# Electronic Supplementary Information: Molecular dynamics study on helium nanobubbles in water 

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## 1 The constant energy simulations

We performed the constant energy simulation after the constant temperature simulation for each of the nanobubble systems with bubble radii of about 2,3 , and 5 nm . Figure 1 shows the energy, temperature, and pressure during the constant energy simulation.


Figure 1: (a) The energy, (b) temperature, and (c) pressure of the nanobubble system during the constant energy simulation.

## 2 Computational details

We calculated the normal pressure profile of the nanobubbles using the Irving-Kirkwood method and the Thompson's algorithm. [1, 2] This algorithm can be used for the pair potential only. We decomposed the forces coming from the each angle term to two pair-forces, and used shifted Coulomb potential with the same cutoff as that of the Lennard-Jones potential. Figure 2 shows that the kinetic, bonded, and non-bonded contributions to the normal pressure profiles.


Figure 2: Components of pressure profile for the bubbles with the initial radii of (a) 2 nm , (b) 3 nm , and (c) 5 nm , respectively.

Surface tensions of bubble $\gamma$ were calculated using the formula,

$$
\begin{equation*}
\gamma^{3}=\frac{3}{8}\left(p_{\text {in }}-p_{\text {out }}\right)^{2} \int_{0}^{R_{\infty}} r^{2}\left(P_{\mathrm{N}}(r)-p_{\text {out }}\right) d r . \tag{1}
\end{equation*}
$$

The $p_{\text {in }}$ and $p_{\text {out }}$ are the inner and outer pressures, respectively, which were determined by fitting the normal pressure profile $P_{\mathrm{N}}(r)$ to the hyperbolic tangent function,

$$
\begin{equation*}
P(r)=\frac{p_{\text {in }}+p_{\text {out }}}{2}-\frac{p_{\text {in }}-p_{\text {out }}}{2} \tanh \left(2\left(r-R_{p}\right) / \xi_{p}\right) . \tag{2}
\end{equation*}
$$

$R_{\infty}$ must be chosen as $P_{\mathrm{N}}(r)=p_{\text {out }}$ for $r>R_{\infty}$. In practice, we used a $R_{\infty}$ of 10 nm for all cases. The fitted parameters for the nanobubbles are shown in Table 1.

Table 1: $R_{\text {ini }}$ is the initial radius of bubble. $p_{\text {in }}$ and $p_{\text {out }}$ are the inner and outer pressures, respectively. $\xi_{p}$ is the interfacial width of pressure.

| $R_{\text {ini }}(\mathrm{nm})$ | 2 | 3 | 5 |
| :--- | :---: | :---: | :---: |
| $p_{\text {in }}(\mathrm{MPa})$ | 61.248 | 38.650 | 24.130 |
| $p_{\text {out }}(\mathrm{MPa})$ | -1.637 | -0.762 | -1.804 |
| $\xi_{p}(\mathrm{~nm})$ | 0.634 | 0.293 | 0.274 |

The surface of tension was calculated by the Laplace equation,

$$
\begin{equation*}
R_{\mathrm{s}}=\frac{2 \gamma}{p_{\text {in }}-p_{\text {out }}} \tag{3}
\end{equation*}
$$

with the determined surface tension $\gamma$.
We also calculated the surface tension for a flat water surface using

$$
\begin{equation*}
\gamma_{\infty}=\frac{L_{z}}{2}\left[\left\langle P_{z z}\right\rangle-\frac{1}{2}\left(\left\langle P_{x x}\right\rangle+\left\langle P_{y y}\right\rangle\right)\right], \tag{4}
\end{equation*}
$$

where $L_{z}$ is the cell length in z-direction, and $P_{\alpha \beta}$ is a component of the stress tensor. [3] The $\langle\ldots\rangle$ refer to time average.

## 3 Additional figures

Figure 3 shows the dipole and quadropole densities around the nanobubble interface. Figure 4 shows the electrostatic potential around the nanobubble interface and its dipole and quadropole contributions.


Figure 3: (a) Dipole and (b) quadropole moment densities for each of bubbles with the initial radii of 2 nm (red), 3 nm (green), and 5 nm (bule). Those for the flat water surface are also shown by violet lines.


Figure 4: (a) Electrostatic potential and (b) its quadropole and (c) dipole contributions for each of bubbles with the initial radii of 2 nm (red), 3 nm (green), and 5 nm (bule). Those for the flat water surface are also shown by violet lines.

## References

[1] J. H. Irving and John G. Kirkwood. The statistical mechanical theory of transport processes. iv. the equations of hydrodynamics. J. Chem. Phys., 18:817-829, 1950.
[2] S. M. Thompson, K. E. Gubbins, J. P. R. B. Walton, R. A. R. Chantry, and J. S. Rowlinson. J. Chem. Phys., 81:530-542, 1984.

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[3] David Brown and Sylvie Neyertz. A general pressure tensor calculation for molecular dynamics simulations. Mol. Phys., 84:577-595, 1995.

