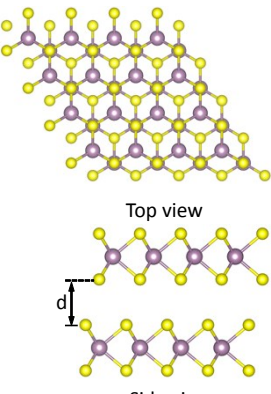
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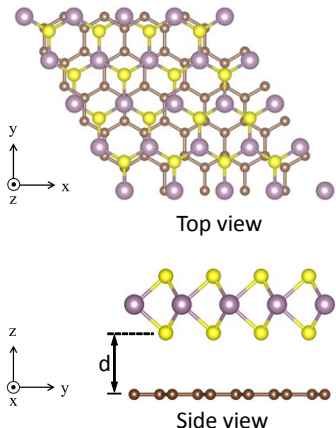
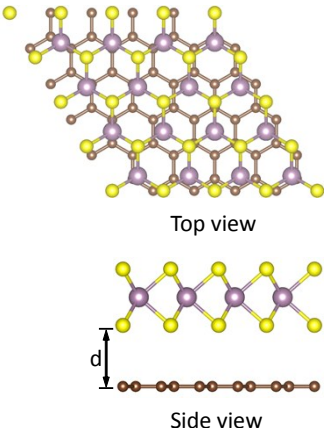
Supplementary Information Outline:

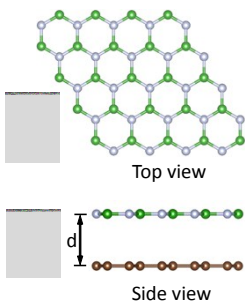
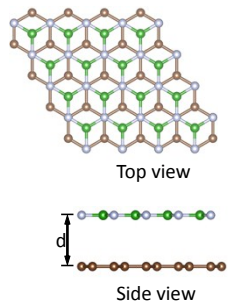
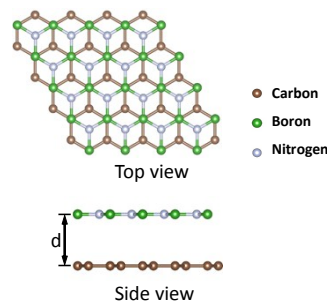
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1. Interlayer binding energies and equilibrium distance results

Table 1. Interlayer binding energies (E_{bind}) and equilibrium distances (d) for bilayer graphene, bilayer h -BN, bilayer MoS₂, G/MoS₂ and G/ h -BN heterostructures.

Bilayer	Graphene		h -BN		2H-MoS ₂	
						
	E_{bind} (meV/Å ²)	d (Å)	E_{bind} (meV/Å ²)	d (Å)	E_{bind} (meV/Å ²)	d (Å)
D2	19.24	3.247	24.03	3.148	17.10	3.103
OptB88	24.76	3.400	23.17	3.340	25.36	3.094
MBD	17.35	3.472	19.62	3.364	16.77	3.219

G/MoS ₂ heterostructure	Stacking mode A		Stacking mode B	
				
	E_{bind} (meV/Å ²)	d (Å)	E_{bind} (meV/Å ²)	d (Å)
D2	14.62	3.373	15.17	3.376
OptB88	22.85	3.386	22.84	3.389
MBD	15.96	3.407	15.94	3.411

G/h-BN heterostructure	Stacking mode C		Stacking mode D		Stacking mode E		
							
	E_{bind} (meV/Å ²)	d (Å)	E_{bind} (meV/Å ²)	d (Å)	E_{bind} (meV/Å ²)	d (Å)	
	D2	17.43	3.366	18.41	3.349	23.26	3.129
	OptB88	21.32	3.443	21.93	3.404	25.11	3.327
	MBD	16.18	3.577	16.75	3.445	19.61	3.345

2. Out-of-plane Poisson's ratio results

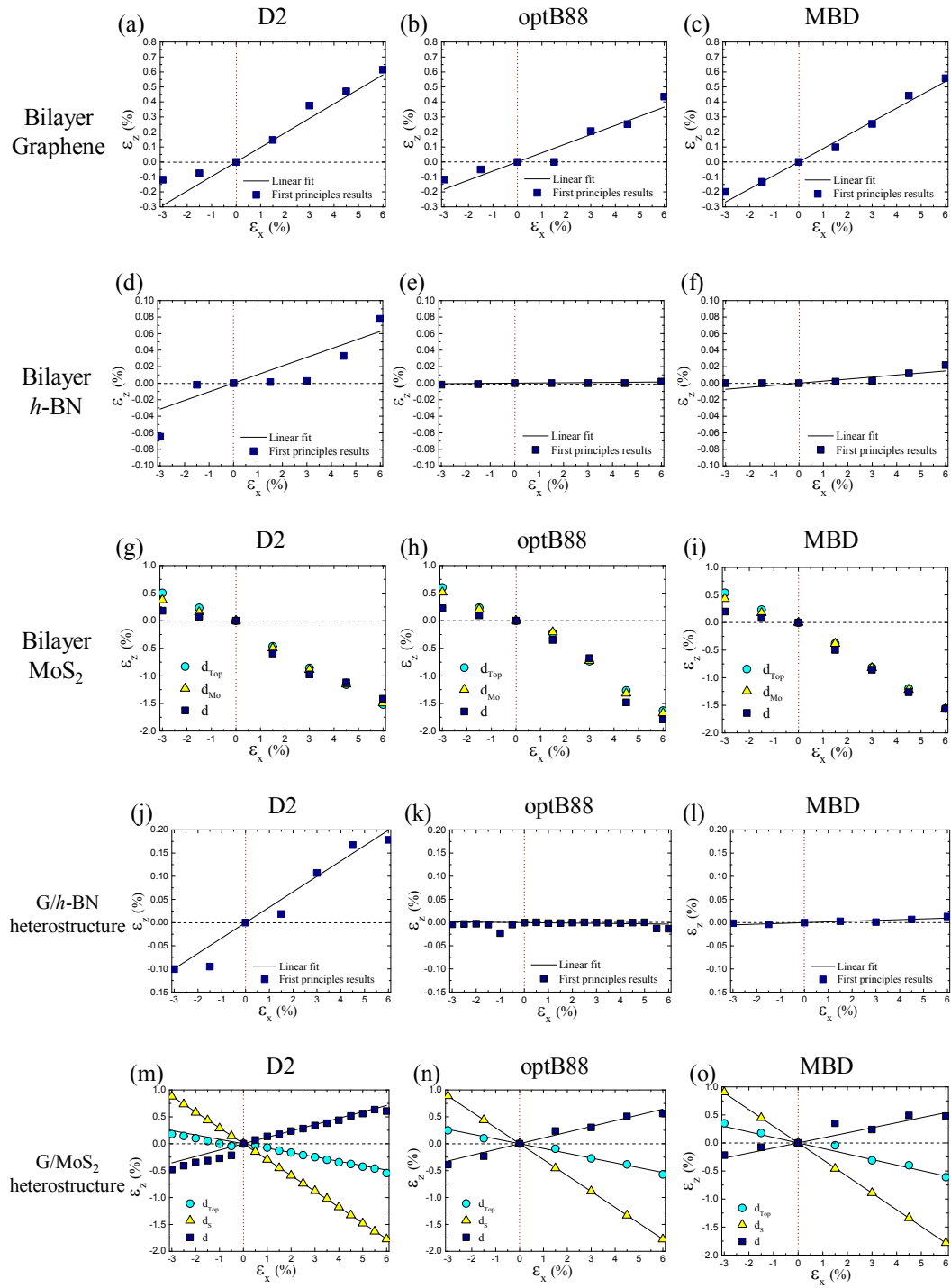


Figure S1. The strain generated in the z direction (ϵ_z) versus the strain applied in the x direction (ϵ_x) data of uniaxial deformation of the (a, b, c) bilayer graphene, (d, e, f) bilayer h -BN, (g, h, i) bilayer MoS_2 , (j, k, l) G/ h -BN and (m, n, o) G/ h -BN heterostructures, calculated by density functional theory with different van der Waals corrections.

Table S2. Computational results of out-of-plane Poisson's ratio (ν_{13}) for the bilayer graphene, bilayer *h*-BN, bilayer MoS₂, G/MoS₂ and G/*h*-BN heterostructures by density functional theory with different van der Waals corrections.

		Out-of-plane Poisson's ratio (ν_{13})		
Bilayer		D2	optB88	MBD
Graphene		-0.097	-0.061	-0.089
<i>h</i> -BN		-0.010	-0.00023	-0.0025
MoS ₂	d_{Top}	0.247	0.256	0.250
	d_{Mo}	0.240	0.258	0.247
	d	0.228	0.264	0.243
heterostructure				
G/ <i>h</i> -BN		-0.033	0.0005	-0.002
G/MoS ₂	d_{Top}	0.080	0.099	0.089
	d_S	0.295	0.297	0.296
	d	-0.120	-0.090	-0.107
d represents interfacial layer equilibrium distance. d_S represents the distance between S and S atoms. d_{Mo} represents the distance between Mo atoms in upper MoS₂ monolayer and Mo atoms in lower MoS₂ monolayer and d_{Top} represents the distance between top S atoms in upper MoS₂ monolayer and S (C) atoms in lower MoS₂ (Graphene) monolayer (Fig. 1).				

3. Out-of-plane Stiffness results

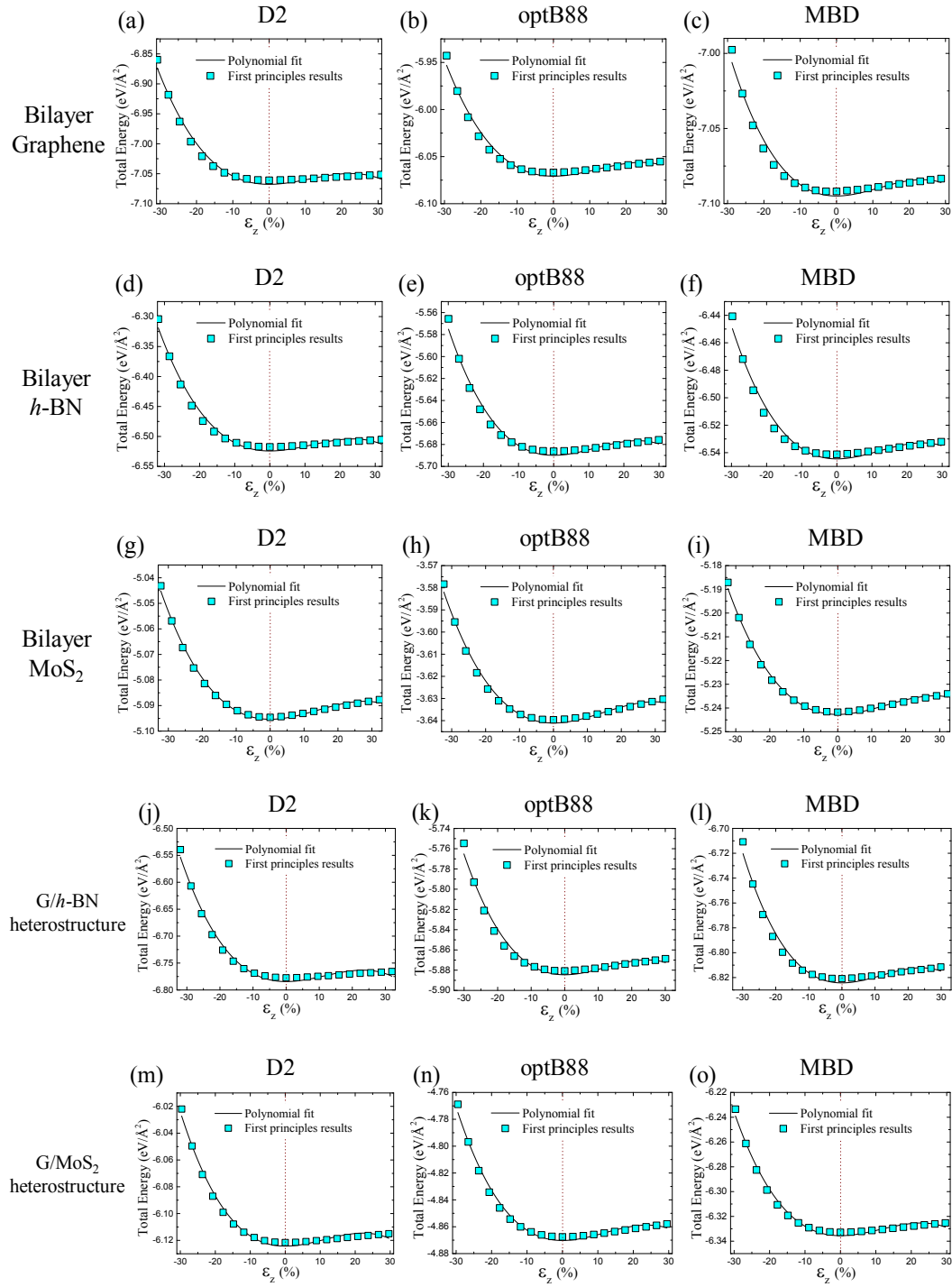


Figure S2. Total energy of the (a, b, c) bilayer graphene, (d, e, f) bilayer *h*-BN, (g, h, i) bilayer MoS₂, (j, k, l) G/MoS₂ and (m, n, o) G/*h*-BN heterostructures with the strain applied in the *z* direction (ϵ_z) calculated by density functional theory with different van der Waals corrections.

Table S3. Computational results of out-of-plane stiffness for the bilayer graphene, bilayer *h*-BN, bilayer MoS₂, G/MoS₂ and G/*h*-BN heterostructures by density functional theory with different van der Waals corrections.

	Out-of-plane Stiffness (Gpa)					
Bilayer	D2		optB88		MBD	
	C ₃₃	C ₃₃₃	C ₃₃	C ₃₃₃	C ₃₃	C ₃₃₃
Graphene	106	-939	71	-586	54	-463
<i>h</i> -BN	109	-463	67	-463	56	-463
MoS ₂	28	-202	34	-202	30	-210
heterostructure						
G/ <i>h</i> -BN	121	-1040	70	-571	61	-508
G/MoS ₂	56	-500	57	-475	56	-489

4. In-plane Poisson's ratio results

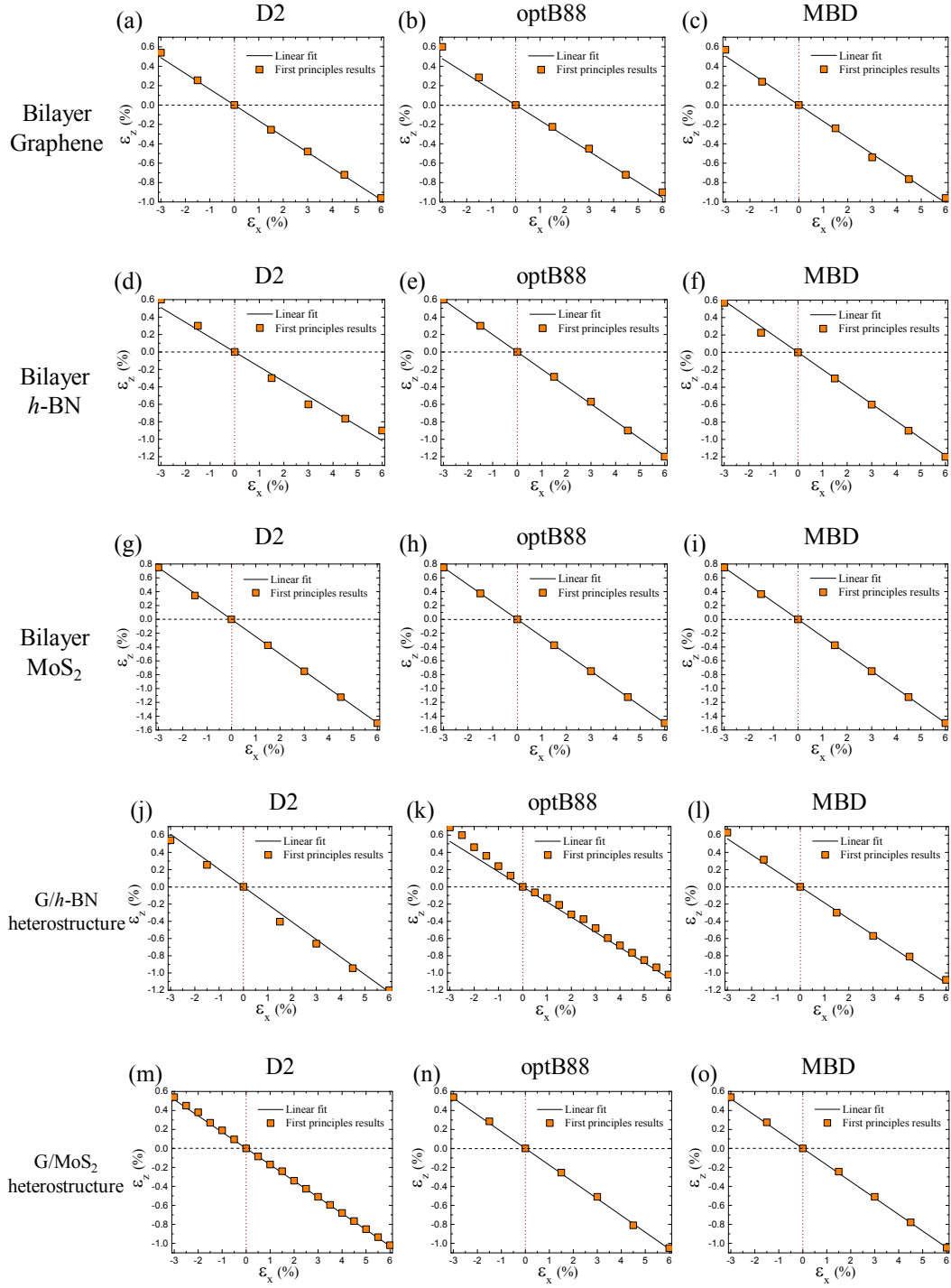


Figure S3. The strain generated in the y direction (ϵ_y) versus the strain applied in the x direction (ϵ_x) data of uniaxial deformation of the (a, b, c) bilayer graphene, (d, e, f) bilayer *h*-BN, (g, h, i) bilayer MoS₂, (j, k, l) G/MoS₂ and (m, n, o) G/*h*-BN heterostructures, calculated by density functional theory with different van der Waals corrections.

Table S4. Computational results of in-plane Poisson's ratio (ν_{12}) for the bilayer graphene, bilayer h -BN, bilayer MoS_2 , G/MoS_2 and G/h -BN heterostructures by density functional theory with different van der Waals corrections.

	In-plane Poisson's ratio (ν_{12})		
Bilayer	D2	optB88	MBD
Graphene	0.163	0.159	0.168
h -BN	0.169	0.199	0.197
MoS_2	0.249	0.250	0.250
heterostructure			
G/h -BN	0.204	0.176	0.186
G/MoS_2	0.171	0.177	0.174

5. In-plane Stiffness results

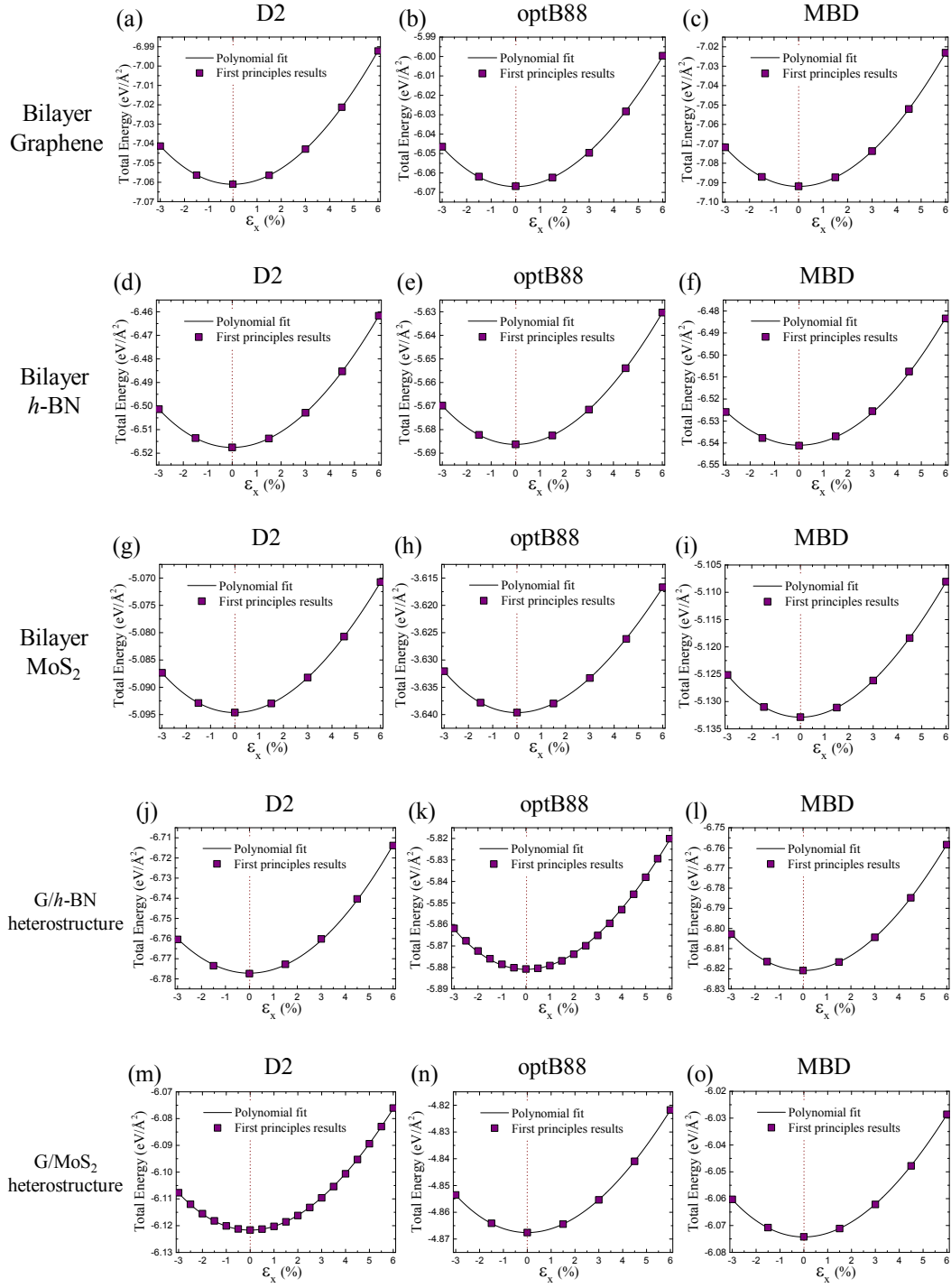


Figure S4. Total energy of the (a, b, c) bilayer graphene, (d, e, f) bilayer *h*-BN, (g, h, i) bilayer MoS₂, (j, k, l) G/MoS₂ and (m, n, o) G/*h*-BN heterostructures with the strain applied in the x direction (ϵ_x) calculated by density functional theory with different van der Waals corrections.

Table S5. Computational results of in-plane stiffness for the bilayer graphene, bilayer *h*-BN, bilayer MoS₂, G/MoS₂ and G/*h*-BN heterostructures by density functional theory with different van der Waals corrections.

	In-plane Stiffness (Nm ⁻¹)					
Bilayer	D2		optB88		MBD	
	Y ₁₁	Y ₁₁₁	Y ₁₁	Y ₁₁₁	Y ₁₁	Y ₁₁₁
Graphene	673	-3034	679	-4124	683	-3471
<i>h</i> -BN	551	-2671	554	-2900	538	-1101
MoS ₂	244	-1548	247	-2157	256	-1808
heterostructure						
G/ <i>h</i> -BN	596	-1385	625	-4352	615	-2974
G/MoS ₂	465	-2969	468	-3053	465	-3004

By analyzing total energy (see Figure 1), the stiffness (Young's Modulus) SOEC and TOEC of the G/MoS₂ and G/*h*-BN heterostructures in the in-plane direction (Y₁₁ and Y₁₁₁) were derived (see Methods section). The calculated Y₁₁ of the G/MoS₂ and G/*h*-BN heterostructures are almost equal to the sum of the stiffness (Young's Modulus) of the two layers of the materials. The Y₁₁ of the two-dimensional Graphene, MoS₂, and *h*-BN are 340 Nm⁻¹, 109 Nm⁻¹ and 238 Nm⁻¹, respectively¹⁻³. This result reveals the reason for the enhanced stability of the carbon-based heterostructures observed in the experiment⁴.

6. The relationship between Poisson's ratio and stiffness

The stiffness tensor of a hexagonal crystal system can be written as follows:

$$C_{\alpha\beta} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{C_{11} - C_{12}}{2} \end{pmatrix}.$$

The compliance tensor can be obtained by taking inverse of the stiffness tensor:

$$S_{\alpha\beta} = C_{\alpha\beta}^{-1} = \begin{pmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{12} & S_{11} & S_{13} & 0 & 0 & 0 \\ S_{13} & S_{13} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(S_{11} - S_{12}) \end{pmatrix}.$$

The in-plane Poisson's ratio ν_{12} and the out-of-plane Poisson's ratio ν_{13} :

$$\begin{cases} \nu_{12} = -\frac{S_{12}}{S_{11}} \\ \nu_{13} = -\frac{S_{13}}{S_{11}}. \end{cases}$$

After simplification,

$$\begin{cases} \nu_{12} = \frac{C_{13}^2 - C_{12}C_{33}}{C_{13}^2 - C_{11}C_{33}} \\ \nu_{13} = \frac{C_{13}(C_{12} - C_{11})}{C_{13}^2 - C_{11}C_{33}}. \end{cases}$$

Therefore, we can verify the accuracy of the results by the relationship between Poisson's ratio and stiffness tensor.

7. the relationship between θ and ν_{13}

When i fixed in the x direction and j varying in the y - z plane by an angle of θ , the transformation

from the stress σ_β (unprimed) to the stress $\sigma_{\beta'}$ in arbitrary system (primed) is described by:

$$\begin{pmatrix} \sigma_{1'} \\ \sigma_{2'} \\ \sigma_{3'} \\ \sigma_{4'} \\ \sigma_{5'} \\ \sigma_{6'} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \cos^2 \theta & \sin^2 \theta & 2\sin \theta \cos \theta & 0 & 0 \\ 0 & \sin^2 \theta & \cos^2 \theta & -2\sin \theta \cos \theta & 0 & 0 \\ 0 & -\sin \theta \cos \theta & \sin \theta \cos \theta & \cos^2 \theta - \sin^2 \theta & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos \theta & -\sin \theta \\ 0 & 0 & 0 & 0 & \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{pmatrix}$$

$$\tilde{\sigma}_{\beta'} = T_{\sigma_{\beta'\beta}} \tilde{\sigma}_\beta$$

Note that we use contracted notations ($11 \rightarrow 1$, $22 \rightarrow 2$, $33 \rightarrow 3$, $13 \rightarrow 4$, $23 \rightarrow 5$, $12 \rightarrow 3$, $C_{ijkl} \rightarrow C_{\alpha\beta}$,

$C_{ijklmn} \rightarrow C_{\alpha\beta\gamma}$) for the tensor indices. The transformation from the strain ε_α (unprimed) to the strain

$\varepsilon_{\alpha'}$ in arbitrary system (primed) is described by:

$$\begin{pmatrix} \varepsilon_{1'} \\ \varepsilon_{2'} \\ \varepsilon_{3'} \\ \varepsilon_{4'} \\ \varepsilon_{5'} \\ \varepsilon_{6'} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \cos^2 \theta & \sin^2 \theta & \sin \theta \cos \theta & 0 & 0 \\ 0 & \sin^2 \theta & \cos^2 \theta & -\sin \theta \cos \theta & 0 & 0 \\ 0 & -2\sin \theta \cos \theta & 2\sin \theta \cos \theta & \cos^2 \theta - \sin^2 \theta & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos \theta & -\sin \theta \\ 0 & 0 & 0 & 0 & \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{pmatrix}$$

$$\tilde{\varepsilon}_{\alpha'} = T_{\varepsilon_{\alpha'\alpha}} \tilde{\varepsilon}_\alpha$$

The compliance coefficients $S_{\alpha'\beta'}$ are defined as the proportionality constants between stress $\sigma_{\beta'}$

and strain by the Hooke's law:

$$\varepsilon_{\alpha'} = S_{\alpha'\beta'} \sigma_{\beta'}$$

It can be expressed by the coordinate transformation equation:

$$T_{\varepsilon_{\alpha'\alpha}} \varepsilon_\alpha = S_{\alpha'\beta'} T_{\sigma_{\beta'\beta}} \sigma_\beta$$

We obtain

$$\varepsilon_\alpha = \frac{T_{\varepsilon_{\alpha'\alpha}}^{-1} S_{\alpha'\beta'} T_{\sigma_{\beta'\beta}}}{S_{\alpha\beta}} \sigma_\beta,$$

and

$$S_{\alpha'\beta'} = T_{\varepsilon_{\alpha'\alpha}} S_{\alpha\beta} T_{\sigma_{\beta'\beta}}^{-1}$$

The out-of-plane Poisson's ratio ν_{13} :

$$v_{13} = -\frac{S_{13}}{S_{11}}.$$

After simplification,

$$v_{13}(\theta) = -\frac{(C_{11} - C_{12})C_{13}\cos^2\theta - (C_{13}^2 - C_{12}C_{13})\sin^2\theta}{C_{13}^2 - C_{11}C_{13}}.$$

Reference

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