

Electronic Supplementary Information
**Mechanically Activated Rupture of Single Covalent Bonds:
Evidence of Force Induced Bond Hydrolysis**

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Data analysis

Theoretical Background

To extract the structural and kinetic parameters of the investigated binding potential, i.e., the bond dissociation energy D_e , the maximum rupture force f_{\max} , and the Arrhenius A -factor, the experimental results were analyzed with a theoretical model based on the combination of the Arrhenius rate equation and a Morse potential.

If a Morse potential $V(x) = D_e(1 - \exp(-\beta x))^2$ is chosen as a representation of the binding potential, the effective potential under mechanical load becomes:

$$V_{\text{eff}}(x) = D_e(1 - \exp(-\beta x))^2 - fx, \quad (1)$$

where D_e is the dissociation energy, $1/\beta$ is the relative width of the potential and fx is the mechanical energy provided by the force transducer. The energy barrier of this effective potential decreases with increasing force, and the barrier height becomes ^{1,2}

$$E_a(f) = D_e \left\{ \sqrt{1 - f/f_{\max}} - f/f_{\max} \cdot \coth\left(\sqrt{1 - f/f_{\max}}\right) \right\} - \frac{\alpha}{2} h\nu \quad (2a)$$

$$\text{with } f_{\max} = V'_{\max} = \frac{1}{2} D_e \beta, \quad (2b)$$

where f_{\max} is the maximum slope of the potential and the term $\frac{\alpha}{2} h\nu$ accounts for the contribution of the zero-point energy.

Inserting equation (2a) into the Arrhenius equation, one can calculate the probability P that a given bond is still intact at time t by solving the first order rate equation

$$\frac{dP}{dt} = -k^{\text{off}} \cdot P = -A \cdot \exp[-E_a(f)/(k_B T)] \cdot P. \quad (3)$$

Assuming that the force increases nearly linearly in time, one can replace t with f

$$\frac{dP}{df} = \frac{-A}{df/dt} \cdot \exp[-E_a(f)/(k_B T)] \cdot P, \quad (4)$$

where df/dt is the rate at which the force is increased. Integration of (4) yields the probability that the bond is still intact when the force f is reached

$$P(f) = \exp\left\{ \frac{A}{df/dt} \cdot \int_{f_0}^f \exp[-E_a(\hat{f})/(k_B T)] \cdot d\hat{f} \right\}. \quad (5)$$

The probability that the bond is already broken at force f is therefore

$$P_r(f) = 1 - P(f) = 1 - \exp\left\{ \frac{A}{df/dt} \cdot \int_{f_0}^f \exp[-E_a(\hat{f})/(k_B T)] \cdot d\hat{f} \right\}, \quad (6)$$

and the rupture force distribution becomes

$$p_r(f) = dP_r(f)/df \quad (7)$$

Because equation (6) cannot be solved analytically, we have solved it by numerical integration and used the result to calculate $p_r(f)$ for the different force-loading rates used in the experiment.

Fitting Procedure

To determine the structural parameters D_e and f_{\max} as well as the Arrhenius A -factor of the Morse potential, we used the maximum likelihood parameter estimation (MLE) procedure. MLE is a method used for fitting a statistical model to data in order to provide estimates for the model's parameters.³ From a statistical standpoint, the rupture force data vector (f_1, f_2, \dots) is a random sample from an unknown population. The goal of data analysis is to identify the population that is most likely to have generated the sample. In statistics, each population is identified by a corresponding probability distribution. If a model for the statistical process is available, there exists a probability distribution p with a certain model parameter w . Our model for the rupture process has parameters for the interatomic interaction as well as temperature and force-loading rate. The model probability to observe the data vector is given from Eq. (7) by $p_r(f_1, f_2, \dots | w)$. As the parameter changes in value, different probability distributions are generated. Formally, a model is defined as the family of probability distributions indexed by the model's parameters. In our model the parameters temperature and force-loading rate are known, the parameters for the interatomic interaction are unknown and therefore represent the estimated values. As a consequence, the parameters temperature and force-loading rate can be treated as data. The model probability to observe the data is therefore $p_r(\{f_1, \dot{f}_1, T_1\}, \{f_2, \dot{f}_2, T_2\}, \dots | w)$ and $w = \{D_e, f_{\max}, A\}$ represents the parameter vector to be estimated. Because the rupture events are statistically independent of one another, probability can be expressed as a product of probabilities for individual observations

$$p_r(\{f_1, \dot{f}_1, T_1\}, \{f_2, \dot{f}_2, T_2\} | w) = p_r(\{f_1, \dot{f}_1, T_1\} | w) \cdot p_r(\{f_2, \dot{f}_2, T_2\} | w)$$

The likelihood function is then given by the product of the individual probabilities and can be written as

$$L(w | \{f_1, \dot{f}_1, T_1\}, \{f_2, \dot{f}_2, T_2\}, \dots) = \prod_{i=1, N} p_r(\{f_i, \dot{f}_i, T_i\} | w) \quad (8)$$

The principle of MLE states that the desired probability distribution is the one that makes the observed data "most likely", which means that one must seek the value of the parameter vector that maximizes the likelihood function. The resulting parameter vector, which is sought by searching the multi-dimensional parameter space, is called the MLE, and is denoted by

$w_{MLE} = \{D_e^{(MLE)}, f_{\max}^{(MLE)}, A^{(MLE)}\}$. Usually, the optimization algorithm is performed with the natural logarithm of the likelihood function, the log-likelihood function, because of the better numerical stability:

$$\log L(w | \{f_1, \dot{f}_1, T_1\}, \{f_2, \dot{f}_2, T_2\}, \dots) = \sum_{i=1, N} \log p_r(\{f_i, \dot{f}_i, T_i\} | w) \quad (9)$$

We have implemented the bond rupture probability distribution given in Eq. (7) as well as the log-likelihood function (Eq. (9)) in Visual Basic. Care was taken to ensure the proper normalization of the distribution. A renormalization was necessary in cases, where the experimental conditions constrained the possible values: because rupture forces below a threshold of 0.5 nN were not included in the data set, the probability distribution in Eq. (7) was multiplied with a resolution function with value of zero below the threshold and one above the threshold. Thereafter the distribution was renormalized such that the integrated probability had again a unit value.

A great advantage of the MLE is that different data sets can be evaluated simultaneously. In our analysis we have evaluated data with different force-loading rates and different temperatures simultaneously, since the effect of these conditions is contained in Eq. (9) parametrically. For the parameter estimation the log-likelihood function was evaluated for the data and the maximum value was searched using the gradient based optimization algorithm implemented in Excel 2003. The MLE estimate corresponds to the maximum of the log-likelihood function in the multidimensional parameter space.

To analyze the experimental data, we have assumed that the interatomic forces can be described with a Morse potential (Eq. (1)) together with an Arrhenius pre-factor A for the bond rupture rate, which may be dependent on the temperature (directly or indirectly due to solvent effects). Therefore we allowed the values of A to vary independently for the three data subsets with different temperatures, but treated the interatomic parameters D_e and f_{\max} as global parameters.

All parameter sets for these five values are summarized in Table 1 together with the corresponding value of the log-likelihood function $\log L$ (Eq. (9)) depending on the factor for the included zero-point energy correction.

Table 1. Structural parameters D_e, f_{\max} together with the obtained temperature-dependent pre-factors A calculated by fitting an Arrhenius kinetics model combined with a Morse potential using the MLE method.

α^a	$\log L^b$	$D_{e,\text{global}}$ [kJ mol ⁻¹]	$f_{\max,\text{global}}$ [nN]	$A_{282\text{K}}$ [s ⁻¹]	$A_{293\text{K}}$ [s ⁻¹]	$A_{320\text{K}}$ [s ⁻¹]
0.0	17720,7971	34.0	3.46	1.36×10^3	5.29×10^3	4.78×10^3
0.2	17720,9680	35.5	3.55	9.02×10^2	3.55×10^3	3.31×10^3
0.4	17720,1934	37.4	3.69	8.82×10^2	3.52×10^3	3.29×10^3
0.6	17718,4624	39.8	4.43	8.20×10^2	3.31×10^3	3.12×10^3
0.8	17716,2214	43.0	5.08	8.03×10^2	3.28×10^3	3.10×10^3
1.0	17713,6910	47.3	6.02	8.42×10^2	3.46×10^3	3.27×10^3

^a Factor for zero-point energy correction, ^b Value of the log-likelihood function calculated from Eq. (9)

Note that for $\alpha = 1$, the zero-point energy contribution $\frac{\alpha}{2} h\nu$ includes only to the ground state oscillation of the one-dimensional Morse potential. The proper zero point contribution consists of the difference between all zero mode contributions from the bound state and the transition state. They are accessible with a vibrational analysis of the bound state and the transition state under applied force conditions and are not known here. We therefore take $h\nu$ as the order of magnitude of the effect and treat α as a fit parameter.

References

1. M. K. Beyer, *Journal of Chemical Physics*, 2000, **112**, 7307-7312.
2. F. Hanke and H. J. Kreuzer, *Physical Review E*, 2006, **74**, 031909.
3. S. Brandt, *Data Analysis: Statistical and Computational Methods for Scientists and Engineers*, Springer Verlag, Berlin, 1998.