Supplementary Information for: Rapid survey of nuclear quadrupole resonance by broadband excitation with comb modulation and dual-mode acquisition Yuta Hibe ${ }^{1}$, Yasuto Noda ${ }^{1}$, K. Takegoshi ${ }^{1}$, and Kazuyuki Takeda ${ }^{1}$ ${ }^{1}$ Division of Chemistry, Graduate School of Science, Kyoto University, 606-8502 Kyoto, Japan

## 1 Derivation of Eqs. (2)-(4)

In general, an excitation pulse $f(t)$ with amplitude $\omega_{1}(t)$ and phase $\phi(t)$ is expressed as

$$
\begin{equation*}
f(t)=A e^{i \phi(t)} . \tag{S1}
\end{equation*}
$$

Now, let us recall Eq. (1):

$$
\begin{align*}
g_{1}(t) & =\omega_{\max }[\operatorname{sech}(\beta t)]^{1-i \mu} \\
& =\omega_{\max } \operatorname{sech}(\beta t)[\operatorname{sech}(\beta t)]^{-i \mu} \tag{S2}
\end{align*}
$$

Comparing Eq. (S2) with Eq. (S1), we obtain

$$
\begin{align*}
\omega_{1}(t) & =\omega_{\max } \operatorname{sech}(\beta t),  \tag{S3}\\
e^{i \phi(t)} & =[\operatorname{sech}(\beta t)]^{-i \mu} . \tag{S4}
\end{align*}
$$

By taking natural logarithm of both sides of Eq. (S4), we obtain

$$
\begin{align*}
\ln [\operatorname{sech}(\beta t)]^{-i \mu} & =\ln e^{i \phi(t)},  \tag{S5}\\
-i \mu \ln [\operatorname{sech}(\beta t)] & =i \phi(t), \tag{S6}
\end{align*}
$$

so that

$$
\begin{equation*}
\phi(t)=-\mu \ln [\operatorname{sech}(\beta t)] . \tag{S7}
\end{equation*}
$$

The frequency in Eq. (3) is obtained by taking time derivative on Eq. (S4). Noting that

$$
\begin{equation*}
\operatorname{sech}(x)=\frac{1}{\cosh x} \tag{S8}
\end{equation*}
$$

and

$$
\begin{equation*}
(\operatorname{sech}(x))^{\prime}=\left(\frac{1}{\cosh x}\right)^{\prime}=\frac{-\sinh c}{\cosh ^{2} x}=-\frac{\tanh x}{\cosh x}=-\tanh x \operatorname{sech} x \tag{S9}
\end{equation*}
$$

we obtain

$$
\begin{align*}
\frac{d \phi(t)}{d t} & =-\mu \frac{(\operatorname{sech}(\beta t))^{\prime}}{\operatorname{sech}(\beta t)} \\
& =-\mu \frac{-\tanh (\beta t) \operatorname{sech}(\beta t) \cdot \beta}{\operatorname{sech}(\beta t)} \\
& =\mu \beta \tanh (\beta t)=\Delta \omega \tag{S10}
\end{align*}
$$

## $2{ }^{35} \mathrm{Cl}$ NQR rapid scan and FID signals of $\mathrm{KClO}_{3}$ with various frequency offsets

All the following figures show the rapid scan signal (upper) and FID signal (lower) in ${ }^{35} \mathrm{Cl}$ NQR experiments of $\mathrm{KClO}_{3}$ when the frequency offset of sweep center from resonance $\Delta \omega_{\mathrm{c}}$ was $\pm 2 \pi \cdot 10,-2 \pi \cdot 50, \pm 2 \pi \cdot 100, \pm 2 \pi \cdot 150$ and $\pm 2 \pi \cdot 200 \mathrm{kHz}$. Red and green lines describe the in-phase and quadrature components of the magnetization, respectively. The combmodulated HS pulse was aborted at time $t$ indicated in the captions and in the figures by the broken lines.


Figure S1: $\Delta \omega_{\mathrm{c}}=2 \pi \cdot 10 \mathrm{kHz}, t=4.68 \mathrm{~ms}$.


Figure S2: $\Delta \omega_{\mathrm{c}}=2 \pi \cdot 100 \mathrm{kHz}, t=5.10 \mathrm{~ms}$.


Figure S3: $\Delta \omega_{\mathrm{c}}=2 \pi \cdot 150 \mathrm{kHz}, t=5.10 \mathrm{~ms}$.


Figure S4: $\Delta \omega_{\mathrm{c}}=2 \pi \cdot 200 \mathrm{kHz}, t=5.10 \mathrm{~ms}$.


Figure S5: $\Delta \omega_{\mathrm{c}}=-2 \pi \cdot 10 \mathrm{kHz}, t=5.46 \mathrm{~ms}$.


Figure S6: $\Delta \omega_{\mathrm{c}}=-2 \pi \cdot 50 \mathrm{kHz}, t=5.10 \mathrm{~ms}$.


Figure S7: $\Delta \omega_{\mathrm{c}}=-2 \pi \cdot 100 \mathrm{kHz}, t=5.10 \mathrm{~ms}$.


Figure S8: $\Delta \omega_{\mathrm{c}}=-2 \pi \cdot 150 \mathrm{kHz}, t=5.10 \mathrm{~ms}$.


Figure S9: $\Delta \omega_{\mathrm{c}}=-2 \pi \cdot 200 \mathrm{kHz}, t=5.10 \mathrm{~ms}$.

