SUPPORTING INFORMATION

Appendix B. Electric potential due to the tessera itself

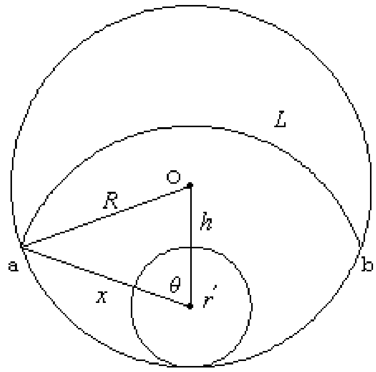

Figure A1 A depiction of the electric potential at $\mathbf{r}'$ within an circular disk R.

In this section we calculate the electric potential due to the surface element itself. The circular disk is depicted in Figure A1. Setting the circular disk with an area s and the radius $R = \sqrt{s/\pi}$. The surface charge q_i of tessera i can be approximately expressed as $q_i = s \cdot \sigma$ where σ is the isodensity of surface charge. For the calculation of the electric potential at position $\mathbf{r}'$, with distance h from the origin, generated by the other continuous charges in the same tessera, the circular disk should be divided into two domains, the one is enveloped by the small solid circle with radius $R-h$, denoted by D_1, and the other is labeled by D_2 which included all the circular disk except D_1. With the approximation of isodensity of surface charge, the potential contributed from D_1 can be easily obtained, i.e.,

$$\int_0^{R-h} \frac{2\pi x\sigma}{x} dx = 2\pi(R-h)\sigma \qquad \text{(B1)}$$

where x is the distance to $\mathbf{r}'$. In order to integrate the potential produced by D_2, we should obtain firstly the length of arc L which is the segment of dashing circle with a radius x from a to b in a clockwise direction. Making use of the cosine theorem, $x^2 + h^2 - 2xh\cos\theta = R^2$, the length of arc L is given by

$$L = 2\theta x = 2x\text{Arccos}[\frac{x^2 + h^2 - R^2}{2xh}] \qquad \text{(B2)}$$

Integrating on D_2, the result can be given as

$$\int_{R-h}^{R+h} \frac{\sigma L dx}{x} = \int_{R-h}^{R+h} 2\sigma \text{Arccos}[\frac{x^2 + h^2 - R^2}{2xh}] dx \tag{B3}$$

Combing eqns (B1) and (B3), the electric potential at position $\mathbf{r}'$ is given by

$$\varphi(\mathbf{r}') = \int_{R-h}^{R+h} 2\sigma \text{Arccos}[\frac{x^2 + h^2 - R^2}{2xh}] dx + 2\pi(R-h)\sigma \tag{B4}$$

Equation (B4) can not be integrated analytically, but the numerical integration shows that the trend of $\varphi(\mathbf{r}')$ varying with h exhibits a linear decrease property from $h = 0$ to $h = R$. Therefore, it is reasonable to approximate $\varphi(\mathbf{r}')$ with the linear interpolation method.

Application of eqn (B4) to the case of $h = 0$, the electric potential at the center of the tessera can be given as

$$\varphi(r_0') = 2\pi R\sigma = 2\sigma\sqrt{\pi s} \tag{B5}$$

The formula is identical with the result deduced from circular convex. The potential at the edge of circular disk, for the case of $h = R$, equals to,

$$\varphi(R) = 4R\sigma = 4\sigma\sqrt{s/\pi} \tag{B6}$$

Employing linear interpolation method, $\varphi(h)$ with h being the distance to can be written as

$$\varphi(\mathbf{r}') = (2\sqrt{\pi s} + \frac{4\sqrt{s/\pi} - 2\sqrt{\pi s}}{R} h)\sigma \tag{B7}$$

Combining the formulation $\varphi(\mathbf{r}')$ with $\sigma = q_i / s$ and integrating over $\mathbf{r}'$, we have

$$\int_i \frac{\sigma(\mathbf{r})\sigma(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r} d\mathbf{r}' = \int_i \varphi(\mathbf{r}')\sigma(\mathbf{r}') d\mathbf{r}' = \int_0^R \varphi(\mathbf{r}') 2\pi h\sigma \, \mathrm{d}h = \frac{q_i^2}{\sqrt{s}}[\frac{\sqrt{4\pi}}{3} + \frac{8}{3\sqrt{\pi}}] \tag{B8}$$