Electronic Supplementary Material (ESI) for Physical Chemistry Chemical Physics.

Synthesis and Mechanical Response of Disordered Colloidal Micropillars

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Supporting Information

Calculation of cohesive force between two spheres bound by a water bridge

Ref [1] derives an analytical solution for the capillary force due to a water-bridge between two particles as a function of the particle/water-bridge geometry:

$$F = \pi\sigma\sqrt{R_1 R_2}\left[c + exp\left(a\frac{D}{R_2} + b\right)\right]$$

where:

$$a = -1.1 \times \left(\frac{V}{{R_2}^3}\right)^{-0.53}$$

$$b = \left(-0.148\, ln\left(\frac{V}{{R_2}^3}\right) - 0.96\right)\theta^2 - 0.0082\, ln\left(\frac{V}{{R_2}^3}\right) + 0.48$$

$$c = 0.0018\, ln\left(\frac{V}{{R_2}^3}\right) + 0.078$$

In these expressions, σ is the surface tension of water, R_1 is the radius of particle 1, R_2 is the radius of particle 2, D is the inter-particle separation distance, V is the volume of the water-bridge, and θ is the particle-water-air contact angle. We set:

$$\sigma = 0.065\frac{N}{m}$$

$$R_1 = 3\ \mu m$$

$$R_2 = 3\ \mu m$$

$$D = 0$$

$$V = 0.03 \times V_{particle} = 0.03 \times \frac{4}{3}\pi R_1^3$$

$$\theta = 90°$$

The resulting force is:

$$F = 4.55 \times 10^{-8} N$$

The gravitational force on each particle is found by:

$$F_g = V_{particle} \times \rho \times a_g$$

where ρ is the density of the particle. We set:

$$\rho = 1.05 \frac{g}{cc}$$

for polystyrene and find:

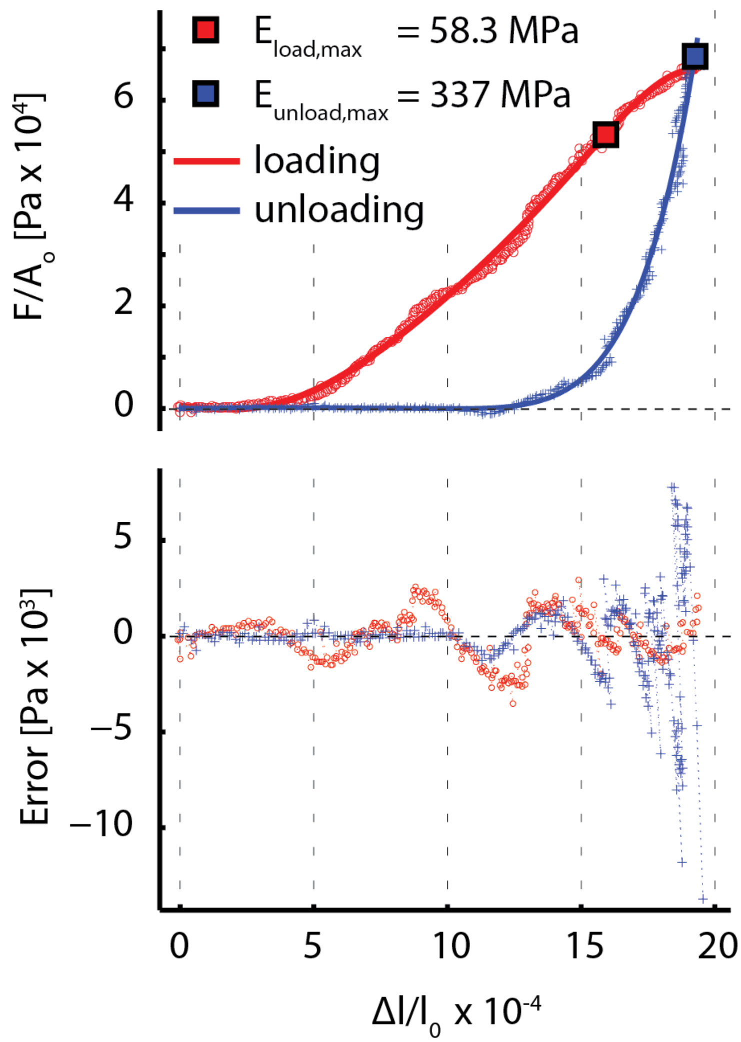
$$F_g = 1.17 \times 10^{-12} N$$

The resulting ratio is:

$$\frac{F}{F_g} = 3.91 \times 10^4$$

Determination of Maximum Instantaneous Stiffness on Loading and Unloading

The elastic limit of granular materials is known to be extremely small [2]. Therefore, we do not expect linear stiffness response in the pillars during the course of a compression cycle. To quantify the stiffness, we fit a sixth-order polynomial to the stress-strain curve on loading and unloading. An example fit and residual error is shown below:



We extract the maximum value of the first derivative of the loading and unloading functions w.r.t. to strain over the strain bounds of the experiment and record these values as the maximum stiffness.

Supplementary Information References

1. Soulié, F., Cherblanc, F., El Youssoufi, M.S., Saix, C., *International Journal of Numerical Analytical Methods in Geomechanics*, 2006, **30**, 213–228.
2. Hardin, B. O. *Journal of Geotechnical Engineering*, 1989, **115**, 788-805.