

Phase Cycles

Phase cycling for PE-WATERGATE (Fig. 1a of the manuscript). Phases are notated as multiples of 90° ($0 = 0^\circ$, $1 = 90^\circ$, $2 = 180^\circ$, $3 = 270^\circ$), with subscripts denoting repetition; the minimum phase cycle is eight transients.

$$\begin{aligned}\Phi_1 &= 02 \\ \Phi_2 &= 2_23_20_21_2 + 0_{32}2_{32} \\ \Phi_{2'} &= \Phi_2 + 2 = 0_21_22_23_2 + 0_{32}2_{32} \\ \Phi_3 &= 0_81_82_83_8 \\ \Phi_4 &= 1_{64}3_{64} \\ \Phi_R &= \Phi_1 + 2\Phi_2 + 2\Phi_3 = 0_82_8 + 0_22_2 + 02\end{aligned}$$

Additionally, for PE-ES-WATERGATE (Fig. 1b of the manuscript).

$$\Phi_{3'} = \Phi_3 + 2 = 2_83_80_81_8$$

Excitation Profile

Wolfram Mathematica 7.0 code for calculation of excitation profile of 180_x hard pulse flanked by two 90_x selective pulses of duration pw_{90shp} using a density matrix approach for a single spin- $1/2$.

```
Id[0] = {{1, 0}, {0, 1}};
Id[1] = {{0, 0.5}, {0.5, 0}};
Id[2] = {{0, -0.5*I}, {0.5*I, 0}};
Id[3] = {{0.5, 0}, {0, -0.5}};
\Phi[a_] := Sin[(Pi/2)*Mod[a, 4]]*Id[2] + Cos[(Pi/2)*Mod[a, 4]]*Id[1];
Pulse[\Nu_i_, RF_, \Phi_] := -(\Nu_i*Id[3]) - RF*\Phi[\Phi];
CalcPulse[\Nu_i_, RF_, \Phi_][t_][\Rho_] := MatrixExp[-(I*2*Pi*Pulse[\Nu_i_, RF, \Phi_]*t)].\Rho_.MatrixExp[I*2*Pi*Pulse[\Nu_i_, RF, \Phi_]*t];
\Omega1[pdur_] := 1/4/pdur
pw90 = 10/10^6;
RFhp = \Omega1[pw90];
pw90shp = 10.5/10^3;
RFshp = \Omega1[pw90shp];
\Nu_i = 0;
Offsets = 1000;
OffRange = 10000;
a = 3;
For[i = 1, i < Offsets + 2, i++, spoffs = (i - 1)*(OffRange/(Offsets - 1)) - Offsets*(OffRange/(Offsets - 1)/2);
\[\Sigma] = -2 Id[3];
\[\Sigma] = CalcPulse[\Nu_i - spoffs, RFshp, 0][pw90shp][\[\Sigma]];
\[\Sigma] = CalcPulse[\Nu_i - spoffs, RFhp, 2][pw90*2][\[\Sigma]];
\[\Sigma] = CalcPulse[\Nu_i - spoffs, RFshp, 0][pw90shp][\[\Sigma]];
MagOut[i] = \[\Sigma];
Res[i] = spoffs; ];
Print["Excitation Profile:"]
ListPlot[Table[{Res[i], Re[0.5*(1 + Tr[Id[3] . MagOut[i]])}], {i, 1, Offsets + 1}], PlotRange -> {{-720 + 1880, -2160 + 1880}, {0, 1}}, Joined -> True]
```



```
settable(t6,16,rec);
settable(t7,16,rec2);
/*Start Sequence*/
status(A);
  delay(d1);
status(B);
  obspower(tpwr);
  rgpulse(pw, t1, rof1, rof1);
  if (es[A] == 'y' ) {
    zgradpulse(gzlv11*0.379,gt2);
    delay(gstab-rof1-rof1-rof1-rof1);
    if (phaseinc < 0.0) {
      phaseinc = 1440+phaseinc;
    }
    stepsize(0.25,OBSch);
    initval(phaseinc,v1);
    obspower(selpwr);
    xmtrphase(v1);
    txphase(t21);
    rgpulse(selpw,t21,rof1,rof1);
    obspower(tpwr);
    xmtrphase(zero);
    txphase(t2);
    rgpulse(pw*2.0,t2,rof1,rof1);
    obspower(selpwr);
    xmtrphase(v1);
    txphase(t21);
    rgpulse(selpw,t21,rof1,rof1);
    obspower(tpwr);
    xmtrphase(zero);
    zgradpulse(gzlv11*0.379,gt2);
    delay(gstab);
  }
  if ((pe[A] == 'y'|dse[A] == 'y') && es[A] != 'y'){
    zgradpulse(gzlv11*0.379,gt2);
    delay(selpw+gstab-rof1-rof1);
    rgpulse(pw*2.0,t2,rof1,rof1);
    zgradpulse(gzlv11*0.379,gt2);
    delay(selpw+gstab-rof1-rof1);
  }
  if (pe[A] == 'y'){
    /* Use 90d refocussing pulse */
    rgpulse(pw,t3,rof1,rof1);
  }
  if (pe[A] == 'y'|dse[A] == 'y'|es[A] == 'y'){
    zgradpulse(gzlv11,gt2);
    delay(gstab-rof1-rof1-rof1-rof1);
    if (phaseinc < 0.0){
      phaseinc = 1440+phaseinc;
    }
    stepsize(0.25,OBSch);
    initval(phaseinc,v1);
    obspower(selpwr);
    xmtrphase(v1);
    txphase(t4);
    rgpulse(selpw,t4,rof1,rof1);
    obspower(tpwr);
    xmtrphase(zero);
    txphase(t5);
    rgpulse(pw*2.0,t5,rof1,rof1);
    obspower(selpwr);
    xmtrphase(v1);
    txphase(t4);
    rgpulse(selpw,t4,rof1,rof1);
    obspower(tpwr);
    xmtrphase(zero);
    delay(gstab/2.0);
    zgradpulse(gzlv11,gt2);
    delay(gstab/2.0);
    setreceiver(t6);
  }
  if (pe[A] != 'y' && dse[A] != 'y' && es[A] != 'y'){
    zgradpulse(gzlv11,gt2);
    delay(gstab-rof1-rof1-rof1-rof1);
    if (phaseinc < 0.0){
      phaseinc = 1440+phaseinc;
    }
  }
}
```

```
        stepsize(0.25,OBSch);  
        initval(phaseinc,v1);  
        xmtrphase(v1);  
        txphase(t21);  
        rgpulse(selpw,t21,rof1,rof1);  
        obspower(tpwr);  
        xmtrphase(zero);  
        txphase(t2);  
        rgpulse(pw*2.0,t2,rof1,rof1);  
        obspower(selpwr);  
        xmtrphase(v1);  
        txphase(t21);  
        rgpulse(selpw,t21,rof1,rof1);  
        obspower(tpwr);  
        xmtrphase(zero);  
        delay(gstab/2.0);  
        zgradpulse(gzlv11,gt2);  
        delay(gstab/2.0);  
        setreceiver(t7);  
    }  
status(C);  
}
```