# Supplemental Information:

# Increasing the Weights in the Molecular Work-out of *cis*- and *trans*-Formic Acid: Extension of the Vibrational Data Base *via* Deuteration

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## 1 Methods

## 1.1 Experimental details

#### H/D exchange and impurities

Isotopic H/D impurities in the spectra were assessed by comparison of integrated intensities of suitable vibrational bands and computed harmonic Raman scattering cross-section ratios as

$$\frac{N_1}{N_2} = \frac{\sigma_2}{\sigma_1} \cdot \frac{\int I_1(\tilde{\nu}) \mathrm{d}\tilde{\nu}}{\int I_2(\tilde{\nu}) \mathrm{d}\tilde{\nu}}$$

For integrated intensities, we assign error bars of  $\pm 20\%$  and calculate resulting impurity errors using the Gaussian error propagation. The acidic H/D exchange for HCOOD was estimated analysing the  $\nu_6$  and  $\nu_1$  spectral regions, where vibrational bands of HCOOH and HCOOD can be found in the same spectral window. The same applies to DCOOH impurities in DCOOD spectra. Detailed analysis and comparison of respective spectral regions of all H/D isotopologues further revealed DCOOD impurities in all recorded spectra of DCOOH, which are nearly twice as high as the acidic H/D impurities in spectra of HCOOD/DCOOH. Therefore, it seems likely that these are partly due to the manufacturing process.

**Table S1:** Detailed information about all H/D isotopologues of formic acid are listed alongside an estimated upper bound of the formic acid concentrations in the helium expansion and observed isotopologic impurities.

Chemical	Lot #	Manufacturer	Concentration in He	Impurities
HCOOH	A0380211	Acros Organics, 98+%	< 0.2%	-
DCOOH	1401769	abcr, 95 wt% in $H_2O$ ; 98 atom%D	< 0.4%	$(6 \pm 2)\%$ DCOOD
HCOOD	PR-19516/07238FA1	Cambridge Isotope Laboratories, Inc. (OD, 98%) ( $<5\%$ D <sub>2</sub> O)	< 0.4%	$(3\pm1)\%$ HCOOH
HCOOD	1424534	abcr, 95 wt% in $D_2O$ ; 98 atom%D	< 0.3%	$(3 \pm 1)\%$ HCOOH
DCOOD	1414704	abcr, 95 wt% in $D_2O$ ; 98 atom%D	< 0.2 - 0.4%	$(2\pm1)\%$ DCOOH

### 1.2 Computational details

#### Geometry optimisation and harmonic frequencies

Geometry optimisation and harmonic vibrational frequency calculations at the  $CCSD(T)^{[1]}$  level have been performed with CFOUR version 1.<sup>[2, 3]</sup> Geometry optimisation, harmonic frequency, (harmonic) IR, and Raman activity calculations at all other levels have been carried out with GAUSSIAN 09 Rev. E.01.<sup>[4]</sup> All specified input parameters are summarised in Tab. S2. Raman scattering cross-sections have been computed according to Eq. 1 (main text) and are shown in Tab. S3 at the B3LYP-D3(BJ)/aVTZ level.

**Table S2:** Keywords which have been specified in GAUSSIAN 09 Rev.  $E.01^{[4]}$  and CFOUR version  $1^{[2, 3]}$  calculations using Dunning's correlation consistent aug-cc-pVTZ<sup>[5]</sup> (aVTZ) basis set.

Method	Program	Symmetry	Keywords
HF, MP2	GAUSSIAN	on	Opt=VeryTight Freq=Raman
B3LYP, PBE0	GAUSSIAN	on	<pre>EmpiricalDispersion=GD3BJ DenFit Integral(Grid=SuperFineGrid)</pre>
B2PLYP			Opt=VeryTight Freq=Raman)
CCSD(T)	CFOUR	on	FROZEN_CORE=ON SCF_CONV=10 CC_CONV=10 CC_PROGRAM=ECC VIB=EXACT
			ABCDTYPE=AOBASIS LINEQ_CONV=10 GEO_CONV=10 GEO_METHOD=NR

#### Quartic force field and Fermi resonance constant W

Expanding the PES as a Taylor series up to fourth order in dimensionless normal coordinates  $\mathbf{q}$  around the equilibrium, one obtains

$$\frac{V(\mathbf{q})}{hc} = \sum_{k} \omega_k q_k^2 + \frac{1}{3!} \sum_{klm} \phi_{klm} q_k q_l q_m + \frac{1}{4!} \sum_{klmn} \phi_{klmn} q_k q_l q_m q_n, \tag{1}$$

where  $\phi$  are the derivatives of the potential energy with respect to dimensionless normal coordinates (using a nonrestrictive summation) and  $\omega$  the harmonic frequencies. From the available formic acid PES (with analytical Hessians) fortran routine by Tew and Mizukami,<sup>[6]</sup> the quartic force field (QFF) was obtained by two-step numerical differentiation in normal coordinates using the PyPES program package<sup>[7]</sup> with default settings and the following reference geometries:

```
5
Equilibrium geometry of trans-formic acid in bohr
С
 0.7813670998 0.000000000 0.1870289330
 2.8443983303 0.000000000 0.0625424285
Η
0 -0.4067012605 0.000000000 2.1159295718
0 -0.2299101242 0.000000000 -2.1389130948
H -2.0445178745 0.000000000 -1.9246987056
5
Equilibrium geometry of cis-formic acid in bohr
  0.7563063927
                0.000000000 0.1917690324
С
Η
 2.8307835834
                0.000000000 0.0574837406
0 -0.3789085156
                0.000000000 2.1376897827
                0.000000000 -2.0713552046
0
 -0.4190401473
  0.8280301033
                0.000000000 -3.3936226467
Η
```

From the cubic force constants  $\phi$ , model off-diagonal matrix elements W are obtained by scaling them with appropriate factors<sup>[8]</sup> for Fermi type 1 ( $\omega_i \approx 2\omega_j$ )

$$W_{ijj} = \frac{\phi_{ijj}}{4} \tag{2}$$

and type 2 resonances  $(\omega_i \approx \omega_j + \omega_k)$ 

$$W_{ijk} = \frac{\phi_{ijk}}{\sqrt{8}}.$$
(3)

# 2 Formic acid fundamentals

**Table S3:** Harmonic wavenumbers ( $\omega$ , in cm<sup>-1</sup>), IR oscillator strengths ( $S_{\rm IR}$ , in km mol<sup>-1</sup>), and Raman scattering cross-sections ( $\sigma_{\rm Ra}$ , in 10<sup>-36</sup> m<sup>2</sup> sr<sup>-1</sup>) for all four H/D isotopologues of *cis*- and *trans*-formic acid, computed at the B3LYP-D3(BJ)/aVTZ level.

			$\nu_1$	$\nu_2$	$\nu_3$	$ u_4 $	$\nu_5$	$\nu_6$	$\nu_7$	$\nu_8$	$\nu_9$
	trans	ω	3716.5	3048.7	1811.4	1401.9	1298.7	1121.6	629.1	1051.6	674.6
		$S_{ m IR}$	60.5	39.7	372.9	2.4	9.0	262.1	41.6	2.0	137.2
нсоон		$\sigma_{ m Ra}$	76.1	171.8	39.7	19.2	9.9	12.5	34.5	6.1	4.4
		ω	3781.5	2962.5	1856.2	1414.1	1269.0	1104.5	659.9	1032.5	530.5
	cis	$S_{ m IR}$	59.0	73.7	309.9	0.4	309.4	57.5	9.4	0.0	82.7
		$\sigma_{ m Ra}$	104.4	172.2	47.5	14.9	8.2	50.8	4.1	7.4	9.7
		ω	3716.3	2267.2	1779.0	984.9	1294.9	1157.5	623.2	885.8	664.0
	trans	$S_{\mathrm{IR}}$	61.4	58.0	367.2	50.0	0.6	200.7	42.0	1.3	132.1
рсоон		$\sigma_{ m Ra}$	78.4	124.3	56.0	9.9	12.2	20.3	32.8	1.4	$\begin{array}{c} \nu_9 \\ \hline 0.74.6 \\ 137.2 \\ 4.4 \\ 530.5 \\ 82.7 \\ 9.7 \\ \hline 0.664.0 \\ 132.1 \\ 5.4 \\ 525.5 \\ 91.3 \\ 10.9 \\ \hline 0.530.5 \\ 84.7 \\ 1.4 \\ \hline 0.399.0 \\ 30.7 \\ 3.2 \\ \hline 77.6 \\ 0.8 \\ 0.393.2 \\ 77.2 \\ 2.9 \\ \hline \end{array}$
DCOOH		ω	3781.4	2206.2	1819.3	988.9	1263.1	1147.9	653.3	876.3	525.5
	cis	$S_{ m IR}$	59.2	93.3	291.5	11.8	274.8	61.3	8.7	1.0	91.3
		$\sigma_{ m Ra}$	104.0	117.8	70.6	31.1	11.1	31.3	3.9	1.6	10.9
	trans	ω	2702.1	3049.3	1805.5	1392.6	999.0	1186.4	561.6	1049.4	530.5
		$S_{\mathrm{IR}}$	40.0	35.9	355.5	2.6	72.1	170.3	41.9	0.5	84.7
UCOOD		$\sigma_{ m Ra}$	75.9	160.9	39.2	17.0	6.7	18.6	31.1	6.9	1.4
псоор		ω	2752.8	2962.5	1849.8	1412.9	913.9	1171.0	630.5	1032.5	399.0
	cis	$S_{ m IR}$	38.2	71.4	335.3	1.6	6.3	290.9	9.8	0.0	30.7
		$\sigma_{ m Ra}$	85.8	175.9	49.6	14.4	37.8	16.6	1.8	7.4	3.2
		ω	2702.7	2266.0	1775.1	1049.6	959.9	1179.9	557.6	885.7	512.8
	trans	$S_{\mathrm{IR}}$	36.3	57.4	351.6	5.3	58.5	158.9	41.9	1.7	77.6
DCOOD		$\sigma_{ m Ra}$	65.3	126.6	54.8	12.5	7.2	23.3	29.8	1.5	0.8
		ω	2753.0	2204.8	1814.6	1045.6	891.4	1172.7	623.2	875.9	393.2
	cis	$S_{\mathrm{IR}}$	36.3	92.2	316.6	37.7	2.9	239.8	9.1	1.7	37.2
		$\sigma_{ m Ra}$	89.5	118.6	71.4	14.1	38.2	16.1	1.9	1.7	2.9

**Table S4:** Comparison of Raman jet (this work) and Raman gas phase band positions (Bertie *et al.*<sup>[9, 10]</sup>) for *trans*formic acid. Differences in nomenclature employed by Bertie *et al.* (Herzberg nomenclature for each isotopologue) and this work (Herzberg nomenclature for HCOOH applied to all isotopologues) are highlighted in red. All reported values are in units of cm<sup>-1</sup>. Fermi resonance doublets are indicated by braces.

											-			
HCOOH				DCOOH			HCOOD			DCOOD				
This work Bertie <i>et al.</i>			This work Bertie <i>et al.</i>		Tł	This work Bertie <i>et al.</i>		Th	is work	Bertie <i>et al.</i>				
						trar	is							
$\nu_1  \begin{cases} 3570\\ 3567 \end{cases}$	3568.9	$\left. \right\}  \nu_1$	$\nu_1$	3569	3566	$ u_1 $	$\nu_1$	2631	2631.4	$ u_2 $	$\nu_1$	2632	2631.9	$\nu_1$
$\nu_2$ 2942	2942.0	$\nu_2$	$\nu_2$	2219	2218	$\nu_2$	$\nu_2$	$\left\{\begin{array}{c}2954\\2938\end{array}\right.$	$2938.2^{b}$	$\big\}  \nu_1$	$\nu_2$	$\left\{\begin{array}{c} 2231\\2194\end{array}\right.$	$2231.8 \\ 2195.1$	$\left. \right\} \nu_2$
$\nu_3$ 1776	1776.6	$ u_3$	$\nu_3$	$ \left\{\begin{array}{c} 1762 \\ 1725 \end{array}\right. $	$\begin{array}{c} 1760 \\ 1724 \end{array}$	$\left. \right\}  \nu_3$	$\nu_3$	1772	1773	$\nu_3$	$\nu_3$	$\begin{cases} 1761 \\ 1725 \end{cases}$	$1760.0 \\ 1723.5$	$\left. \right\} \nu_3$
$\nu_4$ 1379	1380.6	$\nu_4$	$\nu_4$	971		$\nu_6$	$\nu_4$	1365	1368	$ u_4 $	$\nu_4$	1039	1042	$\nu_5$
$\left \begin{array}{c} \nu_5 \\ 2\nu_9 \end{array}\right  \left \begin{array}{c} 1306 \\ 1220 \end{array}\right $	$1307.1^{a}$	$\begin{cases} \nu_5 \\ 2\nu_9 \end{cases}$	$ \frac{\nu_5}{2\nu_9} $	1299 1206	1297	$\begin{cases} \nu_4 \\ 2\nu_9 \end{cases}$	$\begin{array}{c} \nu_5\\ 2\nu_9 \end{array}$	<pre></pre>	$972^{c}$	$\begin{cases} \nu_6 \\ 2\nu_9 \end{cases}$	$\nu_5$	945	945.0	$ u_6$
$\nu_6$ 1104	1103.8	$\nu_6$	$\nu_6$	1142	1140	$\nu_5$	$\nu_6$	1176	1177.9	$\nu_5$	$\nu_6$	1170	1169.7	$ u_4$
$\nu_7$ 626	624.9	$\nu_7$	$\nu_7$	620	620	$\nu_7$	$\nu_7$	558	560.2	$\nu_7$	$\nu_7$	554	555.5	$\nu_7$
$\nu_8$		$\nu_8$	$\nu_8$			$\nu_8$	$\nu_8$			$\nu_8$	$\nu_8$			$\nu_8$
$\nu_9$	642	$\nu_9$	$\nu_9$			$\nu_9$	$\nu_9$			$ u_9$	$\nu_9$			$\nu_9$

<sup>a</sup> Originally assigned to  $2\nu_9$  by Bertie *et al.* 

<sup>b</sup> Bertie *et al.* orignally assigned a Fermi resonance doublet at 2941.8 and 2938.2 to  $\nu_1/\nu_3 + \nu_6$ . The band at 2941.8 is most likely  $\nu_2$  of *trans*-HCOOH.

<sup>c</sup> Tentative assignment by Bertie *et al.* 



**Figure S1:** Harmonic wavenumbers (in cm<sup>-1</sup>) of A' symmetric fundamentals and overtones in the spectral windows between 900 and 1600 cm<sup>-1</sup> as a function of the relative C–H proton mass of *trans*-formic acid (left O–H, right O–D). This figure is the same as Fig. 6 in the main manuscript, showing other electronic structure methods than B3LYP-D3. Grey bars indicate fractional masses which equal the mass of hydrogen or deuterium. Harmonically avoided crossings due to mode-mixing are indicated by a red disk in the interaction region, in which twice the effective coupling constant 2W is printed in bold letters. Green disks highlight the Fermi resonance-coupled states  $\nu_5$  and  $2\nu_9$  in *trans*-HCOOH, -DCOOH, and -HCOOD.

## 3 Spectra of *cis*-formic acid



Figure S2: Raman jet spectra of the four H/D isotopologues of formic acid in the O–H (top) and O–D (bottom) stretching region ( $\nu_1$ ) with increasing nozzle temperature. For each isotopologue, the spectra have been intensity-scaled to the  $\nu_1$  band of *trans*-formic acid with the lowest intensity amongst the four temperatures. For an overview of the overall spectral intensity, the 160 °C spectrum is additionally shown further intensity-scaled (×0.05) in grey. The band position (in cm<sup>-1</sup>) is shown for the  $\nu_1$  fundamentals of the respective *cis*-conformers. For each isotopologue, harmonic  $\nu_1(trans)$ -scaled line positions (calculated at the B3LYP-D3(BJ)/aVTZ level) are shown in green using scaling factors of 0.960, 0.960, 0.974, and 0.974 (top to bottom). In case of Fermi resonance-doubling, as observed for HCOOH, the resonance band centre is used as a scaling reference.



Figure S3: Raman jet spectra of the four H/D isotopologues of formic acid in the C–H (top) and C–D (bottom) stretching region ( $\nu_2$ ) with increasing nozzle temperature. For each isotopologue, the spectra have been intensity-scaled to the  $\nu_2$  band of *trans*-formic acid with the lowest intensity amongst the four temperatures. For an overview of the overall spectral intensity, the 160 °C spectrum is additionally shown further intensity-scaled (×0.05) in grey. The band position (in cm<sup>-1</sup>) is shown for the  $\nu_2$  fundamentals of the respective *cis*-conformers. For each isotopologue, harmonic  $\nu_2(trans)$ -scaled line positions (calculated at the B3LYP-D3(BJ)/aVTZ level) are shown in green using scaling factors of 0.965, 0.966, 0.979, and 0.979 (top to bottom). In case of Fermi resonance-doubling, as observed for HCOOD and DCOOD, the resonance band centre is used as a scaling reference. Bands of isotopic impurities are labelled as  $\ddagger$ .



Figure S4: Raman jet spectra of the four H/D isotopologues of formic acid in the C=O stretching region ( $\nu_3$ ) with increasing nozzle temperature for C-hydrogenated (top) and C-deuterated (bottom) formic acids. For each isotopologue, the spectra have been intensity-scaled to the  $\nu_3$  band of *trans*-formic acid with the lowest intensity amongst the four temperatures. For an overview of the overall spectral intensity, the 160 °C spectrum is additionally shown further intensity-scaled ( $\times 0.05$ ) in grey. The band position (in cm<sup>-1</sup>) is shown for the  $\nu_3$  fundamentals of the respective *cis*-conformers. For each isotopologue, harmonic  $\nu_3(trans)$ -scaled line positions (calculated at the B3LYP-D3(BJ)/aVTZ level) are shown in green using scaling factors of 0.981, 0.982, 0.984, and 0.982 (top to bottom). In case of Fermi resonance-doubling, as observed for DCOOH and DCOOD, the resonance band centre is used as a scaling reference. Bands of isotopic impurities are labelled as  $\ddagger$ .



Figure S5: (top) Raman jet spectra of the O–D bending region ( $\nu_5$ ) with increasing nozzle temperature. For each isotopologue, the spectra have been intensity-scaled to the  $\nu_6$  band (not shown) of *trans*-formic acid with the lowest intensity amongst the four temperatures. For an overview of the overall spectral intensity, the 160 °C spectrum of HCOOD is additionally shown further intensity-scaled (×0.05) in grey. The band position (in cm<sup>-1</sup>) is shown for the  $\nu_5$  fundamentals of the respective *cis*-conformers. For each isotopologue, harmonic  $\nu_5(trans)$ -scaled line positions (calculated at the B3LYP-D3(BJ)/aVTZ level) are shown in green using scaling factors of 0.982 and 0.984 for HCOOD and DCOOD, respectively. In case of Fermi resonance-doubling, as observed for HCOOD, the resonance band centre is used as a scaling reference. (bottom) Raman jet spectra of the O–D bending region ( $\nu_5$ ) with the polarisation of the incident laser (operated at 20 W) perpendicular ( $\perp$ , default for all other measurements) and parallel (||) with respect to the scattering plane,<sup>[11]</sup> and residual after subtraction ( $\perp -7/6$  ||).<sup>[12]</sup> Bands with smaller depolarisation ratios persist more in the residual spectrum. The band position of  $2\nu_9$  has been estimated for DCOOD, see caption to Figure 4 for further details.

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