

Supplementary Information

Variational formulation of perturbative explicitly correlated coupled cluster methods.

Martin Torheyden and Edward F. Valeev*

107 Davidson Hall, Department of Chemistry, Virginia Tech, Blacksburg, VA 24061, USA

*Electronic address: evaleev@vt.edu

TABLE I: Experimental geometries for the test molecules. Taken from Ref.
1. Bond lengths are given in Å and bond angles in degree.

molecule	internal coordinate	value ^{1,2}
H ₂	R _{HH}	0.74144
HF	R _{HF}	0.91680(8)
H ₂ O	R _{OH}	0.9572
	∠ _{HOH}	104.52
HOF	R _{OH}	0.9657
	R _{FO}	1.4350(31)
	∠ _{HOF}	97.54(50)
H ₂ O ₂	R _{OH}	0.967
	R _{OO}	1.4556
	∠ _{HOO}	102.32
	∠ _{HOOH}	113.70 ³
HNC	R _{NH}	0.9940(8)
	R _{CN}	1.1689(2)
NH ₃	R _{NH}	1.011(6)
	∠ _{HNH}	106.7
N ₂ H ₂	R _{NH}	1.029(1)
	R _{NN}	1.247(1)
	∠ _{HNN}	106.3(2)
C ₂ H ₂	R _{CH}	1.06215(17)
	R _{CC}	1.20257(9)
HNO	R _{NH} (R ₀)	1.062(8)
	R _{NO} (R ₀)	1.211(6)
	∠ _{HNO} (θ ₀)	108.5(8)
HCN	R _{CH}	1.06501(8)
	R _{CN}	1.15324(2)
C ₂ H ₄	R _{CH}	1.081(2)
	R _{CC}	1.334(2)
	∠ _{HCH}	117.37(33)
CH ₄	R _{CH}	1.0858(10)
N ₂	R _{NN}	1.09768(5)
CH ₂ O	R _{CH}	1.1005(20)
	R _{CO}	1.2033(10)
	∠ _{HCH}	116.30(25)
CH ₂	R _{CH}	1.107(2)
	∠ _{HCH}	102.4(4)
CO	R _{CO}	1.12832
CO ₂	R _{CO}	1.15995
O ₃	R _{OO}	1.2717(2)
	∠ _{OOO}	116.78(33)
F ₂	R _{FF}	1.41193

¹ Bond lengths from table 15.3 of [1].

² Bond angles from table 15.8 of [1].

³ Dihedral from original literature as cited below table 15.3 of [1].

TABLE IV: The valence correlation contributions to the electronic reaction energies, $\Delta H_e^0(0K)$, (in kJ/mol) for the 12 test reactions¹.

	CCSD(T)		CCSD(T) _{R12}				
	E	Δ_{CBS}^2	<i>d</i>		E	Δ_{CBS}^2	
	E	Δ_{CBS}^2	E	Δ_{CBS}^2	E	Δ_{CBS}^2	
1	aug-cc-pVDZ	-11.86	11.98	-20.41	3.43	-21.12	2.72
	aug-cc-pVTZ	-22.22	1.62	-23.96	-0.12	-23.86	-0.02
	CBS	-23.84		-23.84		-23.84	
2	aug-cc-pVDZ	3.07	19.02	-13.62	2.33	-13.98	1.98
	aug-cc-pVTZ	-14.42	1.53	-17.60	-1.65	-17.11	-1.16
	CBS	-15.95		-15.95		-15.95	
3	aug-cc-pVDZ	5.12	-2.23	4.35	-3.00	4.92	-2.43
	aug-cc-pVTZ	6.35	-1.00	7.24	-0.11	7.23	-0.12
	CBS	7.35		7.35		7.35	
4	aug-cc-pVDZ	13.72	15.43	-1.55	0.16	-2.03	-0.33
	aug-cc-pVTZ	-3.26	-1.55	-2.94	-1.24	-2.42	-0.71
	CBS	-1.71		-1.71		-1.71	
5	aug-cc-pVDZ	7.33	10.17	-1.81	1.03	-2.17	0.68
	aug-cc-pVTZ	-2.53	0.31	-3.34	-0.50	-3.08	-0.24
	CBS	-2.84		-2.84		-2.84	
6	aug-cc-pVDZ	-4.53	22.15	-22.22	4.46	-23.29	3.40
	aug-cc-pVTZ	-24.75	1.93	-27.30	-0.62	-26.94	-0.26
	CBS	-26.69		-26.69		-26.69	
7	aug-cc-pVDZ	21.50	12.51	8.54	-0.45	8.21	-0.79
	aug-cc-pVTZ	8.89	-0.10	8.14	-0.86	8.38	-0.62
	CBS	8.99		8.99		8.99	
8	aug-cc-pVDZ	34.98	15.88	20.19	1.09	20.75	1.65
	aug-cc-pVTZ	22.80	3.71	18.30	-0.79	18.91	-0.18
	CBS	19.10		19.10		19.10	
9	aug-cc-pVDZ	19.91	0.61	15.48	-3.82	16.37	-2.93
	aug-cc-pVTZ	16.51	-2.79	18.04	-1.26	18.14	-1.16
	CBS	19.30		19.30		19.30	
10	aug-cc-pVDZ	-33.94	13.28	-45.38	1.84	-45.76	1.46
	aug-cc-pVTZ	-43.11	4.11	-46.88	0.35	-46.71	0.51
	CBS	-47.22		-47.22		-47.22	
11	aug-cc-pVDZ	62.80	12.04	48.44	-2.32	50.37	-0.39
	aug-cc-pVTZ	56.61	5.86	50.24	-0.52	51.28	0.53
	CBS	50.75		50.75		50.75	
12	aug-cc-pVDZ	-82.68	23.72	-101.89	4.50	-102.97	3.42
	aug-cc-pVTZ	-96.38	10.01	-104.55	1.85	-104.32	2.07
	CBS	-106.39		-106.39		-106.39	

¹ See Table V in the main text.

² The difference between the respective reaction energy correlation contribution and the associated CBS value. The CBS value was obtained by Schwenke extrapolation[2] using the aug-cc-pVQZ and aug-cc-pV5Z basis sets. The CCSD correlation energy and the (T) correction were extrapolated separately.

TABLE II: The valence correlation energies (in mE_h) of the 20 test molecules. CBS refers to the correlation energy obtained by the basis set extrapolation with Schwenke method[2] using aug-cc-pVQZ and aug-cc-pV5Z basis sets.

	basis set	CCSD		CCSD(2) _{R12}			
		E	%	<i>d</i>		E	%
H ₂	aug-cc-pVDZ	-35.83	87.7	-39.97	97.8	-39.99	97.8
	aug-cc-pVTZ	-39.61	96.9	-40.62	99.4	-40.68	99.5
	CBS	-40.88		-40.88		-40.88	
CH ₂	aug-cc-pVDZ	-143.22	81.5	-168.70	96.0	-169.83	96.7
	aug-cc-pVTZ	-166.10	94.6	-174.29	99.2	-174.56	99.4
	CBS	-175.64		-175.64		-175.64	
CH ₄	aug-cc-pVDZ	-191.03	82.0	-225.01	96.6	-226.30	97.2
	aug-cc-pVTZ	-220.69	94.7	-231.32	99.3	-231.59	99.4
	CBS	-232.93		-232.93		-232.93	
H ₂ O	aug-cc-pVDZ	-227.11	76.2	-290.27	97.4	-291.51	97.8
	aug-cc-pVTZ	-273.05	91.6	-294.73	98.9	-295.42	99.1
	CBS	-298.09		-298.09		-298.09	
NH ₃	aug-cc-pVDZ	-214.32	79.5	-261.00	96.9	-262.32	97.3
	aug-cc-pVTZ	-251.86	93.5	-267.24	99.2	-267.64	99.3
	CBS	-269.47		-269.47		-269.47	
HF	aug-cc-pVDZ	-225.97	72.0	-307.61	98.0	-308.81	98.4
	aug-cc-pVTZ	-280.97	89.5	-309.10	98.5	-310.33	98.9
	CBS	-313.90		-313.90		-313.90	
N ₂	aug-cc-pVDZ	-319.81	78.5	-394.39	96.8	-396.84	97.4
	aug-cc-pVTZ	-376.88	92.5	-403.42	99.0	-404.21	99.2
	CBS	-407.57		-407.57		-407.57	
F ₂	aug-cc-pVDZ	-435.45	72.5	-589.12	98.0	-592.25	98.5
	aug-cc-pVTZ	-538.96	89.7	-591.79	98.5	-594.59	98.9
	CBS	-601.00		-601.00		-601.00	
CO	aug-cc-pVDZ	-306.29	77.4	-384.26	97.1	-386.34	97.6
	aug-cc-pVTZ	-363.09	91.7	-391.40	98.9	-392.31	99.1
	CBS	-395.75		-395.75		-395.75	
HCN	aug-cc-pVDZ	-302.63	79.8	-365.92	96.4	-368.36	97.1
	aug-cc-pVTZ	-353.51	93.2	-376.22	99.2	-376.80	99.3
	CBS	-379.43		-379.43		-379.43	
HNC	aug-cc-pVDZ	-295.73	79.5	-359.09	96.6	-361.42	97.2
	aug-cc-pVTZ	-346.00	93.1	-368.52	99.1	-369.07	99.3
	CBS	-371.76		-371.76		-371.76	
C ₂ H ₂	aug-cc-pVDZ	-278.86	80.5	-332.70	96.0	-335.58	96.8
	aug-cc-pVTZ	-325.04	93.8	-343.87	99.2	-344.25	99.3
	CBS	-346.59		-346.59		-346.59	
C ₂ H ₄	aug-cc-pVDZ	-313.83	81.3	-372.11	96.4	-374.79	97.1
	aug-cc-pVTZ	-363.58	94.2	-383.07	99.2	-383.52	99.3
	CBS	-386.07		-386.07		-386.07	
CH ₂ O	aug-cc-pVDZ	-347.02	77.8	-432.39	97.0	-434.76	97.5
	aug-cc-pVTZ	-411.35	92.3	-441.33	99.0	-442.27	99.2
	CBS	-445.85		-445.85		-445.85	
HNO	aug-cc-pVDZ	-379.09	77.6	-475.01	97.2	-477.76	97.8
	aug-cc-pVTZ	-450.22	92.1	-483.55	98.9	-484.75	99.2
	CBS	-488.71		-488.71		-488.71	
N ₂ H ₂	aug-cc-pVDZ	-363.11	79.1	-444.16	96.8	-446.94	97.4
	aug-cc-pVTZ	-426.85	93.0	-454.64	99.1	-455.48	99.3
	CBS	-458.83		-458.83		-458.83	
HOF	aug-cc-pVDZ	-428.91	74.1	-565.84	97.7	-568.91	98.2
	aug-cc-pVTZ	-524.15	90.5	-571.37	98.7	-573.52	99.0
	CBS	-579.14		-579.14		-579.14	
H ₂ O ₂	aug-cc-pVDZ	-428.48	76.2	-547.97	97.4	-551.05	98.0
	aug-cc-pVTZ	-514.97	91.6	-556.16	98.9	-557.70	99.2
	CBS	-562.45		-562.45		-562.45	
CO ₂	aug-cc-pVDZ	-501.12	76.1	-639.02	97.0	-642.55	97.5
	aug-cc-pVTZ	-601.18	91.2	-651.26	98.8	-652.86	99.1
	CBS	-658.84		-658.84		-658.84	
O ₃	aug-cc-pVDZ	-640.95	77.6	-807.34	97.7	-812.12	98.3
	aug-cc-pVTZ	-759.04	91.9	-816.82	98.9	-819.33	99.2
	CBS	-825.94		-825.94		-825.94	

TABLE III: The valence correlation energies (in mE_h) of the 20 test molecules. CBS refers to the correlation energy obtained by basis set extrapolation with Schwenke method[2] using aug-cc-pVQZ and aug-cc-pV5Z basis sets. The CCSD correlation energy and the (T) energy correction were extrapolated separately.

	basis set	CCSD(T)		CCSD(T) $_{RI2}$			
		E	%	d		E	%
H ₂	aug-cc-pVDZ	-35.83	87.7	-39.97	97.8	-39.99	97.8
	aug-cc-pVTZ	-39.61	96.9	-40.62	99.4	-40.68	99.5
	CBS	-40.88		-40.88		-40.88	
CH ₂	aug-cc-pVDZ	-146.74	80.9	-172.22	95.0	-173.35	95.6
	aug-cc-pVTZ	-171.19	94.4	-179.38	98.9	-179.65	99.1
	CBS	-181.31		-181.31		-181.31	
CH ₄	aug-cc-pVDZ	-195.50	81.4	-229.47	95.5	-230.77	96.1
	aug-cc-pVTZ	-227.22	94.6	-237.85	99.0	-238.12	99.1
	CBS	-240.17		-240.17		-240.17	
H ₂ O	aug-cc-pVDZ	-232.31	75.4	-295.47	95.9	-296.71	96.3
	aug-cc-pVTZ	-281.68	91.5	-303.35	98.5	-304.04	98.7
	CBS	-307.96		-307.96		-307.96	
NH ₃	aug-cc-pVDZ	-219.77	78.8	-266.46	95.6	-267.77	96.0
	aug-cc-pVTZ	-260.19	93.3	-275.57	98.8	-275.97	99.0
	CBS	-278.79		-278.79		-278.79	
HF	aug-cc-pVDZ	-230.11	71.3	-311.75	96.6	-312.95	97.0
	aug-cc-pVTZ	-288.48	89.4	-316.61	98.1	-317.84	98.5
	CBS	-322.74		-322.74		-322.74	
N ₂	aug-cc-pVDZ	-333.22	77.7	-407.80	95.1	-410.26	95.7
	aug-cc-pVTZ	-396.04	92.3	-422.58	98.5	-423.38	98.7
	CBS	-428.88		-428.88		-428.88	
F ₂	aug-cc-pVDZ	-448.31	71.9	-601.97	96.5	-605.10	97.0
	aug-cc-pVTZ	-558.91	89.6	-611.75	98.0	-614.54	98.5
	CBS	-623.94		-623.94		-623.94	
CO	aug-cc-pVDZ	-318.59	76.7	-396.56	95.5	-398.64	96.0
	aug-cc-pVTZ	-380.63	91.6	-408.95	98.5	-409.86	98.7
	CBS	-415.34		-415.34		-415.34	
HCN	aug-cc-pVDZ	-315.97	79.0	-379.26	94.9	-381.70	95.5
	aug-cc-pVTZ	-371.96	93.0	-394.67	98.7	-395.24	98.9
	CBS	-399.76		-399.76		-399.76	
HNC	aug-cc-pVDZ	-308.76	78.9	-372.12	95.1	-374.45	95.7
	aug-cc-pVTZ	-363.90	93.0	-386.42	98.7	-386.97	98.9
	CBS	-391.46		-391.46		-391.46	
C ₂ H ₂	aug-cc-pVDZ	-291.08	79.7	-344.92	94.5	-347.80	95.3
	aug-cc-pVTZ	-341.90	93.7	-360.72	98.8	-361.11	98.9
	CBS	-365.06		-365.06		-365.06	
C ₂ H ₄	aug-cc-pVDZ	-324.97	80.6	-383.24	95.1	-385.92	95.7
	aug-cc-pVTZ	-379.09	94.0	-398.58	98.9	-399.04	99.0
	CBS	-403.14		-403.14		-403.14	
CH ₂ O	aug-cc-pVDZ	-358.94	77.1	-444.31	95.5	-446.68	96.0
	aug-cc-pVTZ	-428.71	92.1	-458.70	98.6	-459.63	98.8
	CBS	-465.29		-465.29		-465.29	
HNO	aug-cc-pVDZ	-393.75	76.9	-489.67	95.6	-492.41	96.1
	aug-cc-pVTZ	-471.33	92.0	-504.66	98.5	-505.86	98.7
	CBS	-512.27		-512.27		-512.27	
N ₂ H ₂	aug-cc-pVDZ	-377.14	78.4	-458.19	95.3	-460.97	95.8
	aug-cc-pVTZ	-446.77	92.9	-474.57	98.7	-475.40	98.9
	CBS	-480.93		-480.93		-480.93	
HOF	aug-cc-pVDZ	-441.75	73.4	-578.68	96.2	-581.75	96.7
	aug-cc-pVTZ	-544.00	90.4	-591.22	98.2	-593.37	98.6
	CBS	-601.80		-601.80		-601.80	
H ₂ O ₂	aug-cc-pVDZ	-441.63	75.4	-561.12	95.9	-564.19	96.4
	aug-cc-pVTZ	-535.19	91.4	-576.37	98.5	-577.91	98.7
	CBS	-585.36		-585.36		-585.36	
CO ₂	aug-cc-pVDZ	-522.03	75.4	-659.93	95.4	-663.46	95.9
	aug-cc-pVTZ	-630.88	91.2	-680.96	98.4	-682.56	98.6
	CBS	-691.93		-691.93		-691.93	
O ₃	aug-cc-pVDZ	-678.47	77.1	-844.86	96.0	-849.64	96.6
	aug-cc-pVTZ	-808.38	91.9	-866.16	98.4	-868.67	98.7
	CBS	-879.83		-879.83		-879.83	

TABLE V: The electronic reaction energies, $\Delta H_e^0(0K)$, (in kJ/mol) for the 12 test reactions¹.

	CCSD(T)		CCSD(T) _{RI2}				
	E	$\Delta_{\text{experimental}}^2$	E	$\Delta_{\text{experimental}}^2$	E	$\Delta_{\text{experimental}}^2$	
1	aug-cc-pVDZ ²	-18.20	3.6	-26.74	-4.9	-27.46	-5.7
	aug-cc-pVTZ ²	-22.62	-0.8	-24.37	-2.6	-24.26	-2.5
	experimental ¹	-21.8		-21.8		-21.8	
2	aug-cc-pVDZ ²	-166.06	-0.7	-182.75	-17.3	-183.11	-17.7
	aug-cc-pVTZ ²	-165.52	-0.1	-168.69	-3.3	-168.21	-2.8
	experimental ¹	-165.4		-165.4		-165.4	
3	aug-cc-pVDZ ²	-220.93	-17.0	-221.69	-17.8	-221.13	-17.2
	aug-cc-pVTZ ²	-206.94	-3.0	-206.05	-2.2	-206.06	-2.2
	experimental ¹	-203.9		-203.9		-203.9	
4	aug-cc-pVDZ ²	-261.00	-15.7	-276.27	-31.0	-276.76	-31.5
	aug-cc-pVTZ ²	-248.69	-3.4	-248.37	-3.1	-247.85	-2.5
	experimental ¹	-245.3		-245.3		-245.3	
5	aug-cc-pVDZ ²	-251.16	0.7	-260.30	-8.4	-260.66	-8.8
	aug-cc-pVTZ ²	-250.75	1.1	-251.56	0.3	-251.30	0.6
	experimental ¹	-251.9		-251.9		-251.9	
6	aug-cc-pVDZ ²	-269.36	4.4	-287.05	-13.2	-288.12	-14.3
	aug-cc-pVTZ ²	-273.37	0.4	-275.92	-2.1	-275.57	-1.8
	experimental ¹	-273.8		-273.8		-273.8	
7	aug-cc-pVDZ ²	-323.58	-3.3	-336.54	-16.2	-336.87	-16.6
	aug-cc-pVTZ ²	-322.22	-1.9	-322.97	-2.7	-322.73	-2.4
	experimental ¹	-320.3		-320.3		-320.3	
8	aug-cc-pVDZ ²	-438.17	7.4	-452.96	-7.4	-452.40	-6.8
	aug-cc-pVTZ ²	-442.92	2.7	-447.42	-1.8	-446.81	-1.2
	experimental ¹	-445.6		-445.6		-445.6	
9	aug-cc-pVDZ ²	-464.05	-17.4	-468.48	-21.8	-467.60	-20.9
	aug-cc-pVTZ ²	-451.03	-4.3	-449.51	-2.8	-449.41	-2.7
	experimental ¹	-446.7		-446.7		-446.7	
10	aug-cc-pVDZ ²	-526.86	17.3	-538.30	5.9	-538.68	5.5
	aug-cc-pVTZ ²	-538.26	5.9	-542.02	2.2	-541.85	2.3
	experimental ¹	-544.2		-544.2		-544.2	
11	aug-cc-pVDZ ²	-567.39	-2.5	-581.75	-16.9	-579.82	-14.9
	aug-cc-pVTZ ²	-560.45	4.4	-566.83	-1.9	-565.78	-0.9
	experimental ¹	-564.9		-564.9		-564.9	
12	aug-cc-pVDZ ²	-810.59	35.1	-829.80	15.9	-830.88	14.8
	aug-cc-pVTZ ²	-832.42	13.3	-840.59	5.1	-840.36	5.3
	experimental ¹	-845.7		-845.7		-845.7	

¹ The valence correlation energy plus the Hartree-Fock energy computed in the given basis set plus core correlation energy from Table V in the main text.

² The difference between the computed reaction energy and the experimental value.

TABLE VI: The electronic reaction energies, $\Delta H_e^0(0K)$, (in kJ/mol) for the 12 test reactions¹.

	CCSD(T)		CCSD(T) $_{\overline{RI2}}$			
	E	$\Delta_{\text{experimental}}^2$	E	$\Delta_{\text{experimental}}^2$	E	$\Delta_{\text{experimental}}^2$
1 aug-cc-pVDZ ²	-10.54	11.3	-19.09	2.7	-19.80	2.0
aug-cc-pVTZ ²	-20.90	0.9	-22.64	-0.8	-22.54	-0.7
experimental ¹	-21.8		-21.8		-21.8	
2 aug-cc-pVDZ ²	-145.49	19.9	-162.18	3.2	-162.54	2.9
aug-cc-pVTZ ²	-162.98	2.4	-166.16	-0.8	-165.67	-0.3
experimental ¹	-165.4		-165.4		-165.4	
3 aug-cc-pVDZ ²	-207.74	-3.8	-208.50	-4.6	-207.94	-4.0
aug-cc-pVTZ ²	-206.50	-2.6	-205.62	-1.7	-205.63	-1.7
experimental ¹	-203.9		-203.9		-203.9	
4 aug-cc-pVDZ ²	-228.79	16.5	-244.06	1.2	-244.55	0.8
aug-cc-pVTZ ²	-245.77	-0.5	-245.46	-0.2	-244.93	0.4
experimental ¹	-245.3		-245.3		-245.3	
5 aug-cc-pVDZ ²	-240.15	11.7	-249.29	2.6	-249.65	2.3
aug-cc-pVTZ ²	-250.01	1.9	-250.82	1.1	-250.56	1.3
experimental ¹	-251.9		-251.9		-251.9	
6 aug-cc-pVDZ ²	-250.69	23.1	-268.38	5.4	-269.45	4.4
aug-cc-pVTZ ²	-270.91	2.9	-273.46	0.3	-273.10	0.7
experimental ¹	-273.8		-273.8		-273.8	
7 aug-cc-pVDZ ²	-308.36	11.9	-321.32	-1.0	-321.65	-1.4
aug-cc-pVTZ ²	-320.97	-0.7	-321.73	-1.4	-321.48	-1.2
experimental ¹	-320.3		-320.3		-320.3	
8 aug-cc-pVDZ ²	-430.17	15.4	-444.97	0.6	-444.41	1.2
aug-cc-pVTZ ²	-442.35	3.2	-446.85	-1.3	-446.24	-0.6
experimental ¹	-445.6		-445.6		-445.6	
9 aug-cc-pVDZ ²	-446.45	0.2	-450.88	-4.2	-450.00	-3.3
aug-cc-pVTZ ²	-449.85	-3.2	-448.33	-1.6	-448.23	-1.5
experimental ¹	-446.7		-446.7		-446.7	
10 aug-cc-pVDZ ²	-529.56	14.6	-541.00	3.2	-541.38	2.8
aug-cc-pVTZ ²	-538.73	5.5	-542.49	1.7	-542.32	1.9
experimental ¹	-544.2		-544.2		-544.2	
11 aug-cc-pVDZ ²	-554.64	10.3	-569.01	-4.1	-567.07	-2.2
aug-cc-pVTZ ²	-560.83	4.1	-567.21	-2.3	-566.16	-1.3
experimental ¹	-564.9		-564.9		-564.9	
12 aug-cc-pVDZ ²	-820.40	25.3	-839.61	6.1	-840.69	5.0
aug-cc-pVTZ ²	-834.11	11.6	-842.27	3.4	-842.04	3.7
experimental ¹	-845.7		-845.7		-845.7	

[1] T. Helgaker, P. Jørgensen, and J. Olsen, *Modern Electronic Structure Theory* (Wiley, Chichester, 2000), 1st ed.

[2] D. W. Schwenke, J. Chem. Phys. **122**, 014107 (2005).

¹ The valence correlation energy in the given basis plus the CBS Hartree Fock energy plus the core-correlation energy, both found in Table V in the main text.

² The difference between the computed reaction energy and the corresponding experimental value.