

# The chemical roots of the matching polynomial

Remi Chauvin,<sup>[a,b],\*</sup> Christine Lepetit,<sup>[a,b]</sup> Patrick W. Fowler,<sup>†</sup> Jean-Paul Malrieu.<sup>‡</sup>

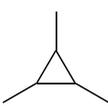
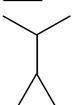
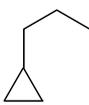
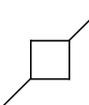
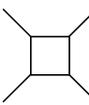
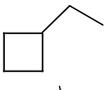
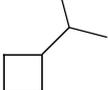
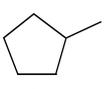
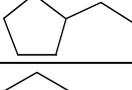
<sup>[a]</sup>CNRS ; LCC (Laboratoire de Chimie de Coordination) ; 205, route de Narbonne, F-31077  
Toulouse, France. Fax: (+33)5 61 55 30 03. E-mail: chauvin@lcc-toulouse.fr. <sup>[b]</sup>Université de  
Toulouse ; UPS, INPT ; LCC ; F-31077 Toulouse, France. <sup>†</sup>*Department of Chemistry, University of  
Sheffield, Sheffield S3 7HF, U.K., and* <sup>‡</sup>*Laboratoire de Chimie et Physique Quantiques IRSAMC,  
UMR 5626 CNRS, 118 route de Narbonne, 31062, Toulouse cedex 09, France.*

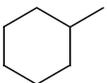
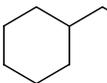
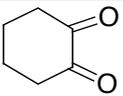
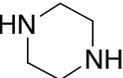
## Supplementary Information

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	$P^0(x)$	$P^1(x)$	$P^{ac}(x)$	TRE*
	$x^3 - 3x - 2$	$x^3 - 3x + 2$	$x^3 - 3x$	-0.464
	$x^4 - 4x^2 - 2x + 1$	$x^4 - 4x^2 + 2x + 1$	$x^4 - 4x^2 + 1$	-0.063
	$x^5 - 5x^3 - 2x^2 + 3x$	$x^5 - 5x^3 + 2x^2 + 3x$	$x^5 - 5x^3 + 3x$	0.023
	$x^6 - 6x^4 - 2x^3 + 6x^2 - 1$	$x^6 - 6x^4 + 2x^3 + 6x^2 - 1$	$x^6 - 6x^4 + 6x^2 - 1$	0.009
	$x^5 - 5x^3 - 2x^2 + 4x + 2$	$x^5 - 5x^3 + 2x^2 + 4x - 2$	$x^5 - 5x^3 + 4x$	-0.111
	$x^6 - 6x^4 - 2x^3 + 7x^2 + 4x$	$x^6 - 6x^4 + 2x^3 + 7x^2 - 4x$	$x^6 - 6x^4 + 7x^2$	0.471
	$x^6 - 6x^4 - 2x^3 + 8x^2 + 4x - 1$	$x^6 - 6x^4 + 2x^3 + 8x^2 - 4x - 1$	$x^6 - 6x^4 + 8x^2 - 1$	0.103
	$x^6 - 6x^4 - 2x^3 + 7x^2 + 2x - 1$	$x^6 - 6x^4 + 2x^3 + 7x^2 - 2x - 1$	$x^6 - 6x^4 + 7x^2 - 1$	0.045
	$x^4 - 4x^2$	$x^4 - 4x^2 + 4$	$x^4 - 4x^2 + 2$	-1.226
	$x^5 - 5x^3 + 2x$	$x^5 - 5x^3 + 6x$	$x^5 - 5x^3 + 4x$	-0.404
	$x^6 - 6x^4 + 5x^2 - 1$	$x^6 - 6x^4 + 9x^2 - 1$	$x^6 - 6x^4 + 7x^2 - 1$	-0.163
	$x^6 - 6x^4 + 5x^2$	$x^6 - 6x^4 + 9x^2$	$x^6 - 6x^4 + 7x^2$	-0.248
	$x^8 - 8x^6 + 14x^4 - 8x^2 + 1$	$x^8 - 8x^6 + 18x^4 - 8x^2 + 1$	$x^8 - 8x^6 + 16x^4 - 8x^2 + 1$	-0.072
	$x^6 - 6x^4 + 6x^2$	$x^6 - 6x^4 + 10x^2 - 4$	$x^6 - 6x^4 + 8x^2 - 2$	-1.060
	$x^7 - 7x^5 + 10x^3$	$x^7 - 7x^5 + 14x^3 - 8x$	$x^7 - 7x^5 + 12x^3 - 4x$	-1.124
	$x^5 - 5x^3 + 5x - 2$	$x^5 - 5x^3 + 5x + 2$	$x^5 - 5x^3 + 5x$	-0.301
	$x^6 - 6x^4 + 8x^2 - 2x - 1$	$x^6 - 6x^4 + 8x^2 + 2x - 1$	$x^6 - 6x^4 + 8x^2 - 1$	0.020
	$x^7 - 7x^5 + 13x^3 - 2x^2 - 6x + 2$	$x^7 - 7x^5 + 13x^3 + 2x^2 - 6x - 2$	$x^7 - 7x^5 + 13x^3 - 6x$	-0.150
	$x^6 - 6x^4 + 9x^2 - 4$	$x^6 - 6x^4 + 9x^2$	$x^6 - 6x^4 + 9x^2 - 2$	0.273

	$x^7 - 7x^5 + 13x^3 - 7x$	$x^7 - 7x^5 + 13x^3 - 3x$	$x^7 - 7x^5 + 13x^3 - 5x$	0.155
	$x^8 - 8x^6 + 19x^4 - 16x^2 + 4$	$x^8 - 8x^6 + 19x^4 - 12x^2$	$x^8 - 8x^6 + 19x^4 - 14x^3 + 2$	0.249
	$x^8 - 0.44x^7 - 7.912x^6 + 3.071x^5 + 17.511x^4 - 5.685x^3 - 12.327x^2 + 3.054x + 0.767$	$x^8 - 0.44x^7 - 7.912x^6 + 3.071x^5 + 17.511x^4 - 5.685x^3 - 8.327x^2 + 1.293x + 0.961$	$x^8 - 0.44x^7 - 7.912x^6 + 3.071x^5 + 17.511x^4 - 5.685x^3 - 10.327x^2 + 2.174x + 0.864$	0.053
	$x^6 - 0.76x^5 - 3.816x^4 + 2.265x^3 + 3.372x - 1.117x - 0.96$	$x^6 - 0.76x^5 - 3.816x^4 + 2.265x^3 + 3.372x - 1.117x$	$x^6 - 0.76x^5 - 3.815x^4 + 2.267x^3 + 3.370x - 1.118x - 0.48$	0.193
	$x^5 - 1.88x^4 - 3.03x^3 + 3.966x^2 - 1.92x - 1.837$	$x^5 - 1.88x^4 - 3.03x^3 + 3.966x^2 + 1.92x - 0.249 - 1.837$	$x^5 - 1.88x^4 - 3.03x^3 + 3.964x^2 - 1.92x - 1.043$	0.196
	$x^6 - 3x^5 - 2.99x^4 + 10.86x^3 + 2.364x^2 - 7.86x - 0.374$	$x^6 - 3x^5 - 2.99x^4 + 10.86x^3 + 2.364x^2 - 7.86x + 2.25$	$x^6 - 3x^5 - 2.99x^4 + 10.86x^3 + 2.364x^2 - 7.86x - 0.938$	-0.132

**Table S1.** Characteristic and acyclic polynomials of some Hückel and Möbius unicycles with ring size 3-6. The *o*-benzoquinone,<sup>1</sup> 1,4-dihydropyrazine,<sup>2</sup> 1,3-pyrimidine and imidazole graphs are here weighted by the Hess-Schaad parameters:  $h_{O\bullet} = 0.22$ ,  $k_{C-O\bullet} = 0.99$ ,<sup>3</sup>  $h_{N\bullet} = 0.38$ ,  $k_{C-N\bullet} = 0.70$ ,<sup>4</sup>  $h_{N\bullet} = 1.50$ ,  $k_{C-N\bullet} = 0.90$ .<sup>5</sup> \* TRE values for the neutral ground state (in  $\beta$  units).

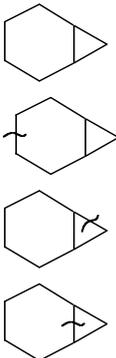
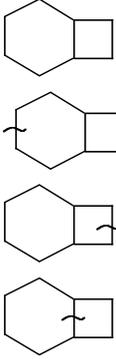
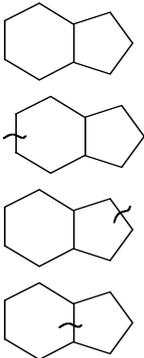
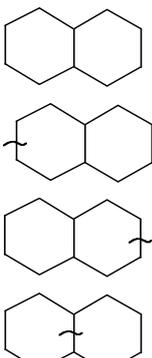
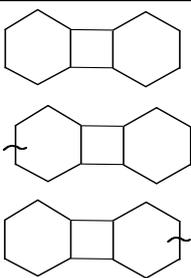
<sup>1</sup> J. Aihara, *J. Am. Chem. Soc.*, 1976, **98**, 2750-2758.

<sup>2</sup> J. Aihara, H. Ichikawa, *Bull. Chem. Soc. Jpn.*, 1988, **61**, 223-228.

<sup>3</sup> B. A. Hess, Jr, L. J. Schaad, C. W. Holyoke, Jr. *Tetrahedron*, 1972, **28**, 5299.

<sup>4</sup> B. A. Hess, Jr, L. J. Schaad, C. W. Holyoke, Jr. *Tetrahedron*, 1975, **31**, 295.

<sup>5</sup> B. A. Hess, Jr, L. J. Schaad, C. W. Holyoke, Jr. *Tetrahedron*, 1972, **28**, 3657.

	Hückel and Möbius characteristic polynomials	Acyclic polynomial
	$P^0(x) = x^7 - 8x^5 - 2x^4 + 17x^3 + 6x^2 - 10x - 4$ $P^1(x) = x^7 - 8x^5 - 2x^4 + 17x^3 + 6x^2 - 6x$ $P^2(x) = x^7 - 8x^5 + 2x^4 + 17x^3 - 6x^2 - 10x + 4$ $P^3(x) = x^7 - 8x^5 + 2x^4 + 17x^3 - 6x^2 - 6x$	$P^{ac}(x) = x^7 - 8x^5 + 17x^3 - 8x$
	$P^0(x) = x^8 - 9x^6 + 22x^4 - 16x^2 + 1$ $P^1(x) = x^8 - 9x^6 + 22x^4 - 12x^2 + 1$ $P^2(x) = x^8 - 9x^6 + 26x^4 - 28x^2 + 9$ $P^3(x) = x^8 - 9x^6 + 26x^4 - 24x^2 + 1$	$P^{ac}(x) = x^8 - 9x^6 + 24x^4 - 20x^2 + 3$
	$P^0(x) = x^9 - 10x^7 + 32x^5 - 2x^4 - 39x^3 + 6x^2 + 15x - 4$ $P^1(x) = x^9 - 10x^7 + 32x^5 - 2x^4 - 35x^3 + 6x^2 + 7x$ $P^2(x) = x^9 - 10x^7 + 32x^5 + 2x^4 - 39x^3 - 6x^2 + 15x + 4$ $P^3(x) = x^9 - 10x^7 + 32x^5 + 2x^4 - 35x^3 - 6x^2 + 7x$	$P^{ac}(x) = x^9 - 10x^7 + 32x^5 - 37x^3 + 11x$
	$P^0(x) = x^{10} - 11x^8 + 41x^6 - 65x^4 + 43x^2 - 9$ $P^1(x) = x^{10} - 11x^8 + 41x^6 - 61x^4 + 31x^2 - 1$ $P^2(x) = x^{10} - 11x^8 + 41x^6 - 61x^4 + 31x^2 - 1$ $P^3(x) = x^{10} - 11x^8 + 41x^6 - 57x^4 + 19x^2 - 1$	$P^{ac}(x) = x^{10} - 11x^8 + 41x^6 - 61x^4 + 31x^2 - 3$
	$P^0(x) = x^{12} - 14x^{10} + 69x^8 - 154x^6 + 162x^4 - 72x^2 + 9$ $P^1(x) = x^{12} - 14x^{10} + 69x^8 - 150x^6 + 142x^4 - 48x^2 + 1$ $P^2(x) = x^{12} - 14x^{10} + 69x^8 - 150x^6 + 142x^4 - 48x^2 + 1$	$P^{ac}(x) = x^{12} - 14x^{10} + 71x^8 - 162x^6 + 164x^4 - 60x^2 + 5$

	$P^3(x) = x^{12} - 14x^{10} + 73x^8 - 178x^6 + 214x^4 - 120x^2 + 25$ $P^4(x) = x^{12} - 14x^{10} + 73x^8 - 174x^6 + 186x^4 - 72x^2 + 1$ $P^5(x) = x^{12} - 14x^{10} + 73x^8 - 174x^6 + 186x^4 - 72x^2 + 1$ $P^6(x) = x^{12} - 14x^{10} + 69x^8 - 146x^6 + 122x^4 - 24x^2 + 1$ $P^7(x) = x^{12} - 14x^{10} + 73x^8 - 170x^6 + 158x^4 - 24x^2 + 1$	
	$P^0(x) = x^9 - 1.5x^8 - 9.62x^7 + 12x^6 + 29.34x^5 - 28.619x^4 - 34.059x^3 + 24.360x^2 + 12.339x - 4.74$ $P^1(x) = x^9 - 1.5x^8 - 9.62x^7 + 12x^6 + 29.34x^5 - 28.618x^4 - 30.059x^3 + 18.359x^2 + 5.859x - 1.5$ $P^2(x) = x^9 - 1.5x^8 - 9.62x^7 + 12x^6 + 29.341x^5 - 25.379x^4 - 34.061x^3 + 14.639x^2 + 12.34x + 1.74$ $P^3(x) = x^9 - 1.50x^8 - 9.62x^6 + 12x^6 + 29.338x^5 - 25.381x^4 - 30.057x^3 + 8.640x^2 + 5.859x - 1.5$	$P^{ac}(x) = x^9 - 1.5x^8 - 9.62x^7 + 12x^6 + 29.34x^5 - 27x^4 - 32.06x^3 + 16.5x^2 + 9.10x - 1.5$

**Table S2.** Characteristic and acyclic polynomials of some Hückel and Möbius polycyclic systems. Each Möbius circuit is marked by a tilde: the edge where it is located is weighted by a factor  $-1$ , while all other edges are non-weighted. The isoindole graph is weighted by the Hess-Schaad Hückel parameters:  $h_{N_i} = 1.50$ ,  $k_{C-N_i} = 0.90$ .<sup>4</sup>