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## Supporting Information for

# Nanoparticle Dispersion in Polymer Nanocomposites by Spin-Diffusion-Averaged Paramagnetic Enhanced NMR Relaxometry: Scaling Relations and Applications

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References

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**Figure S1.** Normalized magnetization,  $M(t)/M_o$ , versus recovery time, *t* for pure poly(vinyl alcohol) and poly(vinyl alcohol)/montmorillonite (PVA/MMT<sub>STx-1b</sub>) nanocomposites at weight ratios (PVA/MMT<sub>STx-1b</sub>) of 100/1, 100/2, 100/4, 100/6, 100/8, and 100/10. The inset displays the same data plotted as  $\ln[1 - M(t)/M_o]$  versus recovery time, *t*, the slopes of which reflect the inverse  $T_1$ s. The relaxation rate increases upon increasing the MMT<sub>STx-1b</sub> content; all nanocomposites exhibit faster relaxation (shorter  $T_1^H$ ) than the corresponding pure PVA. The calculated  $T_1$  values are 11.64 ± 0.23 s, 9.38 ± 0.23 s, 9.21 ± 0.11 s, 7.01 ± 0.0.07 s, 6.67 ± 0.12 s, 5.00 ± 0.11 s and 3.10 ±0.23 s for weight ratios from 100/0 to 100/10, respectively.

### Model analysis: sinks with infinitely fast relaxation

As discussed in the main text, we recently reported an analytical relationship between NMR magnetization growth and interparticle spacings (IPS) in lamellar polymer/paramagnetic clay nanocomposites:<sup>1</sup>

$$\frac{M(t)}{M_o} = 1 - \left(\frac{4D}{\beta\Delta^2}\right)^{1/2} \tan\left(\frac{\beta\Delta^2}{4D}\right)^{1/2} \exp(-t/T_{1,s}) \\ - \frac{8}{\pi^2} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} \left[\frac{1}{1 - (2n+1)^2 \pi^2 D / (\beta\Delta^2)}\right] \exp\left[-\left(\frac{(2n+1)^2 \pi^2 D}{\Delta^2} + \frac{1}{T_{1,m}}\right)t\right]$$
(5)

where  $M_0$  is the total equilibrium magnetization, D is the bulk spin diffusion coefficient (uniform, not a function of spatial position),  $1/T_{1,m}$  is the bulk matrix nuclear relaxation rate,  $1/T_{1,s}$  is the relaxation rate of the clay surface nuclei, and  $\beta$  is the difference between  $1/T_{1,s}$  and  $1/T_{1,m}$  (i.e.,  $\beta = 1/T_{1,s} - 1/T_{1,m}$ ). In the case of the sinks with infinitely fast relaxation (e.g.,  $T_{1,s} \rightarrow$ 0, i.e.,  $\beta \rightarrow \infty$ ), we can simplify eq 5:

$$\frac{M(t)}{M_o} = 1 - \sum_{n=0}^{\infty} \beta_n^{-1} \exp\left[-\left(\frac{8\beta_n D}{\Delta^2} + \frac{1}{T_{1,m}}\right)t\right]$$
(S1)

where  $\beta_n = (2n + 1)^2 \pi^2/8$ . The summation in eq S1 converges quite rapidly with *n*; numerical calculation using just two iterations yields errors less than 5% (see Figure S2). Taking only the first term of the summation, equation S1 can be recast:

$$\frac{M(t)}{M_o} = 1 - \frac{8}{\pi^2} f(t) \exp\left[-\left(\frac{\pi^2 D}{\Delta^2} + \frac{1}{T_{1,m}}\right)t\right]$$
(S2)

where  $f(t) = 1 + 1/9 \exp(-8B t) + 1/25 \exp(-24B t) + ...,$  and  $B = \pi^2 D/\Delta^2$ . The value of f(t)approaches 1 if  $t > (8B)^{-1} = 8\Delta^2/(\pi^2 D)$ . Note that this approximation is valid when spin diffusion lengths,  $(D \times 5T_1)^{1/2}$ , are greater than interparticle separations,  $\Delta$ . In other words, the interparticle distance is such that magnetization throughout the entire sample may equilibrate due to spin diffusion during the  $T_1$  relaxation process. Thus, samples must be characterized by  $T_1 >$  $\Delta^2/(20D)$ . Since this is approximately  $(8B)^{-1} = 8\Delta^2/(\pi^2 D)$ , eq S2 should sufficiently describe long-time relaxation behavior for f(t) = 1 (i.e., n = 0 in summation of eq S1). This was

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confirmed by numerically generating relaxation curves for the first four *n* values of the summation (n = 0, 1, 2 and 3) using parameter values similar to those for a PCN with 5 wt% MMT and a spin diffusion coefficient, D = 0.7 nm<sup>2</sup>/ms. These are shown in Figure S2(a) and reveal no difference in the long-time relaxation behavior when t > ~180 ms  $\approx 8\Delta^2/(\pi^2 D)$ . Although differences are observed in the short-time behavior, Figure S2(b) shows that these do not significantly affect the overall  $T_1$  values determined from plots of  $\ln[\pi^2/8(1-M(t)/M_0)]$  versus recovery time. As a result, from eq S2 with f(t) = 1, the observed  $1/T_{1,PCN}$  can be obtained

$$\frac{1}{T_{1,PCN}} \approx \frac{\pi^2 D}{\Delta^2} + \frac{1}{T_{1,m}}$$
(S3)

Equation S3 can be compared to the semi-empirical equation used to compute the paramagnetic contribution to the spin-lattice relaxation rate:<sup>2-4</sup>

$$R_{1,\text{para}} = 1/T_{1,\text{para}} = 1/T_{1,\text{PCN}} - 1/T_{1,\text{polymer}}$$
(S4)

if the relaxation rate of the pure polymer,  $1/T_{1,polymer}$ , is taken to be the relaxation rate of the bulk polymer in the nanocomposite,  $1/T_{1,m}$ . In this case, the paramagnetic contribution to the relaxation is

$$R_{1,\text{para}} \approx \pi^2 D / \Delta^2 \tag{S5}$$

Thus,  $R_{1,\text{para}} \sim \Delta^{-2}$ , for sinks with infinitely fast relaxation (e.g.,  $T_{1,s} \rightarrow 0$ , *i.e.*,  $\beta \rightarrow \infty$ ).



**Figure S2.** Relaxation curves numerically calculated using eq S1 and the first four terms of f(t), corresponding to eq S2 (first term of summation only) and  $n = 0, \le 1, \le 2$  and  $\le 3$ : (a)  $M(t)/M_0$ , and (b)  $\ln[\pi^2/8(1 - M(t)/M_0)]$  versus recovery time. The following parameters were used in the calculation: spin diffusion coefficient,  $D = 0.7 \text{ nm}^2/\text{ms}$ , bulk polymer relaxation time,  $T_{1,\text{m}} = 1.635 \text{ s}$ ,  $\Delta = 50 \text{ nm}$  and recovery time range from 0.5 to 10000 ms. Calculated values of  $T_{1, \text{PCN}}$  in (b), 296 ms (n = 0), 293 ms ( $n \le 1$ ), and 292 ms ( $n \le 2$  and  $\le 3$ ), are consistent with the relaxation constant of 296 ms determined by fitting the data points in (b) to a conventional exponential recovery.



**Figure S3.** X-ray diffraction patterns of octadecylamine-modified  $MMT_{STx-1b}$  (C18-MMT<sub>STx-1b</sub>) and a PVA/MMT<sub>STx-1b</sub> nanocomposite (PVA/MMT<sub>STx-1b</sub> = 100/10, w/w). This PVA/MMT<sub>STx-1b</sub> nanocomposite contains 10 wt% clay and does not exhibit a basal peak (001) reflection, indicating the clay is exfoliated.



**Figure S4.** Normalized magnetization,  $M(t)/M_o$  versus recovery time for octadecylaminemodified MMT<sub>STx-1b</sub> (C18-MMT<sub>STx-1b</sub>) measured at 300 MHz.  $T_1^{\rm H} = 21.4 \pm 1.3$  ms.



**Figure S5.** (a) Normalized and corrected magnetization versus the square root of recovery time for poly(vinyl alcohol)/montmorillonite (PVA/MMT<sub>STx-1b</sub>) nanocomposites with different clay contents. PVA/MMT<sub>STx-1b</sub> weight ratios are 100/*x* where x = 1, 2, 4, 6, 8 and 10. The data were measured at 300 MHz and are vertically displaced to prevent overlap. Lines are linear leastsquare fits. Slopes of these lines,  $S_i$  are plotted in (b) as a function of clay content,  $w_c$ . These initial slopes, which reflect the effective clay/polymer interfacial area, are linearly proportional to the clay weight fraction and therefore suggest similar degrees of exfoliation in these samples.

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