

# **Turing Patterns Modulation by Chemical Gradient in Isothermal and Non-isothermal Conditions**

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# Supplementary Information

## 1. Determination of the equilibrium points

As exposed in the main text, the IT model is described in the following equations:

$$\frac{\partial u}{\partial \tau} = f(u, v) + d_u \nabla^2 u \quad (1)$$

$$\frac{\partial v}{\partial \tau} = g(u, v) + d_v \nabla^2 v \quad (2)$$

with  $f(u, v) = \xi(a - u) - r_1 + r_2$ ,  $g(u, v) = r_1 - r_2$ , and

$$r_1 = -k_{f_1} e^{e_1(1-\frac{1}{\theta})} u^2 v + k_{r_1} e^{e_2(1-\frac{1}{\theta})} u^3$$

$$r_2 = -k_{f_2} e^{e_3(1-\frac{1}{\theta})} u b + k_{r_2} e^{e_4(1-\frac{1}{\theta})} v d$$

The equilibrium point  $(u^*, v^*)$  is defined through the following conditions  $f(u^*, v^*) = g(u^*, v^*) = 0$ . In the case of IT model:

$$u^* = a \quad (3)$$

$$v^* = \frac{abk_{f_2} + a^3k_{r_1}}{a^2k_{r_1} + dk_{r_2}} \quad (4)$$

In the case of NIT model, the equilibrium point is  $(u^*, v^*, \theta^*)$  with,

$$\theta^* = \frac{1}{(a\gamma\xi + \phi)(a^2k_{f_1} + dk_{r_2})} \left( -a^3b\Delta H_1k_{f_1}k_{f_2} - a^3b\Delta H_2k_{f_1}k_{f_2} + \right. \\ \left. a^3d\Delta H_1k_{r_1}k_{r_2} + a^3d\Delta H_2k_{r_1}k_{r_2} + a^3\gamma k_{f_1}\xi + \right. \\ \left. a^2k_{f_1}\theta_\Omega\phi + a\gamma dk_{r_2}\xi + dk_{r_2}\theta_\Omega\phi \right) \quad (5)$$

## 2. Spatial solutions

We obtain the diffusion process orientation for three different concentrations at the boundary as shown in Fig.1. There is no diffusion process at the equilibrium point concentration. The diffusion process assumes different orientation for each chemical when the concentration is either lesser or greater than the equilibrium points concentration. That is, the concavity in space of the  $u(x, y)$  and  $v(x, y)$  changes, this is illustrates in the Fig. 1:

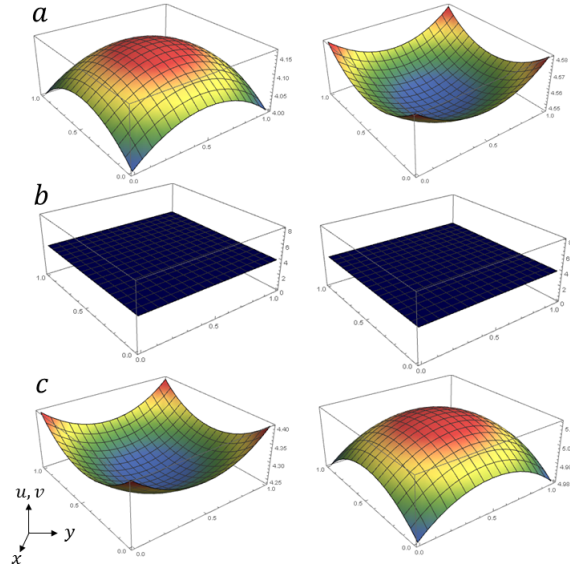


Figure 1: Graphical representation of the spatial solution of  $u(x, y)$  (left) and  $v(x, y)$  (right).  
a)  $(U_\Omega, V_\Omega) = (2.00, 2.29)$  b)  $(U_\Omega, V_\Omega) = (2.10, 2.39)$  c)  $(U_\Omega, V_\Omega) = (2.20, 2.49)$ .

### 3. Symmetry breaking

From the Fig. 8 (main article) the reader cannot realize that the symmetry breaking process happens as DBC approaches to the EP of the homogeneous system, represented by letter "d". Therefore, in order to define precisely the point where the phenomenon occurs, we performed calculations with DBC between the cases "c" and "d" presented in Fig. 8 for the IT model. In that region, an intermediary spatial structure emerges spontaneously, see Fig. 2. From this figure, it is easy to see that the pattern obtained shares structural similarities with the patterns emerged before and after the break of symmetry.

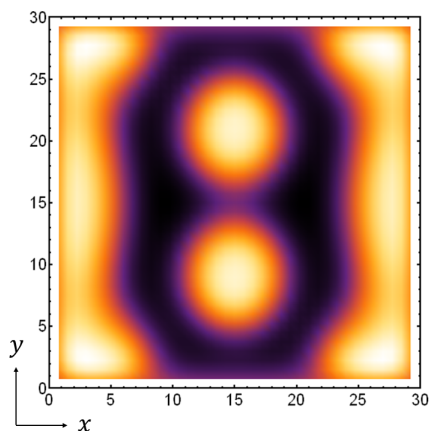


Figure 2: IT model with  $(U_{\Omega}, V_{\Omega}) = (2.0666, 2.3567)$ .

Through the analysis of the total rate of entropy production as function of the sources of chemicals at the boundaries, we can realize that the process of symmetry breaking takes place around  $U_{\Omega} = u^*$  and  $V_{\Omega} = v^*$ , slightly before. Moreover, considering the abrupt change of its behavior, we are able to note that this process is quite similar with a first order transition.

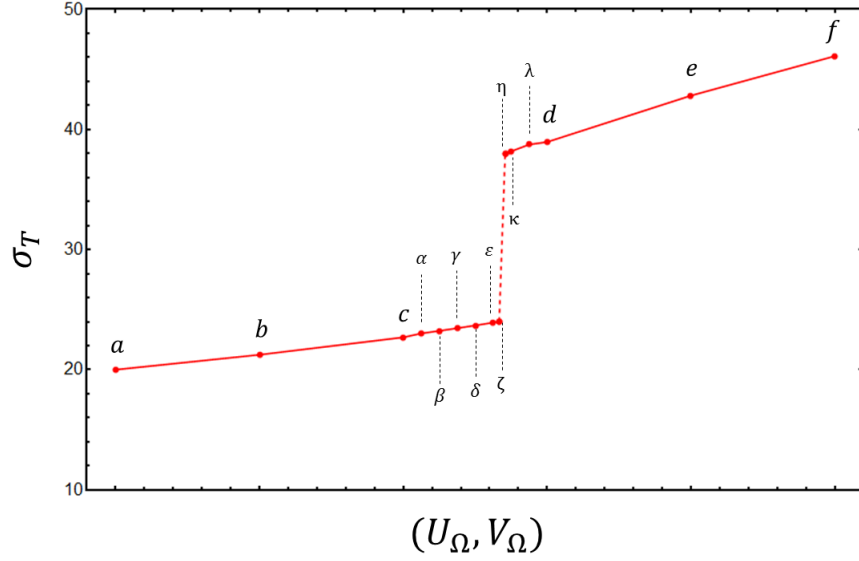


Figure 3: The total rate of entropy production for IT, where  $(U_\Omega, V_\Omega)$  is for  $\alpha$ ) (2.0125, 2.3025),  $\beta$ ) (2.025, 2.315),  $\gamma$ ) (2.0375, 2.3275),  $\delta$ ) (2.05, 2.34),  $\epsilon$ ) (2.0625, 2.3525),  $\zeta$ ) (2.0666, 2.3566),  $\eta$ ) (2.0708, 2.3608),  $\kappa$ ) (2.075, 2.365) and  $\lambda$ ) (2.0875, 2.3775). The letters ( $a, b, c, d, e, f$ ) indicate the same conditions considered in the Fig. 6 (main article).

#### 4. Graphical representation of the CGIB

The Fig. 4 is the graphical representation of the CGIB for considered in the simulations presented in the main text of the paper. Note that, the CGIB for  $30 \times 30$  dimension system is surrounded by the dotted vertical axis.

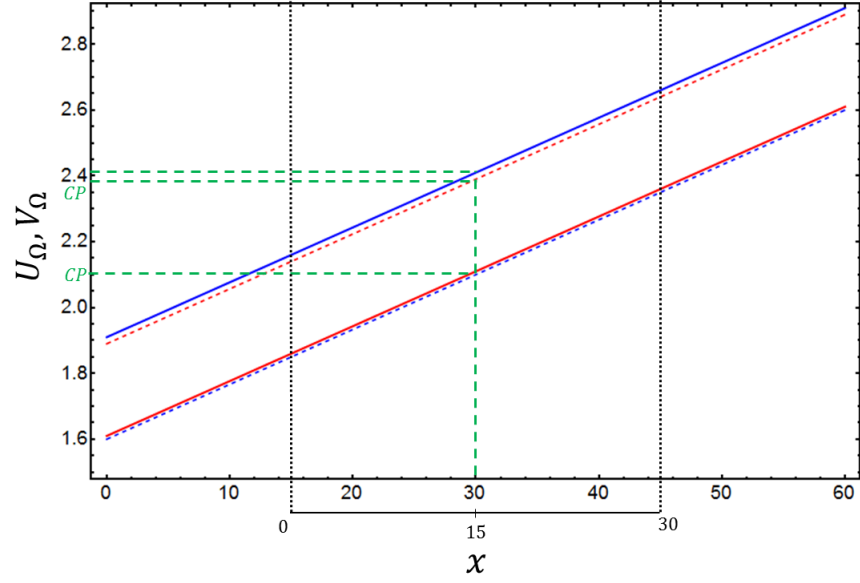


Figure 4: Linear CGIB in the  $x$ -direction. equilibrium point concentrations are indicated by green lines. The concentration of  $U_\Omega$  and  $V_\Omega$  are represented by blue and red color, respectively. The IT and NIT systems are given by dotted and solid lines, respectively.