

Dielectric spectroscopy and Time Dependent Stokes Shift: Two Faces of the Same Coin?

Supporting Information

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1 Molecules and Force Field

We selected Wand's novel surfactant mixture consisting of the zwitterionic lauryldimethylamine-N-oxide (LDAO) and the neutral monoglyceride 1-decanoyl-*rac*-glycerol (DMAG). This mixture was shown to safely encapsulate various diverse proteins and even t-RNA and it approximates realistic biomembranes more closely than the more common anionic surfactant bis(2-ethylhexyl)-sulfosuccinate (aerosol-OT, AOT).^{1,2} The surfactant hydrocarbon tails were modelled with united atoms representing CH, CH₂ and CH₃ groups in order to save computational resources. The atomistic head groups were parametrized with appropriate CHARMM36 atom types³⁻⁶ except for the charges, which were evaluated quantum-chemically. The aliphatic tails were parametrized with GROMOS 45A3 atom types, which were specifically designed for coarse-grained hydrocarbon structures such as lipid aggregates and micelles.⁷ In practice, this combination works well and was demonstrated to accurately reproduce dielectric absorption spectra,⁸ nuclear quadrupole resonance⁹ and terahertz spectra.¹⁰ For details regarding the surfactant model and force field see ref. 11. The numerically dominant but dielectrically near-inert immersion medium isoctane was modelled analogously to the surfactant hydrocarbon tails with charge-less GROMOS 45A3 united-atoms.

2 Simulation setup

We constructed reverse micelles with five different water loadings

$$w_0 = \frac{[\text{H}_2\text{O}]}{[\text{surfactant}]} = \frac{N_{\text{H}_2\text{O}}}{N_{\text{surfactant}}} \quad (\text{S-1})$$

with $w_0 = 3, 7.5, 18, 30$ and 40 . The first three were constructed in analogy to a previous study,¹⁰ while the biggest two were simulated to test the range of possible water loadings as determined by Wand¹ since physico-chemical properties of RMs vary depending on the RM nanopool volume.¹²⁻¹⁸ The respective size of the RMs (water count) was decided according to light-scattering experiments.¹⁹ For statistical reasons, we produced 2 independent replica of each water loading. Table 1 summarizes the systems investigated in the frame of this study.

The reverse micelle intermolecular geometries were preassembled with the software PACKMOL²⁰ using numerical seeds. The specified amount of water molecules were packed into a sphere, then surrounded by LDAO and DMAG molecules with the head groups oriented toward the water sphere. This proto-micelle was then suspended in a box of isoctane molecules as the immersion medium.

All subsequent simulation steps were carried out using DOMDEC CHARMM.^{21,22} The initial intermolecular geometries were energetically minimized by 2000 steps of steepest descent, then equilibrated as isobaric-isothermal ensembles (NpT) with periodic boundary conditions until the box length converged. The trajectory was then produced as an isochoric-isothermal ensemble (NVT) using periodic boundary conditions. The equations of motion were integrated with the leap-frog algorithm²³ and the temperature was kept constant at 300K using a Nosé-Hoover thermostat.^{24,25} Non-bonded electrostatic interactions were calculated with the Particle-Mesh Ewald (PME) method^{26,27} on a $128 \times 128 \times 128$ grid using cubic splines of order 6 and tinfoil boundary conditions ($\kappa = 0.41 \text{ \AA}^{-1}$).

Table 1: Overview of the molecular dynamics simulations reused from the dielectric study (Ref. 11).

	# replicas	N _{H₂O}	N _{LDAO}	N _{DMAG}	N _{isooctane}	timestep /fs	write freq. /ps	trajectory length /ns	box length /Å
reverse micelle, $w_0 = 3$									
a)	2	96	10	22	9000	2	1	200	138.36
b)	10	96	10	22	9000	2	0.01	1	138.36
reverse micelle $w_0 = 7.5$									
c)	2	525	24	46	9000	2	1	200	138.85
d)	10	525	24	46	9000	2	0.01	1	138.85
reverse micelle, $w_0 = 18$									
e)	2	1800	35	65	9000	2	1	200	139.80
f)	10	1800	35	65	9000	2	0.01	1	139.80
reverse micelle, $w_0 = 30$									
g)	2	3040	45	90	9000	2	1	200	140.94
h)	10	3040	45	90	9000	2	0.01	1	140.94
reverse micelle, $w_0 = 40$									
i)	2	4960	78	156	9000	2	1	200	144.23
j)	10	4960	78	156	9000	2	0.01	1	144.23
bulk water									
k)	1	9999	-	-	-	2	1	100	66.97
l)	10	9999	-	-	-	2	0.01	1	66.97

3 Dipolar description of solvation energy

Describing the difference in solvation energy ΔU , see eq. (3), solely in electrostatic terms, it may be written as the interaction of the changed charge distribution of the solute $\Delta\rho_{\text{solute}}$ and the solvent charge distribution ρ_{solvent} as

$$\Delta U = \int \int d\vec{r}_1 d\vec{r}_2 \frac{\Delta\rho_{\text{solute}}(\vec{r}_1)\rho_{\text{solvent}}(\vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|} \quad (\text{S-2})$$

which can be reformulated as

$$\Delta U = \int d\vec{r}_1 \Delta\rho_{\text{solute}}(\vec{r}_1)V(\vec{r}_1) \quad (\text{S-3})$$

in terms of the electrostatic potential

$$V(\vec{r}_1) = \int d\vec{r}_2 \frac{\rho_{\text{solvent}}(\vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|} \quad (\text{S-4})$$

According to the small spatial extension of the solute charge distribution $\Delta\rho_{\text{solute}}$ in relation to the solvent charge distribution ρ_{solvent} , the electrostatic potential may be replaced by a Taylor series

$$V(\vec{r}) = V(r=0) + \vec{r} \cdot (\vec{\nabla}V)_{(r=0)} + \text{quadrupolar terms} \quad (\text{S-5})$$

Inserting this expression into eq. (S-3) and using $\vec{E}(r=0) = -\vec{\nabla}V_{(r=0)}$, we get

$$\Delta U = V(r=0) \int d\vec{r} \Delta\rho_{\text{solute}}(\vec{r}) - \vec{E}_{\text{solvent}}(r=0) \int d\vec{r} \vec{r} \Delta\rho_{\text{solute}}(\vec{r}). \quad (\text{S-6})$$

Since the overall change in the solute charge distribution is zero we finally get

$$\Delta U = -\Delta\vec{\mu} \cdot \vec{E}_{\text{solvent}} \quad (\text{S-7})$$

with the change of the solute dipole moment

$$\Delta\vec{\mu} = \int d\vec{r} \vec{r} \Delta\rho_{\text{solute}}(\vec{r}). \quad (\text{S-8})$$

Separating the length and the direction (*i.e.* strength and orientation) of $\Delta\vec{\mu} = |\Delta\vec{\mu}|\vec{n}_\mu$, the change in solvation energy may be written as

$$\Delta U = -|\Delta\vec{\mu}|\vec{n}_\mu \cdot \vec{E}_{\text{solvent}} \quad (\text{S-9})$$

Since the length of the vector cancels out in eq. (3) and eq. (4), the TDSS in our case probes the direction of the electric field generated by the surrounding solvent at the position of the solute, the so-called local internal field.

In this study, we have chosen three different orientations of $\Delta\vec{\mu}$ (cf. Fig. 2) to keep our analysis general and independent of possible artifacts brought about by a single specific dipole orientation.

4 Derivation of the conversion of dielectric spectra to the TDSS mediated by the reaction field

Based on this one-to-one correspondence of $\Delta\vec{\mu}$ and \vec{E} on the one hand and of $\vec{E}_{solvent}(t)$ and $\vec{M}(t)$ on the other hand, we can transcribe the relations (5), (6) and (7) relevant for DRS to those corresponding to TDSS

$$\langle \vec{E}_{solvent}^{LRT}(\omega) \rangle \propto \chi_{solv}(\omega) \Delta\vec{\mu}(\omega) \quad (S-10)$$

$$\chi_{solv}(\omega) \propto \int_0^\infty dt e^{i\omega t} \left(-\frac{\partial C_s(t)}{\partial t} \right) \quad (S-11)$$

$$C_s(t) = \frac{\langle \vec{E}_{solvent}(0) \vec{E}_{solvent}(t) \rangle}{\langle \vec{E}_{solvent}(0) \vec{E}_{solvent}(0) \rangle} \quad (S-12)$$

The counterpart of the constitutive relation (7) is given by the Reaction Field

$$\vec{E}_{solvent}^{RF}(\omega) \propto \frac{2(\epsilon(\omega) - 1)}{2\epsilon(\omega) + 1} \Delta\vec{\mu}(\omega) \quad (S-13)$$

of dielectric continuum theory.

Starting from the frequency-dependent version of eq. S-7

$$\Delta U(\omega) = -\Delta\vec{\mu}(\omega) \cdot \vec{E}_{solvent}(\omega) \quad (S-14)$$

we can derive two equivalent expressions for the change in solvation energy either from continuum theory

$$\Delta U_{solvent}^{RF}(\omega) \propto -\Delta\vec{\mu}(\omega) \cdot \vec{E}_{solvent}^{RF}(\omega) \propto -\frac{2(\epsilon(\omega) - 1)}{2\epsilon(\omega) + 1} \Delta\mu^2(\omega) \quad (S-15)$$

or from linear response theory

$$\Delta U_{solvent}^{LRT}(\omega) \propto -\Delta\vec{\mu}(\omega) \cdot \vec{E}_{solvent}^{LRT}(\omega) \propto -\chi_{solv}(\omega) \Delta\mu^2(\omega) \quad (S-16)$$

Comparing the result of LRT with the reaction field result we get

$$\chi_{solv}(\omega) \propto \frac{2(\epsilon(\omega) - 1)}{2\epsilon(\omega) + 1} \quad (S-17)$$

Using the explicit expression for the susceptibility (S-11) we further get

$$\int_0^\infty dt e^{i\omega t} \left(-\frac{\partial C_s(t)}{\partial t} \right) \propto \frac{2(\epsilon(\omega) - 1)}{2\epsilon(\omega) + 1} \quad (S-18)$$

$$C_s(0) + i\omega \int_0^\infty dt' e^{i\omega t'} C_s(t') \propto \frac{2(\epsilon(\omega) - 1)}{2\epsilon(\omega) + 1} \quad (S-19)$$

For $\omega = 0$,

$$C_s(0) \propto \frac{2(\epsilon(0) - 1)}{2\epsilon(0) + 1} \quad (S-20)$$

and thus

$$i\omega \int_0^\infty dt' e^{i\omega t'} C_s(t') \propto \frac{2(\epsilon(\omega) - 1)}{2\epsilon(\omega) + 1} - \frac{2(\epsilon(0) - 1)}{2\epsilon(0) + 1} \quad (S-21)$$

In all previous formulae concerning the analogy of DRS and TDSS we have omitted constant prefactors, which is permissible since they cancel out when calculating the normalized TDSS curve.

Based on Euler's formula, the relation

$$i\omega e^{i\omega t'} = i\omega \cos(\omega t') - \omega \sin(\omega t') \quad (S-22)$$

holds and we obtain the cosine transform by taking the imaginary part of the right-hand side

$$\begin{aligned} \int_0^\infty dt' \cos(i\omega t') C_s(t') &\propto \frac{1}{i\omega} \text{Im} \left[\frac{2(\epsilon(\omega) - 1)}{2\epsilon(\omega) + 1} - \frac{2(\epsilon(0) - 1)}{2\epsilon(0) + 1} \right] \\ &= \frac{1}{i\omega} \text{Im} \left[\frac{2(\epsilon(\omega) - 1)}{2\epsilon(\omega) + 1} \right] \end{aligned} \quad (S-23)$$

Note that

$$\begin{aligned}\epsilon(0) &= \text{Re}[\epsilon(0)] + i \text{Im}[\epsilon(0)] \\ &= \epsilon'(0) + i \epsilon''(0) = \epsilon'(0)\end{aligned}\tag{S-24}$$

since the imaginary part of the complex permittivity spectrum vanishes at $\omega = 0$. Multiplication with $\cos(\omega t)$ and integration over $d\omega$

$$\begin{aligned}&\int_0^\infty dt' \left(\int d\omega \cos(\omega t) \cos(\omega t') \right) C_s(t') \\ &\propto \int d\omega \frac{\cos(\omega t)}{i\omega} \text{Im} \left[\frac{2(\epsilon(\omega) - 1)}{2\epsilon(\omega) + 1} \right]\end{aligned}\tag{S-25}$$

using orthogonality

$$\int d\omega \cos(\omega t) \cos(\omega t') = \delta(t - t')\tag{S-26}$$

yields the expression for the normalized TDSS

$$C_S(t) \approx C_S^{\text{conv}}(t) = \int_0^\infty d\omega \frac{\cos(\omega t)}{\omega} \text{Im} \left[\frac{\epsilon(\omega) - 1}{\epsilon(\omega) + \frac{1}{2}} \right].\tag{S-27}$$

Using $\epsilon(\omega) = \epsilon'(\omega) + i \epsilon''(\omega)$, we obtain a pure real-part expression to convert complex dielectric spectra of nonionic dipolar matter to TDSS curves:

$$C_S^{\text{conv}}(t) = \int_0^\infty d\omega \frac{\cos(\omega t)}{\omega} \frac{\frac{3}{2}\epsilon''(\omega)}{\epsilon'^2(\omega) + \epsilon''^2(\omega) + \epsilon'(\omega) + \frac{1}{4}}\tag{S-28}$$

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