Supplementary Information

Table 1 Parameters of the radial symmetry functions ¹ of type G^2 describing the environment of the hydrogen and oxygen atoms, respectively, for each possible neighboring element. A radial shift $R_{\text{shift}} = 0.0$ Bohr and a cutoff radius $R_{\text{c}} = 19.0$ Bohr have been used for all symmetry functions.

central atom	neighbor	η [Bohr ⁻²]
Н	Н	0.200
		0.070
		0.030
		0.010
	Ο	0.200
		0.070
		0.030
		0.010
0	Н	0.200
		0.070
		0.030
		0.010
	Ο	0.030
		0.010
		0.001

Table 2 Parameters of the angular symmetry functions ¹ of type G^4 employed to describe the environment of each atom with respect to the elements of the neighboring atoms. A cutoff radius $R_c = 19.0$ Bohr has been used to define the atomic environments.

$ \begin{array}{c c} \hline central atom & neighbors & \eta [Bohr^{-2}] \\ \hline H & HH & 0.200 \\ & 0.070 \\ & 0.010 \\ & 0.001 \\ OH & 0.200 \\ & 0.070 \\ \end{array} $	λ 1.0 -1.0 1.0 -1.0 1.0 1.0 1.0	$\frac{\zeta}{1.0}$ 1.0 4.0 4.0
H HH 0.200 0.070 0.010 0.001 OH 0.200 0.070	1.0 -1.0 1.0 -1.0 1.0 1.0	1.0 1.0 4.0 4.0
0.070 0.010 0.001 OH 0.200 0.070	-1.0 1.0 -1.0 1.0 1.0	1.0 4.0 4.0
0.010 0.001 OH 0.200 0.070	1.0 -1.0 1.0 1.0	4.0 4.0
0.001 OH 0.200 0.070	-1.0 1.0 1.0	4.0
OH 0.200	1.0 1.0	1.0
0.070	1.0	1.0
0.070		1.0
0.030	1.0	1.0
0.010	1.0	4.0
0.070	-1.0	1.0
0.030	-1.0	1.0
0.010	-1.0	4.0
OO 0.030	1.0	1.0
0.030	-1.0	1.0
0.001	1.0	4.0
0.001	-1.0	4.0
O HH 0.070	1.0	1.0
0.030	1.0	1.0
0.010	1.0	4.0
0.070	-1.0	1.0
0.030	-1.0	1.0
0.010	-1.0	4.0
OH 0.030	1.0	1.0
0.030	-1.0	1.0
0.001	1.0	4.0
0.001	-1.0	4.0
OO 0.030	1.0	1.0
0.030	-1.0	1.0

Table 3 Theoretical vibrational frequencies of the neural network (NN) and DFT-optimized protonated water clusters along with the experimental values. No frequency scaling has been applied. For a visualization of the cluster geometries cf. Table 2 in main text. For comparison also the values of the neutral water monomer are given.

cluster	description	calculated [cm ⁻¹]		experimental [cm ⁻¹]		
		NN	DFT	Ref. ²	Ref. ³	Ref. ⁴
	H ₂ O bend	1583	1590			1595
H ₂ O	H_2O symmetric	3706	3706			3652
	H_2O asymmetric	3809	3814			3756
H ₃ O ⁺	H_3O^+ bend	844	847			526
	H_3O^+ symmetric	3483	3478			3390
	H_3O^+ asymmetric	3565	3566			3519
$H_5O_2^+$	proton oscillation	1076	1111	1085	920	
	H ₂ O symmetric	3634	3637	3520	3609	
	H ₂ O asymmetric	3726	3722	3660	3684	
	H ₃ O ⁺ asymmetric	2139	2147	1880		
	H_3O^+ symmetric	2428	2373	2420		
$H_7O_3^+$	H ₂ O symmetric	3680	3681	3639	3637	
1 5	H_3O^+ free OH	3723	3704	3580	3667	
	H_2O asymmetric	3768	3772	3742	3722	
H ₉ O ₄ ⁺	H ₃ O ⁺ asymmetric	2666	2638	2665		
	H_3O^+ symmetric	2851	2805			
	H_2O symmetric	3698	3690	3644	3645	
	H_2O asymmetric	3775	3781	3730	3730	
	H ₃ O ⁺ asymmetric	2515	2570	2860	2879	
	H_3O^+ symmetric	2852	2839		2967	
	H bond asymmetric	3480	3459	3195	3208	
и o+	H bond symmetric	3525	3497			
$\Pi_{11} O_5$	H ₂ O symmetric	3690	3684	3647	3647	
	OH asymmetric	3760	3754	3712	3736	
	OH symmetric	3764	3755			
	H ₂ O asymmetric	3761	3772	3740	3817	
H ₁₃ O ₆ ⁺	H ₃ O ⁺ asymmetric	2546	2575			
	H_3O^+ symmetric	2919	2886			
	H bond asymmetric	3298	3294			
	H bond symmetric	3379	3363			
	H ₂ O symmetric	3696	3700	3650	3651	
	H ₂ O asymmetric	3782	3796	3740	3741	
H ₁₅ O ⁺ ₇	H ₃ O ⁺ asymmetric	2505	2524			
	H ₃ O ⁺ symmetric	3738	2760			
	H bond stretch	3443	3431			
	OH stretch	3739	3749			
H ₁₇ O ⁺ ₈	H_3O^+ asymmetric	2505	2518			
	H ₃ O ⁺ symmetric	2767	2754			
	H bond stretch	3275	3300			
	OH stretch	3737	3753			

References

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