## Supplementary Materials for Towards Accurate Prediction for Laser-Coolable <br> Molecules: Relativistic Coupled-Cluster Calculations for Yttrium Monoxide and Prospects for Improving its Laser Cooling Efficiencies

The supplementary information includes a demonstration of the sensitivity of computed Franck-Condon factors (FCFs) with respect to harmonic frequencies (Table I) and FranckCondon factors for a complete list of transitions between electronic states (Table II).

TABLE I. Calculated FCFs using Morse potentials with parameters obtained at the EOM$\mathrm{CCSD} / \infty \mathrm{Z} / \mathrm{sc}+\Delta \mathrm{T}$ level and with the use of various values for the harmonic frequency of the $X^{2} \Sigma$ state. The original EOM-CCSD $/ \infty \mathrm{Z} / \mathrm{sc}+\Delta \mathrm{T}$ harmonic frequency for the $X^{2} \Sigma$ state is 853.5 $\mathrm{cm}^{-1}$. As can be seen, a shift of harmonic frequencies by $50 \mathrm{~cm}^{-1}$ only changes individual computed FCFs by less than $1 \%$.

| transitions | $\nu$ | Harmonic frequencies for $X^{2} \Sigma$ state ( $\mathrm{cm}^{-1}$ ) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 803.5 | 823.5 | 843.5 | 853.5 | 863.5 | 883.5 | 903.5 |
| $A^{2} \Pi(0) \rightarrow X^{2} \Sigma(\nu)$ | 0 | 99.699\% | 99.689\% | 99.665\% | 99.648\% | 99.627\% | 99.577\% | 99.515\% |
|  | 1 | 0.295\% | 0.310\% | 0.326\% | 0.333\% | 0.340\% | 0.354\% | 0.368\% |
|  | 2 | 0.006\% | <0.001\% | 0.009\% | 0.019\% | 0.032\% | 0.069\% | 0.117\% |
| $B^{2} \Sigma(0) \rightarrow X^{2} \Sigma(\nu)$ | 0 | 82.819\% | 82.552\% | 82.280\% | 82.142\% | 82.003\% | 81.724\% | 81.441\% |
|  | 1 | 16.083\% | 16.129\% | 16.163\% | 16.176\% | 16.187\% | 16.202\% | 16.208\% |
|  | 2 | 1.066\% | 1.265\% | 1.472\% | 1.579\% | 1.687\% | 1.909\% | 2.136\% |
|  | 3 | 0.031\% | 0.053\% | 0.082\% | 0.098\% | 0.116\% | 0.155\% | 0.199\% |
|  | 4 | <0.001\% | 0.001\% | 0.003\% | 0.004\% | 0.006\% | 0.010\% | 0.015\% |
|  | 5 | <0.001\% | <0.001\% | <0.001\% | <0.001\% | <0.001\% | $<0.001 \%$ | 0.001\% |
| $A^{\prime 2} \Delta(0) \rightarrow X^{2} \Sigma(\nu)$ | 0 | 87.448\% | 87.239\% | 87.023\% | 86.912\% | 86.800\% | 86.571\% | 86.338\% |
|  | 1 | 12.061\% | 12.122\% | $12.174 \%$ | 12.197\% | $12.218 \%$ | $12.253 \%$ | 12.281\% |
|  | 2 | 0.485\% | 0.623\% | 0.773\% | 0.853\% | 0.934\% | 1.105\% | 1.284\% |
|  | 3 | 0.006\% | 0.015\% | 0.029\% | 0.037\% | 0.046\% | 0.067\% | 0.092\% |
|  | 4 | $<0.001 \%$ | $<0.001 \%$ | 0.001\% | 0.001\% | 0.002\% | 0.003\% | 0.005\% |

TABLE II. Calculated Franck-Condon factors using EOM-CCSD $/ \infty \mathrm{Z} / \mathrm{sc}+\Delta \mathrm{T}+\Delta \mathrm{SO}$ ab initio potential energy surfaces.

| transitions $/ \nu$ | 0 | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $A^{2} \Pi_{1 / 2}(0) \rightarrow X^{2} \Sigma_{1 / 2}(\nu)$ | $99.563 \%$ | $0.416 \%$ | $0.020 \%$ | $<0.001 \%$ | $<0.001 \%$ | $<0.001 \%$ |
| $A^{2} \Pi_{3 / 2}(0) \rightarrow X^{2} \Sigma_{1 / 2}(\nu)$ | $99.687 \%$ | $0.293 \%$ | $0.019 \%$ | $<0.001 \%$ | $<0.001 \%$ | $<0.001 \%$ |
| $A^{\prime 2} \Delta_{3 / 2}(0) \rightarrow X^{2} \Sigma_{1 / 2}(\nu)$ | $86.600 \%$ | $12.464 \%$ | $0.895 \%$ | $<0.040 \%$ | $0.001 \%$ | $<0.001 \%$ |
| $A^{\prime 2} \Delta_{5 / 2}(0) \rightarrow X^{2} \Sigma_{1 / 2}(\nu)$ | $87.209 \%$ | $11.936 \%$ | $0.820 \%$ | $<0.035 \%$ | $0.001 \%$ | $<0.001 \%$ |
| $B^{2} \Sigma_{1 / 2}(0) \rightarrow X^{2} \Sigma_{1 / 2}(\nu)$ | $82.411 \%$ | $15.954 \%$ | $1.536 \%$ | $<0.094 \%$ | $0.004 \%$ | $<0.001 \%$ |
| $A^{2} \Pi_{1 / 2}(0) \rightarrow A^{\prime 2} \Delta_{3 / 2}(\nu)$ | $90.762 \%$ | $8.623 \%$ | $0.580 \%$ | $0.034 \%$ | $0.002 \%$ | $<0.001 \%$ |
| $A^{2} \Pi_{3 / 2}(0) \rightarrow A^{\prime 2} \Delta_{3 / 2}(\nu)$ | $90.169 \%$ | $9.139 \%$ | $0.650 \%$ | $0.040 \%$ | $0.002 \%$ | $<0.001 \%$ |
| $A^{2} \Pi_{1 / 2}(0) \rightarrow A^{\prime 2} \Delta_{5 / 2}(\nu)$ | $91.279 \%$ | $8.167 \%$ | $0.523 \%$ | $0.029 \%$ | $0.001 \%$ | $<0.001 \%$ |
| $A^{2} \Pi_{3 / 2}(0) \rightarrow A^{\prime 2} \Delta_{5 / 2}(\nu)$ | $90.700 \%$ | $8.675 \%$ | $0.589 \%$ | $0.034 \%$ | $0.002 \%$ | $<0.001 \%$ |
| $B^{2} \Sigma_{1 / 2}(0) \rightarrow A^{2} \Pi_{1 / 2}(\nu)$ | $87.118 \%$ | $12.233 \%$ | $0.633 \%$ | $0.015 \%$ | $<0.001 \%$ | $<0.001 \%$ |
| $B^{2} \Sigma_{1 / 2}(0) \rightarrow A^{2} \Pi_{3 / 2}(\nu)$ | $86.443 \%$ | $12.834 \%$ | $0.706 \%$ | $0.018 \%$ | $<0.001 \%$ | $<0.001 \%$ |
| $B^{2} \Sigma_{1 / 2}(0) \rightarrow A^{\prime 2} \Delta_{3 / 2}(\nu)$ | $99.635 \%$ | $0.365 \%$ | $<0.001 \%$ | $<0.001 \%$ | $<0.001 \%$ | $<0.001 \%$ |
| $B^{2} \Sigma_{1 / 2}(0) \rightarrow A^{\prime 2} \Delta_{5 / 2}(\nu)$ | $99.514 \%$ | $0.485 \%$ | $0.001 \%$ | $<0.001 \%$ | $<0.001 \%$ | $<0.001 \%$ |

