Power Spectrum Density Method

Weierstrass-Mandelbrot (W-M) showed that many engineered nanosurfaces possess fractal geometry. Fractal dimensions are inversely proportional to the spatial frequency ω (equation 1),

$$\mathrm{P(w)} = 1/\,\omega\,(5 - 2\mathrm{D})\ln\gamma' \qquad (1)$$

where, γ' is the parameter that determines the density of frequencies in the profile. Ausloos and Burman generalized the W-M algorithm by introducing multiple variables to account for higher dimensionally stochastic processes and developed the following equation 2,

$$z(\rho,\theta) = (\ln\gamma' \div \mathrm{M})\sum_{m=1}^{M} A_m \sum_{n=-\infty}^{\infty} (k\gamma^n)^{(D-3)}\{\cos\phi_{m,n} - \cos[k\gamma^n \rho\cos(\theta - \alpha_m) + \phi_{m,n}]\} \qquad (2)$$

where, ρ and θ are planar coordinates of a surface point with height z and are related with planar Cartesian coordinates (x,y).

The power spectral density (denoted as p) of an isotropic 2-D fractal Brownian function which is denoted as f varies as:

$$Pf\,(x,y)\ \alpha\ p^{-\beta} \qquad (3)$$

$$\theta^{\ 1/2} = tan\ (y/x) \qquad (4)$$

where $\beta \geq 0$, and (x,y) are the planar Cartesian coordinates, and $p = (x^2 + y^2)^{1/2}$ is the radial frequency.

The height function z of a fractal surface, exhibiting randomness in all planar direction making the surface three dimensional, is the real part of the Ausloos-Berman function. D is the fractal dimension ($2>D>3$) and is a variant of the spatial frequencies. The parameter M denotes the number of superimposed ridges used to construct the surface. For instance, surfaces possessing cylindrical corrugations have $M = 1$. The anisotropy of the surface geometry is controlled by the magnitude of A_m. For example, $A_m = A$ is for isotropic surface for all m values and A_m varies with m for anisotropic surfaces. The arbitrary angle α_m is used to offset the ridges in the azimuthal direction and equals $\pi\, m/M$ when all ridges are equally offset. The parameter k is a wave number related to the sample size, $k = 2\pi /L$. The uniform distribution of the values of random phase $\phi_{m,n}$ in the interval $[0, \pi]$ can be generated using a random number generator.

The exponent β in equation 3 is related to the fractal dimension, D_f:

$$\beta = 8 - 2D_f \qquad (5)$$

Values of β and D_f are calculated from the slope of the linear part of a plot where the X-axis is expressed in linear measurement units that represent the spatial frequency or the wavelength of the fast Fourier transform (FFT) 2-D image and the Y-axis shows the relative intensity of the image components or PSD in arbitrary units.

The relationship between the fractal dimension and the assembly of nano-features on surfaces is relatively complicated for the 3-D features on the surface. In general, D_f values obtained using 2-D fractal analysis vary between 2 and 3, with D_f of 3 representing the densed or filled surface and D_f less than 3 represents departure from completely filled or dense structure to open or branch-like structures.