# Electronic Supplementary Information: Molecular dynamics study on helium nanobubbles in water

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### 1 The constant energy simulations

We performed the constant energy simulation after the constant temperature simulation for each of the nanobubble systems with bubble radii of about 2, 3, and 5 nm. Figure 1 shows the energy, temperature, and pressure during the constant energy simulation.



Figure 1: (a) The energy, (b) temperature, and (c) pressure of the nanobubble system during the constant energy simulation.

### 2 Computational details

We calculated the normal pressure profile of the nanobubbles using the Irving-Kirkwood method and the Thompson's algorithm. [1, 2] This algorithm can be used for the pair potential only. We decomposed the forces coming from the each angle term to two pair-forces, and used shifted Coulomb potential with the same cutoff as that of the Lennard-Jones potential. Figure 2 shows that the kinetic, bonded, and non-bonded contributions to the normal pressure profiles.



Figure 2: Components of pressure profile for the bubbles with the initial radii of (a) 2 nm, (b) 3 nm, and (c) 5 nm, respectively.

Surface tensions of bubble  $\gamma$  were calculated using the formula,

$$\gamma^3 = \frac{3}{8} (p_{\rm in} - p_{\rm out})^2 \int_0^{R_\infty} r^2 \left( P_{\rm N}(r) - p_{\rm out} \right) dr.$$
(1)

The  $p_{\rm in}$  and  $p_{\rm out}$  are the inner and outer pressures, respectively, which were determined by fitting the normal pressure profile  $P_{\rm N}(r)$  to the hyperbolic tangent function,

$$P(r) = \frac{p_{\rm in} + p_{\rm out}}{2} - \frac{p_{\rm in} - p_{\rm out}}{2} \tanh\left(2(r - R_p)/\xi_p\right).$$
(2)

 $R_{\infty}$  must be chosen as  $P_{\rm N}(r) = p_{\rm out}$  for  $r > R_{\infty}$ . In practice, we used a  $R_{\infty}$  of 10 nm for all cases. The fitted parameters for the nanobubbles are shown in Table 1.

Table 1:  $R_{\text{ini}}$  is the initial radius of bubble.  $p_{\text{in}}$  and  $p_{\text{out}}$  are the inner and outer pressures, respectively.  $\xi_p$  is the interfacial width of pressure.

$R_{\rm ini} \ ({\rm nm})$	2	3	5
$p_{\rm in}$ (MPa)	61.248	38.650	24.130
$p_{\rm out}$ (MPa)	-1.637	-0.762	-1.804
$\xi_p \ (\mathrm{nm})$	0.634	0.293	0.274

The surface of tension was calculated by the Laplace equation,

$$R_{\rm s} = \frac{2\gamma}{p_{\rm in} - p_{\rm out}},\tag{3}$$

with the determined surface tension  $\gamma$ .

We also calculated the surface tension for a flat water surface using

$$\gamma_{\infty} = \frac{L_z}{2} \left[ \langle P_{zz} \rangle - \frac{1}{2} \left( \langle P_{xx} \rangle + \langle P_{yy} \rangle \right) \right], \tag{4}$$

where  $L_z$  is the cell length in z-direction, and  $P_{\alpha\beta}$  is a component of the stress tensor. [3] The  $\langle \ldots \rangle$  refer to time average.

## 3 Additional figures

Figure 3 shows the dipole and quadropole densities around the nanobubble interface. Figure 4 shows the electrostatic potential around the nanobubble interface and its dipole and quadropole contributions.



Figure 3: (a) Dipole and (b) quadropole moment densities for each of bubbles with the initial radii of 2 nm (red), 3 nm (green), and 5 nm (bule). Those for the flat water surface are also shown by violet lines.



Figure 4: (a) Electrostatic potential and (b) its quadropole and (c) dipole contributions for each of bubbles with the initial radii of 2 nm (red), 3 nm (green), and 5 nm (bule). Those for the flat water surface are also shown by violet lines.

#### References

- J. H. Irving and John G. Kirkwood. The statistical mechanical theory of transport processes. iv. the equations of hydrodynamics. J. Chem. Phys., 18:817–829, 1950.
- [2] S. M. Thompson, K. E. Gubbins, J. P. R. B. Walton, R. A. R. Chantry, and J. S. Rowlinson. J. Chem. Phys., 81:530–542, 1984.

[3] David Brown and Sylvie Neyertz. A general pressure tensor calculation for molecular dynamics simulations. *Mol. Phys.*, 84:577 – 595, 1995.