

## $sp^3$ -bonded silicon allotropes based on the Kelvin problem

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### 1. Structural information for Kelvin Silicons

Table S1 Crystallographic data for Kelvin Silicons at 0 GPa.

$K^I$ Space group $Im\bar{3}m$ $a = b = c = 6.7348 \text{ \AA}$				
Atomic coordinates				
Si	12d	0.25000	0.00000	0.50000
$K^{II}$ Space group $Pm\bar{3}n$ $a = b = c = 10.2350 \text{ \AA}$				
Atomic coordinates				
Si1	16i	0.18370	0.18370	0.18370
Si2	24k	0.50000	0.80767	0.61714
Si3	6d	0.25000	0.50000	0.00000
$K^{III}$ Space group $C2/c$ $a = 25.4751 \text{ \AA}$ , $b = 15.2199 \text{ \AA}$ , $c = 9.8931 \text{ \AA}$				
Atomic coordinates				
Si1	8f	0.05400	0.59918	0.60088
Si2	8f	0.13876	0.53982	0.60624
Si3	8f	0.13356	0.40954	0.73362
Si4	8f	0.16893	0.28447	0.61924
Si5	8f	0.94914	0.39437	0.64040
Si6	8f	0.06563	0.11076	0.49278
Si7	8f	0.10566	0.17538	0.67808
Si8	8f	0.97505	0.15188	0.49502
Si9	8f	0.03689	0.24295	0.81261
Si10	8f	0.73340	0.38206	0.59602
Si11	8f	0.81136	0.46341	0.60383
Si12	8f	0.19369	0.59274	0.76670
Si13	8f	0.17645	0.72646	0.87847
Si14	8f	0.75572	0.26910	0.75327
Si15	8f	0.27309	0.63291	0.64193
Si16	8f	0.04053	0.74752	0.68228
Si17	8f	0.10809	0.81609	0.80305
Si18	8f	0.59132	0.45966	0.49777
Si19	8f	0.64846	0.43387	0.67450
Si20	8f	0.64715	0.56654	0.80097
Si21	4e	0.00000	0.50187	0.75000

$K^{IV}$ Space group $I\bar{4}3m$ $a = b = c = 12.4874 \text{ \AA}$				
Atomic coordinates				
Si1	48h	0.63106	0.76376	0.02279
Si2	8c	0.39135	0.60865	0.39135
Si3	2a	0.00000	0.00000	0.00000
Si4	24g	0.70878	0.93204	0.06796

$K^V$ Space group $Cmmm$ $a = 17.6979 \text{ \AA}$ , $b = 10.2316 \text{ \AA}$ $c = 10.3782 \text{ \AA}$				
Atomic coordinates				
Si1	16r	0.10476	0.81429	0.68774
Si2	16r	0.80974	0.69027	0.81308
Si3	8n	0.00000	0.88065	0.81309
Si4	4h	0.86770	0.50000	0.50000
Si5	8o	0.79075	0.50000	0.68778
Si6	8o	0.66677	0.50000	0.61237
Si7	8q	0.06628	0.69894	0.50000
Si8	4i	0.00000	0.74243	0.00000
Si9	8p	0.87888	0.62114	0.00000

$K^{VI}$ Space group $Fd\bar{3}m$ $a = b = c = 25.9753 \text{ \AA}$				
Atomic coordinates				
Si1	192i	0.82200	0.38630	0.50908
Si2	192i	0.81466	0.74944	0.62069
Si3	192i	0.30632	0.49257	0.37073
Si4	192i	0.85952	0.53257	0.46743
Si5	96g	0.30267	0.30267	0.19733
Si6	32e	0.25000	0.25000	0.25000

$K^{VII}$ Space group $P6_3/mmc$ $a = b = 10.4154 \text{ \AA}$ , $c = 17.0590 \text{ \AA}$				
Atomic coordinates				
Si1	24l	0.04984	0.33042	0.06454
Si2	12k	0.08433	0.54216	0.36473
Si3	12k	0.12352	0.87648	0.86128
Si4	4f	0.33333	0.66667	0.31962
Si5	4e	0.00000	0.00000	0.31818
Si6	6h	0.06939	0.53469	0.75000
Si7	6h	0.20224	0.79776	0.75000

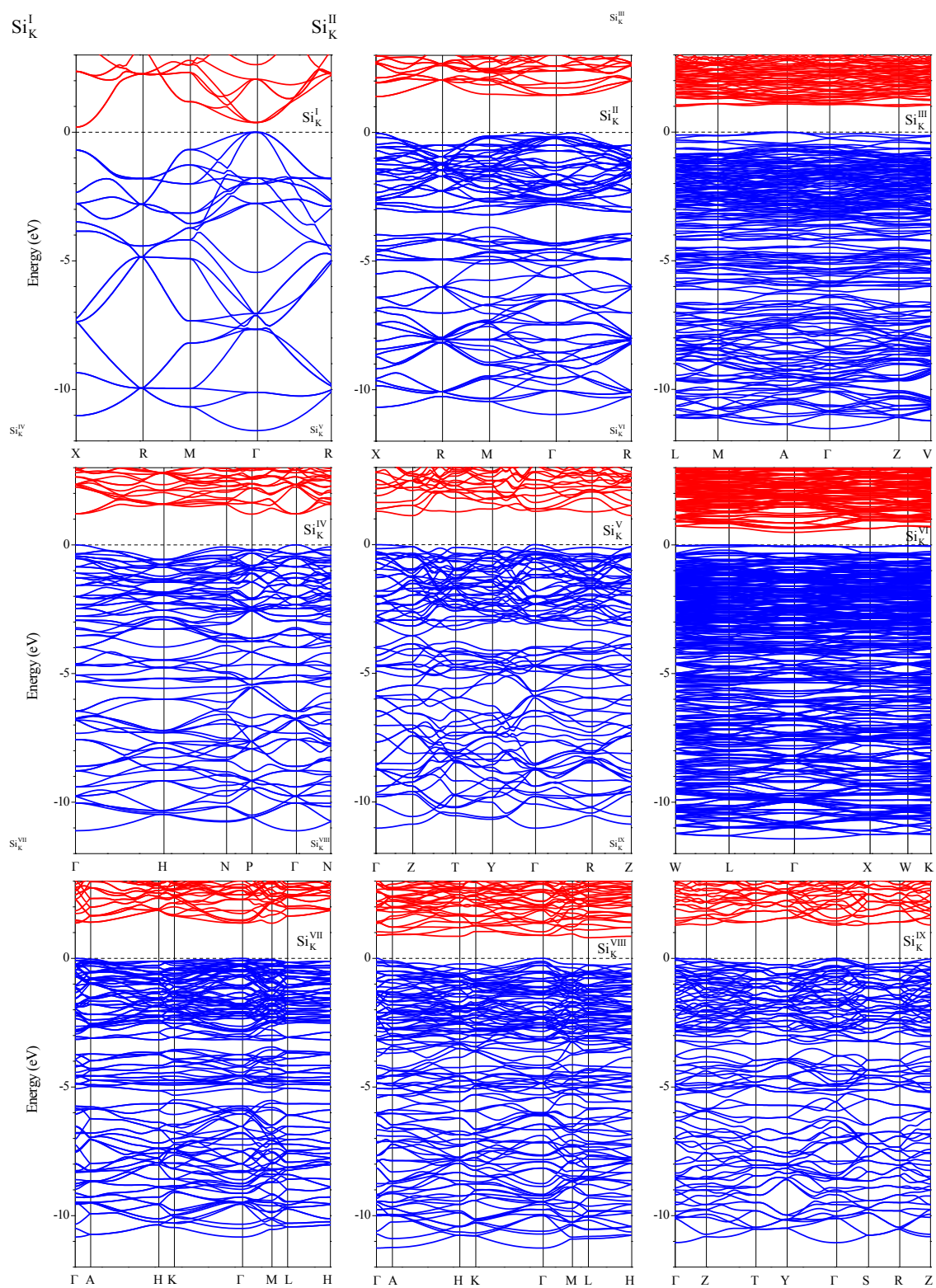
$K^{VIII}$ Space group $P\bar{3}m1$ $a = b = 10.0979 \text{ \AA}$ , $c = 16.0108 \text{ \AA}$				
Atomic coordinates				
Si1	12j	0.00116	0.38093	0.29528
Si2	12j	0.75683	0.00078	0.17352
Si3	12j	0.91169	0.33349	0.05811
Si4	6i	0.86691	0.13309	0.49945
Si5	6i	0.79104	0.20896	0.62182
Si6	6i	0.79117	0.20883	0.37672
Si7	2d	0.66667	0.33333	0.42649
Si8	2d	0.66667	0.33333	0.57190

$K^{IX}$ Space group $Cmcm$ $a = 16.7904 \text{ \AA}$ , $b = 12.9014 \text{ \AA}$ , $c = 10.0253 \text{ \AA}$				
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Atomic coordinates				
Si1	16h	0.38084	0.71275	0.44098
Si2	16h	0.32399	0.87541	0.37336
Si3	16h	0.19229	0.91851	0.44092
Si4	8e	0.39901	0.00000	0.50000
Si5	8f	0.00000	0.24825	0.56097
Si6	8f	0.00000	0.42098	0.63107
Si7	8g	0.11579	0.04402	0.75000
Si8	8g	0.42917	0.37152	0.75000
Si9	4c	0.00000	0.14370	0.75000
$K^X$ Space group $P4_2/nmc$ $a = b = 10.2614 \text{ \AA}$ , $c = 24.3468 \text{ \AA}$				
Atomic coordinates				
Si1	16h	0.18346	0.31598	0.47133
Si2	16h	0.20578	0.12227	0.61131
Si3	16h	0.32019	0.24563	0.68328
Si4	16h	0.18019	0.38278	0.73473
Si5	16h	0.50000	0.31000	0.55372
Si6	16h	0.50000	0.36958	0.64783
Si7	16h	0.00000	0.38028	0.52430
Si8	16h	0.00000	0.23569	0.60339
Si9	16h	0.00000	0.38399	0.67690
Si10	8e	0.00000	0.00000	0.00000
$K^{XI}$ Space group $R\bar{3}$ $a = b = c = 17.6221 \text{ \AA}$				
Atomic coordinates				
Si1	24d	0.34597	0.41824	0.23752
Si2	24d	0.31547	0.40412	0.60584
Si3	24d	0.30775	0.40162	0.10815
Si4	24d	0.30260	0.29669	0.52667
Si5	24d	0.28632	0.38982	0.73659
Si6	24d	0.27115	0.51309	0.54110
Si7	24d	0.26841	0.51848	0.91664
Si8	24d	0.10412	0.52669	0.41579
Si9	24d	0.10644	0.43729	0.04476
Si10	8c	0.31039	0.31039	0.31039
Si11	8c	0.26609	0.76609	0.73391

## 2. Electronic band structures for Kelvin Silicons



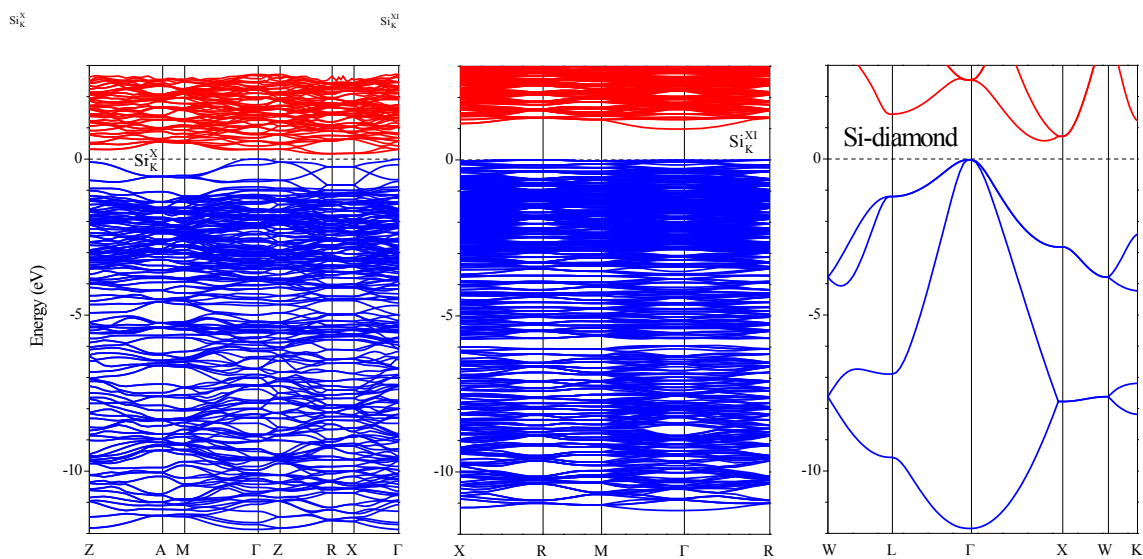


Figure S1 Electronic band structures of Kelvin Silicons and diamond phase of silicon.

### 3. X-ray diffraction spectra for Kelvin Silicons

To provide information and characters for possible experimental observation, the simulated XRD patterns at zero pressures are obtained by the REFLEX program implemented in Materials Studio 5.0 package.<sup>S1</sup>

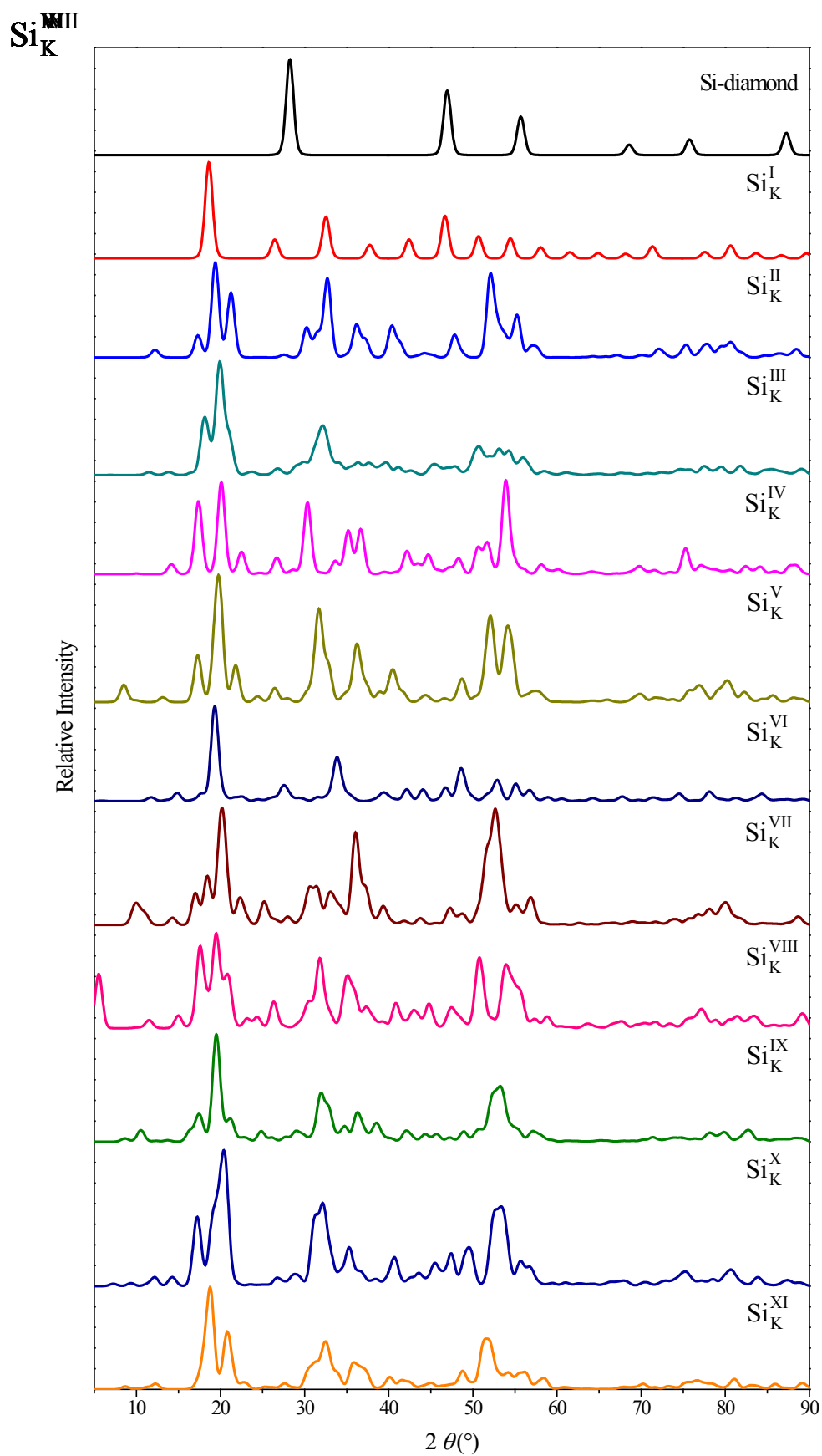


Figure S2 Simulated XRD patterns of diamond silicon and Kelvin Silicons at zero pressure. The X-ray wavelength is 1.5406 Å.

#### 4. The calculated elastic constants for Kelvin Silicons

Table S2 The calculated elastic constants  $C_{ij}$  (GPa) for the Kelvin Silicons at 0 GPa.

$Si_K^I$			$Si_K^{II}$									
$C_{11}$	$C_{44}$	$C_{12}$	$C_{11}$	$C_{44}$	$C_{12}$							
117	38	43	133	48	48							
$Si_K^{III}$												
$C_{11}$	$C_{22}$	$C_{33}$	$C_{44}$	$C_{55}$	$C_{66}$	$C_{12}$	$C_{13}$	$C_{15}$	$C_{23}$	$C_{25}$	$C_{35}$	$C_{46}$
133	120	130	43	43	39	47	45	-1	49	0	-1	-1
$Si_K^{IV}$						$Si_K^V$						
$C_{11}$	$C_{44}$	$C_{12}$	$C_{11}$	$C_{22}$	$C_{33}$	$C_{44}$	$C_{55}$	$C_{66}$	$C_{12}$	$C_{13}$	$C_{23}$	
132	43	45	135	135	126	48	48	45	44	42	42	
$Si_K^{VI}$			$Si_K^{VII}$									
$C_{11}$	$C_{44}$	$C_{12}$	$C_{11}$	$C_{33}$	$C_{44}$	$C_{12}$	$C_{13}$					
123	38	44	137	137	47	45	45					
$Si_K^{VIII}$												
$C_{11}$	$C_{33}$	$C_{44}$	$C_{12}$	$C_{13}$								
122	108	44	39	37								
$Si_K^{IX}$												
$C_{11}$	$C_{22}$	$C_{33}$	$C_{44}$	$C_{55}$	$C_{66}$	$C_{12}$	$C_{13}$	$C_{23}$				
132	140	129	47	44	42	44	47	43				
$Si_K^X$						$Si_K^{XI}$						
$C_{11}$	$C_{33}$	$C_{44}$	$C_{66}$	$C_{12}$	$C_{13}$	$C_{11}$	$C_{44}$	$C_{12}$				
109	116	36	36	38	35	132	41	45				

The mechanical stability of these Kelvin-Silicon structures studied in this work was also examined by calculations of the elastic constants using the CASTEP program implemented in Materials Studio 5.0 package.<sup>S1</sup> The corresponding values of the elastic constants for the Kelvin Silicons at 0 GPa were listed in Table S2. From Table S2, both structures are mechanically stable since their elastic constants satisfy the elastic stability criteria,<sup>S2</sup> i.e., the strain energy must be positive against any homogeneous elastic deformation. Remarkably, several elastic constants  $C_{15}$ ,  $C_{35}$ , and  $C_{46}$  of the monoclinic  $Si_K^{III}$  are negative, while other Kelvin-Silicon structures have

no negative elastic constants. Based on the arguments given by Söderlind and Klepeis,<sup>S3</sup> these negative values do not mean an instability because the actual applied distortions all yield positive elastic coefficients.

**References:**

- S1. M. Segall, P. Lindan, M. Probert, C. Pickard, P. Hasnip, S. Clark, and J. Payne, *J. Phys.: Condens. Matter*, 2002, **14**, 2717.
- S2. J. F. Nye, Oxford University Press, Oxford, 1985.
- S3. P. Söderlind, and J. E. Klepeis, *Phys. Rev. B*, 2009, **79**, 104110.