

SUPPLEMENTARY INFORMATION TO  
**Fingerprint region of the formic acid dimer: variational  
vibrational computations in curvilinear coordinates**

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TABLE I. Normal coordinate definition of FAD corresponding to the QB16-PES: equilibrium structure,  $c_{i\alpha}$  in bohr with  $i = 1(\text{H}^{(A)}), 2(\text{H}^{(A)}), 5(\text{O}^{(A)}), 6(\text{O}^{(A)}), 9(\text{C}^{(A)}), 3(\text{H}^{(B)}), 4(\text{H}^{(B)}), 7(\text{O}^{(B)}), 8(\text{O}^{(B)}), 10(\text{C}^{(B)})$  and  $\alpha = x, y, z$ , and  $l_{\alpha i, j}$  elements used to calculate the  $Q_j$  dimensionless normal coordinates ( $j = 1, \dots, 24$ ). The harmonic frequencies,  $\tilde{\nu}^{\text{HO}}$  in  $\text{cm}^{-1}$ , are also given in the table.  $r_{i\alpha} = c_{i\alpha} + \sum_{j=1}^{24} Q_j l_{\alpha i, j}$

	$Q_1$	$Q_2$	$Q_3$	$Q_4$	$Q_5$	$Q_6$	$Q_7$	$Q_8$
$\tilde{\nu}^{\text{HO}}$	70.079	169.973	170.393	208.841	253.821	275.148	692.643	715.565
	$l_{\alpha i, j}$							
$c_{1x}$	0	-0.0550489068	0	-0.0221897688	0	0.1027336927	0.0106811053	0.0078803994
$c_{1y}$	0.0888640999	0	-0.3653231756	0	-0.2807202366	0	0	0
$c_{1z}$	0	0.0195732544	0	-0.0809832850	0	0.0110746966	-0.0364919882	-0.0370286239
$c_{2x}$	-2.1440160385	0.0678179381	0	0.0208023614	0	-0.0608445441	0.1078648560	0.0925684085
$c_{2y}$	0	0.0306454227	0	0.1749988335	0	0.0906722954	0	0
$c_{2z}$	-0.8915699548	0	0.1081730397	0	-0.0595051283	0	-0.0790462710	0.0205304089
$c_{3x}$	-2.1349725967	0.0508401068	0	0.01271590328	0	-0.0029861332	0.0369000130	0.0364542139
$c_{3y}$	0.1530793141	0	0.0551154157	0	0.0477433605	0	0	0
$c_{3z}$	-2.7675548795	0.1027221555	0	-0.0512142095	0	-0.0756204105	0.0175902254	0.0155037351
$c_{6x}$	2.1002784911	0.0706620794	0	0.0178761714	0	-0.0029895689	-0.0407039395	-0.0425354587
$c_{6y}$	0	-0.1715444148	0	0.0324005323	0	0.0615425686	0	0
$c_{6z}$	-2.2930907605	0.0503750205	0	-0.0967866341	0	0.0748243895	0.0136926724	0.0136488812
$c_{9x}$	0.2020375225	0.0182073300	0	0.0111036095	0	0.0279448602	-0.0002598622	-0.0003293917
$c_{9y}$	0.0156158809	0	-0.1007967531	0	-0.0661305556	0	0	0
$c_{9z}$	-3.5929593001	0.0223423640	0	-0.0795590666	0	0.0067828274	-0.0376791855	-0.0372458161
$c_{3x}$	-0.2868649484	0.0550477261	0	0.0221796487	0	0.1027432996	-0.0106806258	0.0078747894
$c_{3y}$	0	0.0881415843	0	-0.3655592754	0	0.2804082358	0	0
$c_{3z}$	5.6564793125	0.0195862102	0	0.0809913977	0	-0.0110710456	0.0364889326	-0.0370303283
$c_{4x}$	2.1440156985	0.0678024450	0	-0.0208138591	0	-0.0608542506	0.1078602975	0.0925736468
$c_{4y}$	0.0306891267	0	0.1751452961	0	-0.0905441001	0	0	0
$c_{4z}$	0.8915695806	0.01081715038	0	0.0595137351	0	-0.0790584397	-0.0205293799	0.0182076727
$c_{7x}$	2.1349725840	0.0508269012	0	-0.0127196509	0	-0.0029895689	-0.0368983293	0.0364558854
$c_{7y}$	0.1528925491	0	0.0552385319	0	0.0477007959	0	0	0
$c_{7z}$	2.7675545069	0.1027219292	0	0.0512231448	0	-0.0756320465	-0.0175891492	0.0155038045
$c_{8x}$	-2.1002785866	0.0706452307	0	-0.0178800062	0	-0.0206223309	0.0407027015	-0.0425372096
$c_{8y}$	-0.1720339853	0.0325403496	0	0.0967988803	0	-0.0615074767	0	0
$c_{8z}$	2.2930911271	0.0503506096	0	0.0967988803	0	0.0748289406	-0.0136918211	0.01366503538
$c_{10x}$	-0.2020373912	0.0181990479	0	-0.0111065313	0	0.0279449116	0.0002605854	-0.0003293482
$c_{10y}$	0.0151332334	0.1007865686	0	0.1007865686	0	-0.0660394431	0	0
$c_{10z}$	3.5929593354	0.0223551486	0	0.0795697925	0	0.0067792673	0.0376772774	-0.0372472399

(Table I continued.)

	$Q_9$	$Q_{10}$	$Q_{11}$	$Q_{12}$	$Q_{13}$	$Q_{14}$	$Q_{15}$	$Q_{16}$	
$i^{\text{HO}}$	955.687	969.635	1083.808	1099.884	1254.880	1257.923	1405.225	1407.720	
	$l_{\alpha_i, j}$ :								
$c_{1x}$	0	0	0	0	0.0662994950	0.0769799509	0.1502666881	0.1765604364	
$c_{1y}$	-0.0247296491	-0.0473246894	0.1775679438	0.1730731049	0	0	0	0	
$c_{1z}$	0	0	0	-0.0249575215	-0.02444924822	0.0323523955	0.0266266895	0.0266266895	
$c_{2x}$	0	0	0	0	0.0087695806	0.07928842517	0.0766030777	0.0349121374	
$c_{2y}$	0.2324148735	0.2089462462	0.0600001271	0.0997235577	0	0	0	0	
$c_{2z}$	0	0	0	0	0.0146295325	0.01710606874	-0.0023710258	0.0054412768	
$c_{3x}$	0	0	0	-0.0336695746	-0.03296837622	0.0047141652	0.0035566887	0.0035566887	
$c_{3y}$	-0.0161034747	-0.0173582869	0.0050915586	0.0020338197	0	0	0	0	
$c_{3z}$	0	0	0	0	0.0137056925	0.01166829919	-0.0053390700	-0.0016912302	
$c_{6x}$	0	0	0	0	0.0109360953	0.00973205227	-0.0059207954	-0.0050318977	
$c_{6y}$	-0.0024295085	-0.0128374699	0.0103028971	0.0062705186	0	0	0	0	
$c_{6z}$	0	0	0	0	0.0090163634	0.00873518298	-0.0129775345	-0.0137694463	
$c_{9x}$	0	0	0	0	0.0212475911	0.01783137223	-0.0174254883	-0.0147863586	
$c_{9y}$	0.0170175144	0.0266679760	-0.0386850254	-0.0339475088	0	0	0	0	
$c_{9z}$	0	0	0	0	-0.0261151862	-0.02655809320	0.0218634733	0.014948154	
$c_{3x}$	0	0	0	0	-0.0662239388	0.07701977435	0.1486994568	-0.177910191	
$c_{3y}$	0.0248697099	-0.0472670316	-0.1776665890	0.1729598319	0	0	0	0	
$c_{3z}$	0	0	0	0	0.0249324436	-0.02447678207	0.0321180788	-0.026904760	
$c_{4x}$	0	0	0	0	-0.0686967410	0.07935782541	0.0763111573	-0.035554441	
$c_{4y}$	-0.2330859635	0.2081969713	-0.0600960861	0.0996775320	0	0	0	0	
$c_{4z}$	0	0	0	0	-0.0146126018	0.01711941275	-0.0024192361	-0.005424255	
$c_{7x}$	0	0	0	0	0.0336369793	-0.03300183542	0.0046828284	-0.0003595615	
$c_{7y}$	0.0161600581	-0.0173094723	-0.0050906410	0.0020308739	0	0	0	0	
$c_{7z}$	0	0	0	0	-0.0136938047	0.01168167391	-0.0053256537	0.001732898	
$c_{8x}$	0	0	0	0	-0.0109262153	0.00974346038	-0.0058748507	0.005085181	
$c_{8y}$	0.0024723498	-0.0128328944	-0.0103056654	0.0062635652	0	0	0	0	
$c_{8z}$	0	0	0	0	-0.00900076076	0.00874641848	-0.0128548521	0.013883743	
$c_{10x}$	0	0	0	0	-0.0212296035	0.01785354480	-0.0172972349	0.014933437	
$c_{10y}$	-0.0171025155	0.0266218433	0.0387047148	-0.0339245550	0	0	0	0	
$c_{10z}$	0	0	0	0	0.0260887832	-0.02658517678	0.0217335486	-0.015133642	

(Table I continued.)

	$Q_{17}$	$Q_{18}$	$Q_{19}$	$Q_{20}$	$Q_{21}$	$Q_{22}$	$Q_{23}$	$Q_{24}$
$p^{\text{HO}}$	1447.463	1480.190	1713.868	1778.743	3094.003	3095.465	3231.252	3325.172
	$l_{\alpha_i, j}$ :							
$c_{1x}$	0.2868654398	0.0539458530	-0.0271137400	0.0294157509	-0.0068577124	0.00690398212	-0.0036229233	-0.0006193951
$c_{1y}$	0	0	0	0	0	0	0	0
$c_{1z}$	-5.6564792624	-0.0095596860	0.0230512400	-0.0176567618	0.1336263016	0.13096390621	0.0074510990	0.0132394726
$c_{2x}$	-2.1440160385	-0.1520891779	-0.1613601303	0.0507020049	-0.0012160887	0.00024199369	-0.0056173447	-0.0058632848
$c_{2y}$	0	0	0	0	0	0	0	0
$c_{2z}$	-0.8915699548	0.0015851962	-0.0125699147	-0.0021036311	-0.0078113561	0.01374766678	0.1310048828	0.1290683147
$c_{3x}$	-2.1349725967	0.0017714739	-0.0059881835	0.0071063221	-0.0000570353	0.00007000256	-0.0004064703	0.0004206479
$c_{3y}$	0	0	0	0	0	0	0	0
$c_{3z}$	-2.7675548795	0.0134229069	-0.0009824420	0.0007835486	0.0005762451	-0.00101054142	-0.0083322496	-0.0082008171
$c_{6x}$	2.1002784911	0.0086457275	-0.0125342624	-0.0203379601	0.0003841836	-0.00021343281	-0.0011630504	0.0001235220
$c_{6y}$	0	0	0	0	0	0	0	0
$c_{6z}$	-2.2930907605	0.0073392025	-0.0038895304	-0.0146475416	0.0003754058	-0.00018770156	-0.0008181932	0.0001143304
$c_{9x}$	0.2020375225	0.0194227157	0.0345250080	-0.0371287112	0.0002009448	-0.00040886758	0.0025824167	-0.0001807620
$c_{9y}$	0	0	0	0	0	0	0	0
$c_{9z}$	-3.5929593001	0.0085878496	-0.0099552073	0.0225977040	-0.0117622828	0.01143198020	-0.0005022856	-0.0011721340
$c_{3z}$	-0.2868649484	-0.1014723075	-0.0539182353	0.0271121764	0.0067396231	0.00701871764	0.0036224445	-0.0006193446
$c_{3y}$	0	0	0	0	0	0	0	0
$c_{4x}$	5.6564793125	0.0048307456	0.0095549130	-0.0230507487	-0.1313978519	-0.13319919832	-0.0074485992	0.0132389046
$c_{4z}$	2.1440156985	0.1521756753	0.1612764888	-0.0506980876	0.0012117706	0.00026242782	0.0056168386	-0.0058647308
$c_{4y}$	0	0	0	0	0	0	0	0
$c_{4z}$	0.8915695806	-0.0071893064	-0.0015809639	0.0125692787	0.0075769931	0.01387625596	-0.1309923064	0.1290810446
$c_{7x}$	2.1349725840	-0.0027695478	-0.0017703710	0.0059882525	0.0000558376	0.00007096286	0.0004065357	0.0004206417
$c_{7y}$	0	0	0	0	0	0	0	0
$c_{7z}$	2.7675545069	-0.0136455848	-0.0134166519	0.0009823348	-0.0005590306	-0.00102001048	0.0083313875	-0.0082015189
$c_{8x}$	-2.1002785866	0.0086495091	0.0125302330	0.0203386057	-0.0003804949	-0.00021986770	0.0011630137	0.0001234232
$c_{8y}$	0	0	0	0	0	0	0	0
$c_{8z}$	2.2930911271	0.0073371909	0.0038873291	0.0146478186	-0.0003721938	-0.00019400829	0.0008181618	0.0001142268
$c_{10x}$	-0.2020373912	-0.0120840852	-0.0194152472	-0.0345258830	-0.0001939268	-0.00041227494	-0.0025823641	-0.0001805188
$c_{10y}$	0	0	0	0	0	0	0	0
$c_{10z}$	3.5929593354	0.0085976123	0.0099501442	-0.0225979539	0.0115676507	0.01162842543	0.0005020965	-0.0011721186

TABLE II. Curvilinear normal coordinate definition of FAD corresponding to the QB16-PES and the internal coordinate definition of Sec. II.A: equilibrium value of the internal coordinates  $\xi_i^{(\text{eq})}$  with  $i = 1, \dots, 6$  and  $\mathcal{L}_{i,j}$  coefficients ( $j = 1, \dots, 6$ ) corresponding to the  $\mathcal{Q}_j$  curvilinear normal coordinates. The harmonic frequencies,  $\tilde{\nu}^{\text{HO}}$  in  $\text{cm}^{-1}$ , are also given in the table.  $\xi_i = \xi_i^{(\text{eq})} + \sum_{j=1}^6 \mathcal{Q}_j \mathcal{L}_{i,j}$

		$\mathcal{Q}_1$	$\mathcal{Q}_2$	$\mathcal{Q}_3$	$\mathcal{Q}_4$	$\mathcal{Q}_5$	$\mathcal{Q}_6$
$\tilde{\nu}^{\text{HO}}$		65.757	155.438	160.447	222.384	262.267	328.281
	$\mathcal{L}_{i,j}$ :						
$R$	5.6805167586	0	-0.0000000490	0	0.1480298121	0.0078114080	0
$\cos \theta$	-0.3330272401	0	-0.0344289689	0	0.0119751350	-0.0482796603	0
$\phi$	4.7123889804	0.0890745125	0	0.0926450828	0	0	0.0616821979
$\alpha$	3.1415926536	0.0101279780	0	-0.1158548265	0	0	0.0947600155
$\cos \beta$	0.3330274046	0	-0.0344289590	0	-0.0119753490	0.0482796630	0
$\gamma$	1.5707963268	-0.0931639847	0	0.0759836591	0	0	0.0692761166

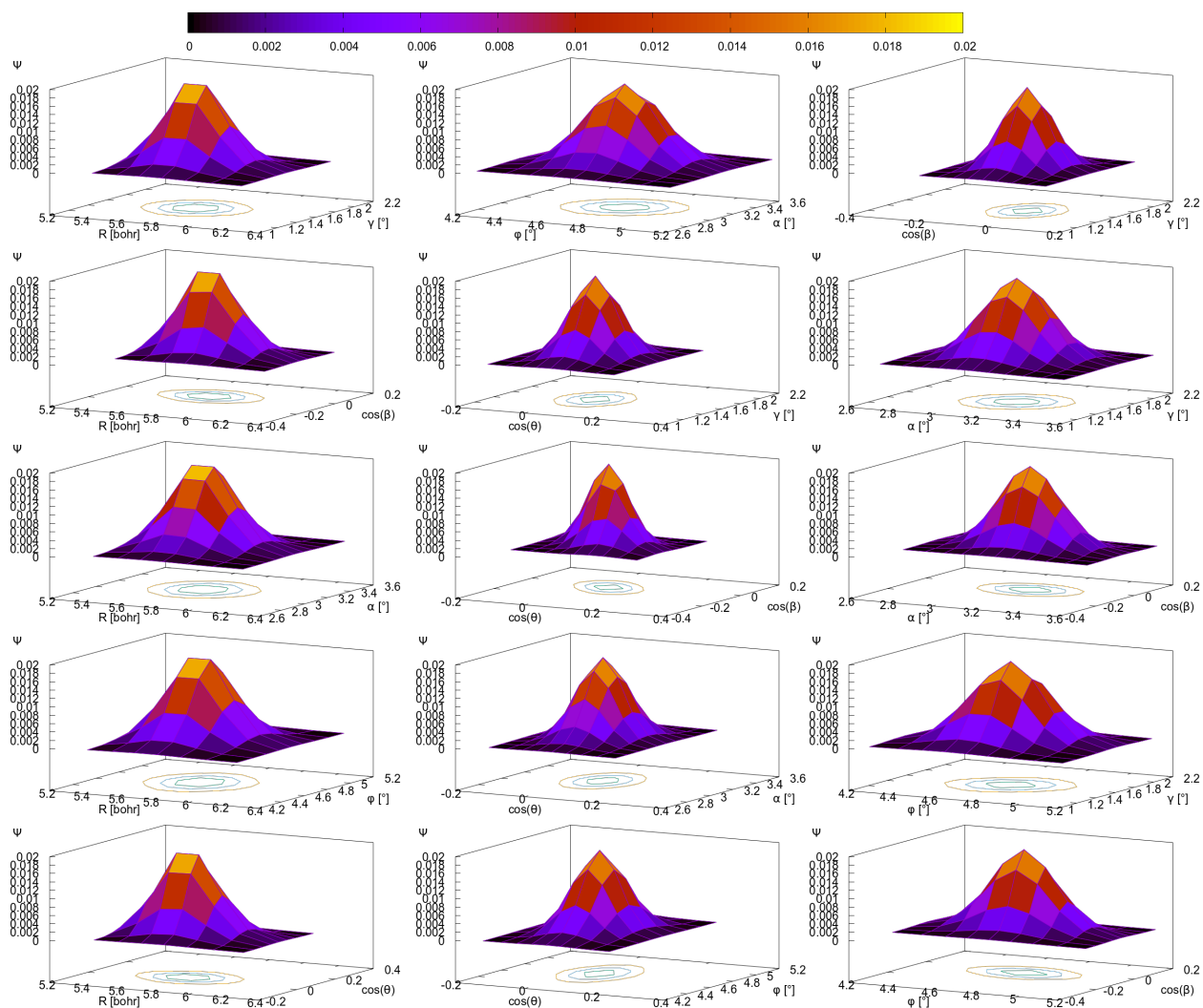


FIG. 1.  $\Psi_0$  with  $\tilde{\nu}_0 = 1532.9 \text{ cm}^{-1}$  obtained with GENIUSH using the  $8D(\mathcal{I}t)$  curvilinear KEO and the QB16-PES.

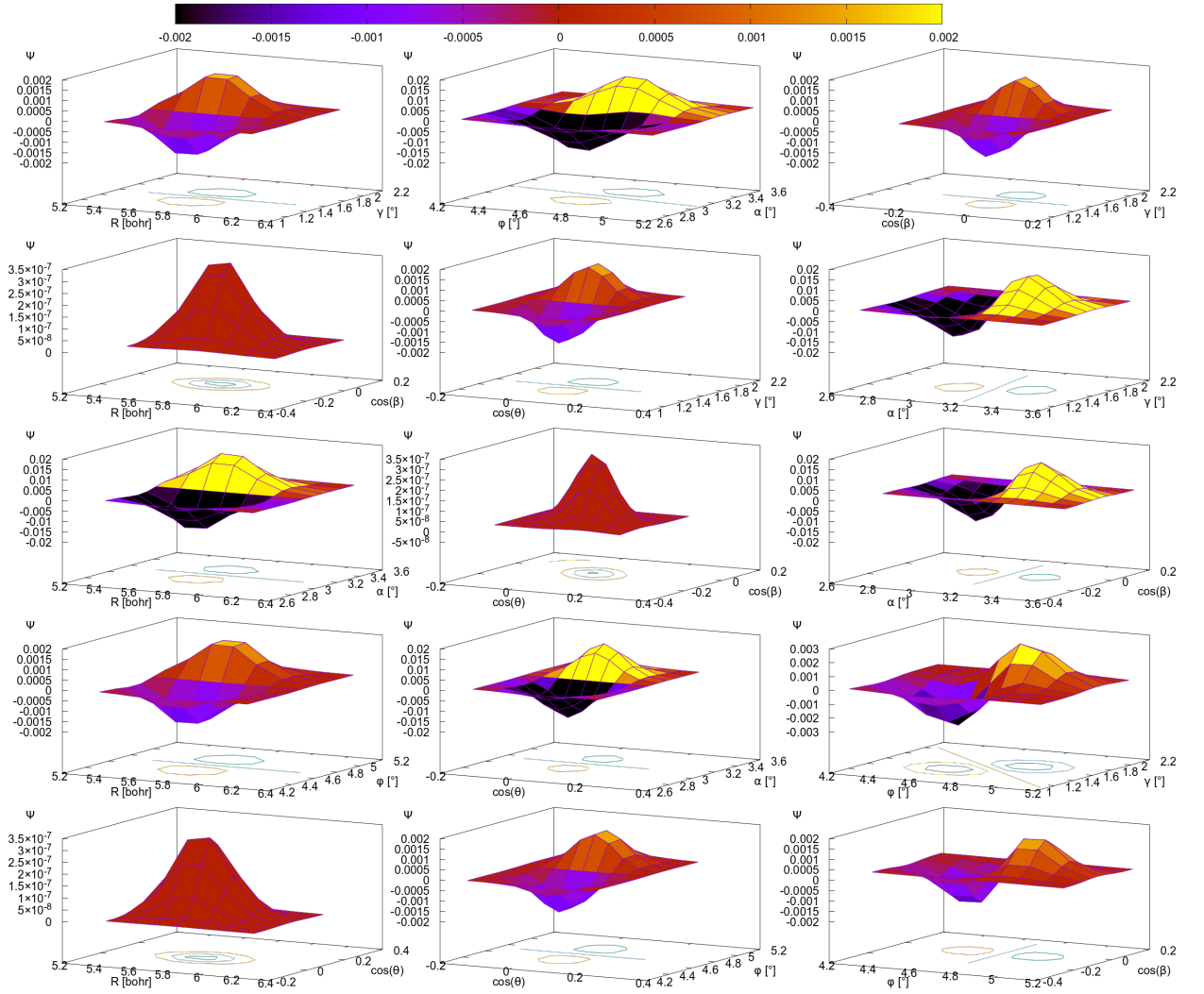


FIG. 2.  $\Psi_1$  with  $\tilde{\nu}_1 - \tilde{\nu}_0 = 70.5 \text{ cm}^{-1}$  (see also caption to Figure 1).

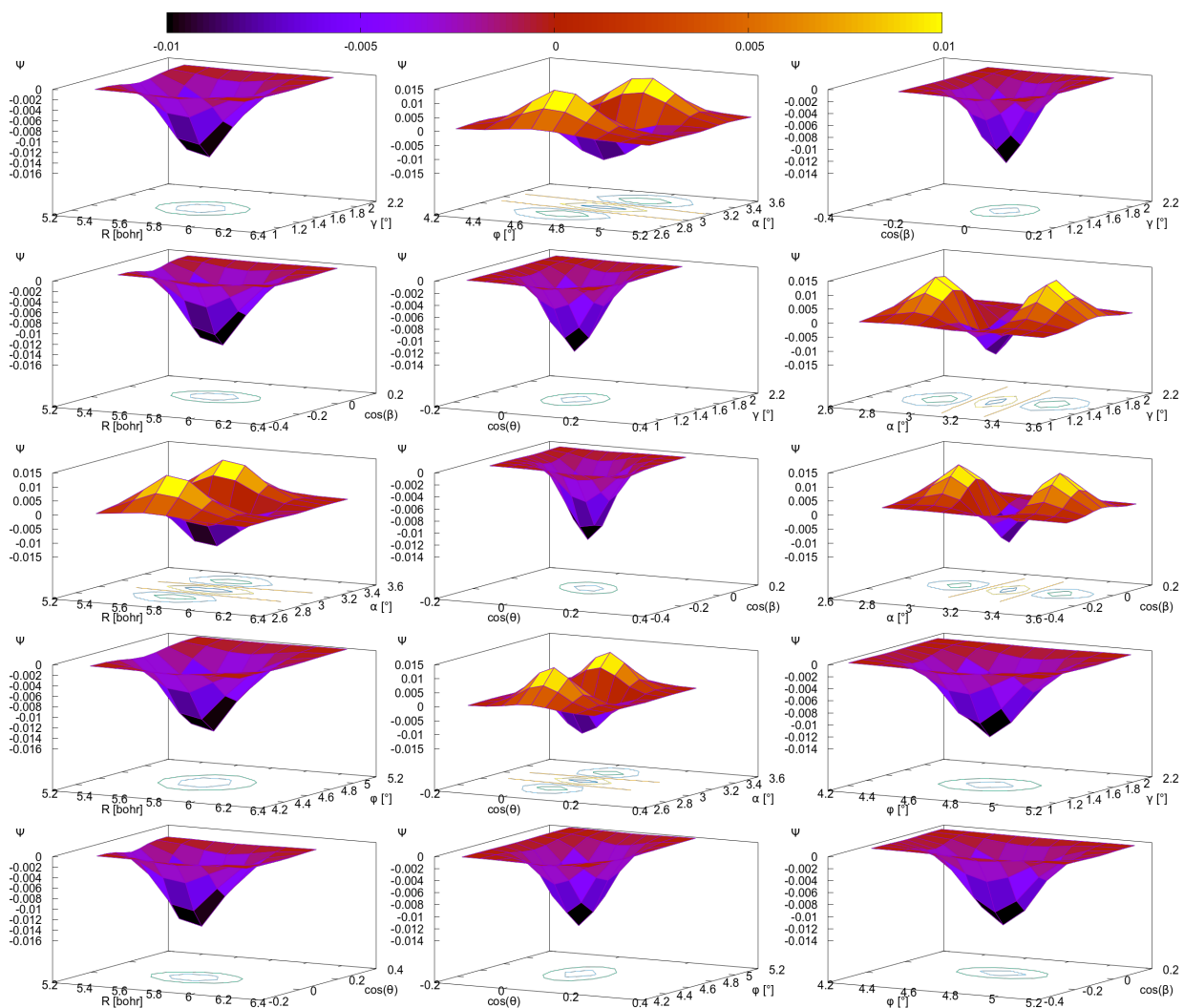


FIG. 3.  $\Psi_2$  with  $\tilde{\nu}_2 - \tilde{\nu}_0 = 140.9 \text{ cm}^{-1}$  (see also caption to Figure 1).



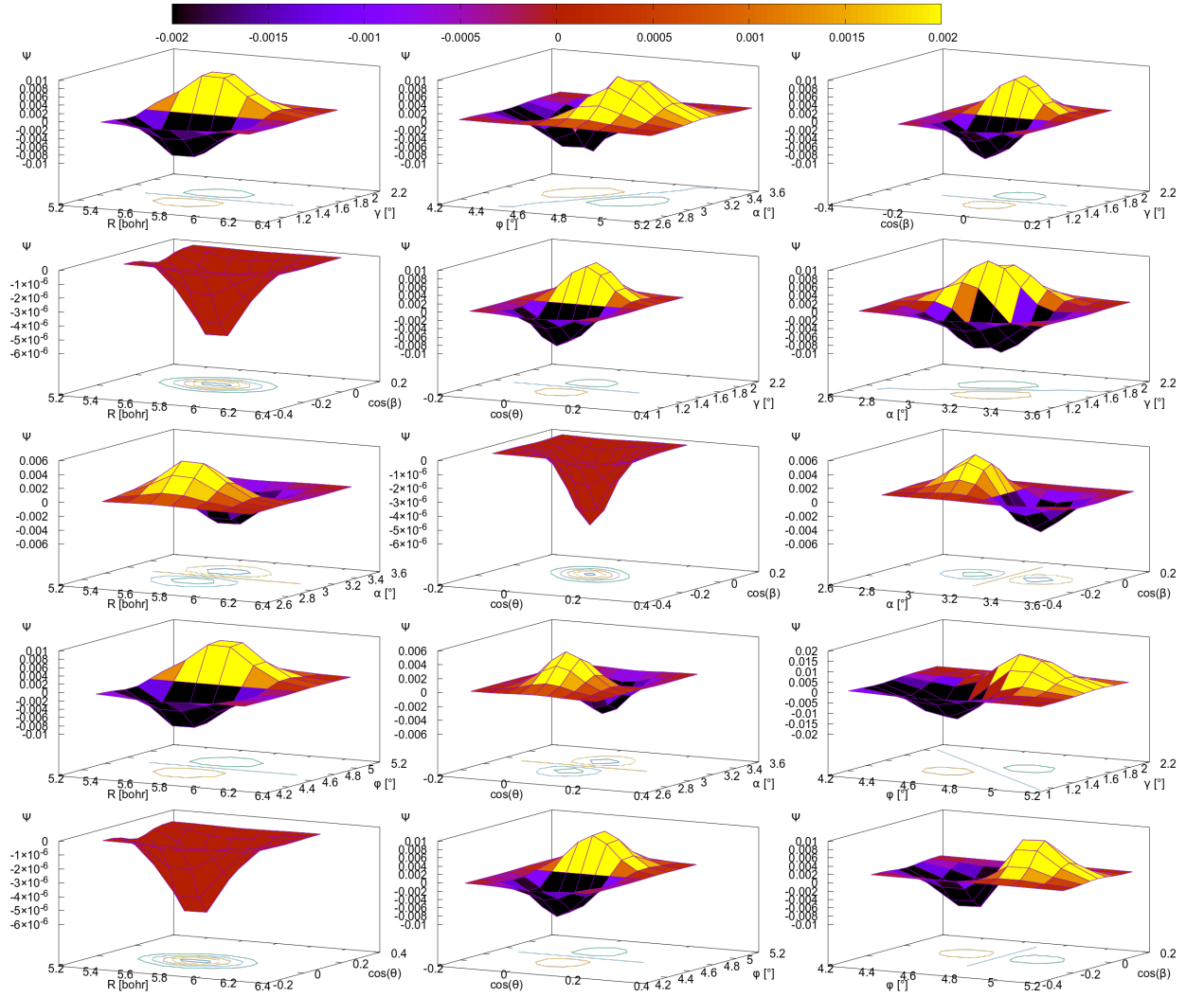


FIG. 4.  $\Psi_3$  with  $\tilde{\nu}_3 - \tilde{\nu}_0 = 162.2 \text{ cm}^{-1}$  (see also caption to Figure 1).

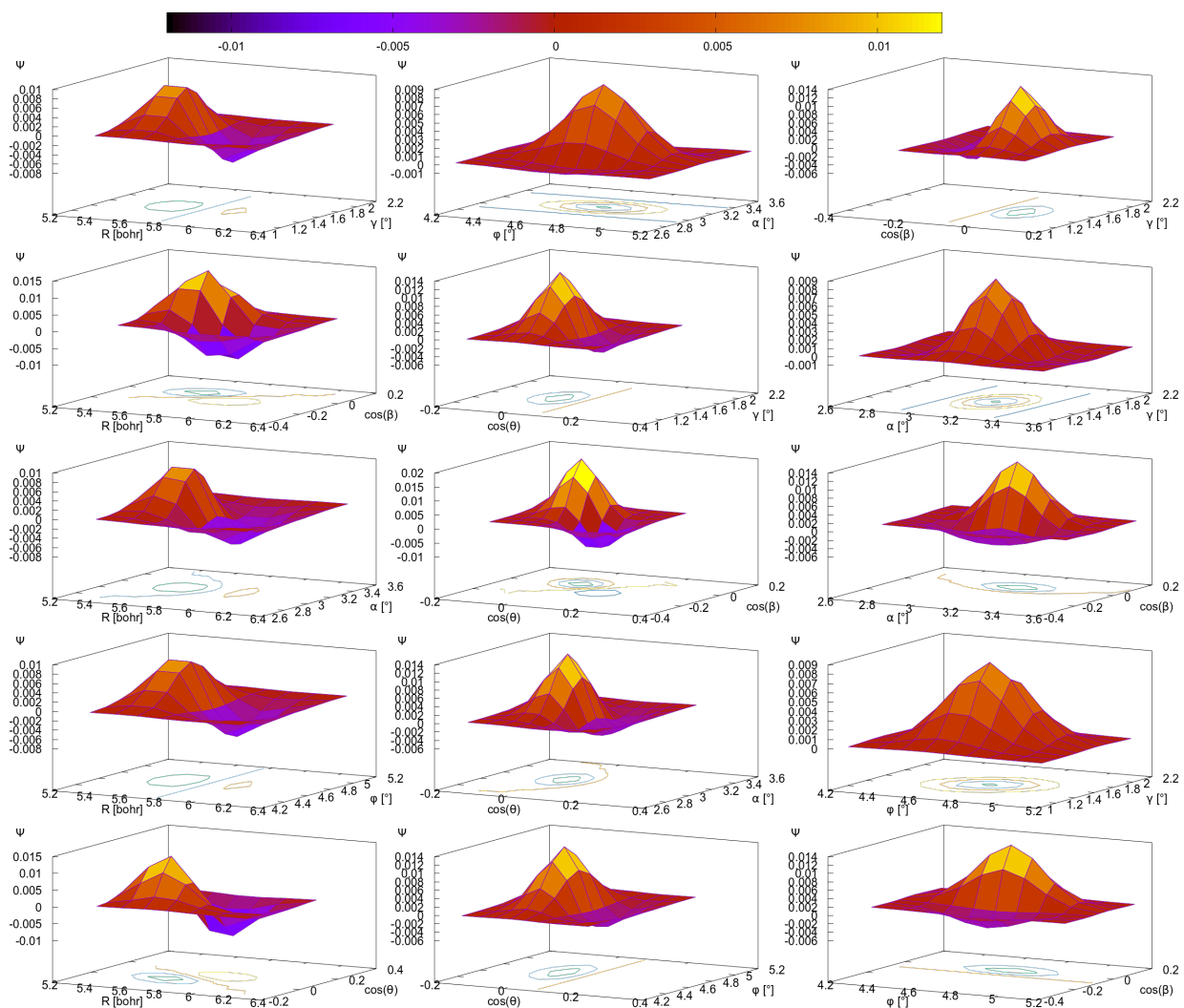


FIG. 5.  $\Psi_4$  with  $\tilde{\nu}_4 - \tilde{\nu}_0 = 191.4 \text{ cm}^{-1}$  (see also caption to Figure 1).

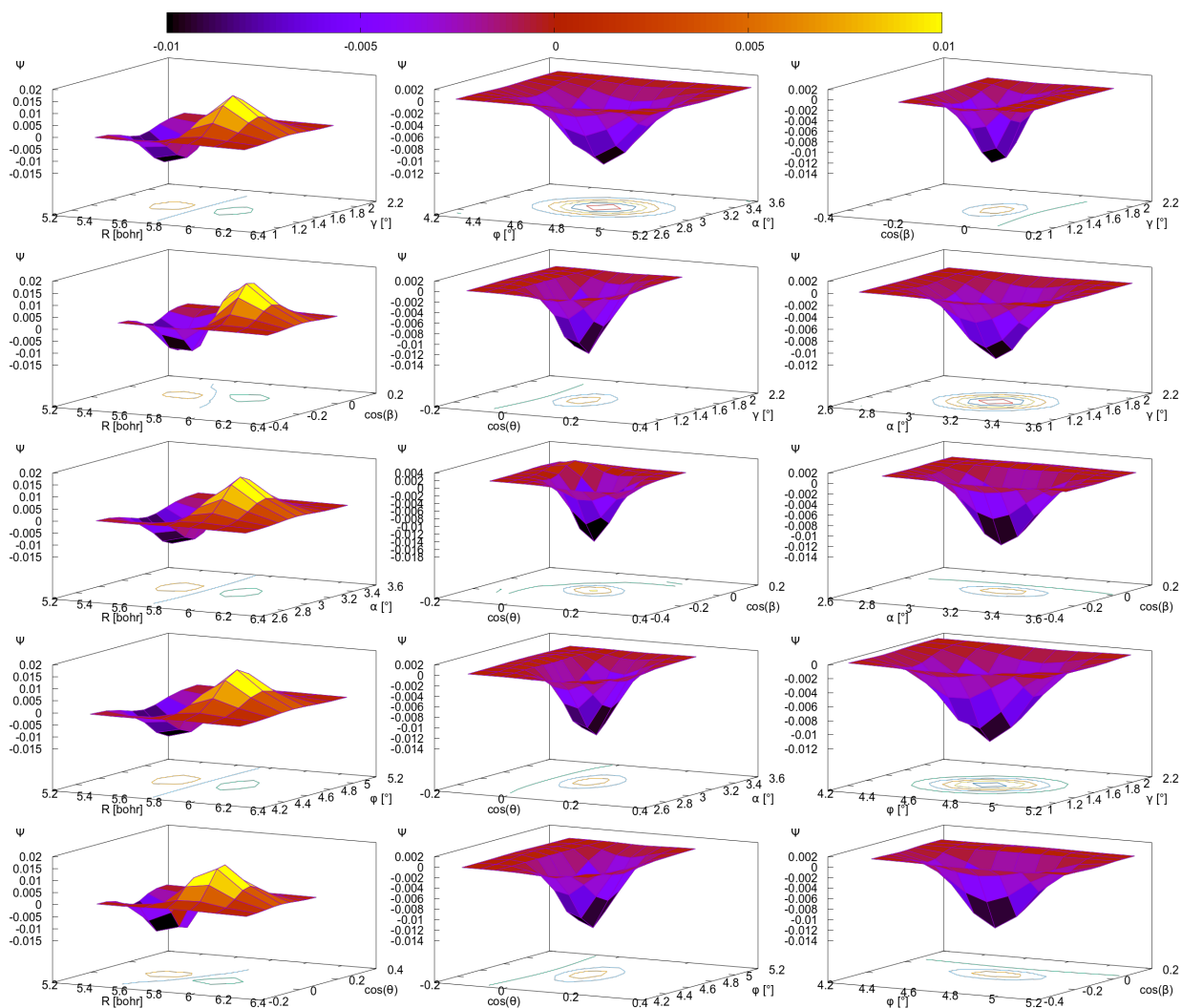


FIG. 6.  $\Psi_5$  with  $\tilde{\nu}_5 - \tilde{\nu}_0 = 207.7 \text{ cm}^{-1}$  (see also caption to Figure 1).

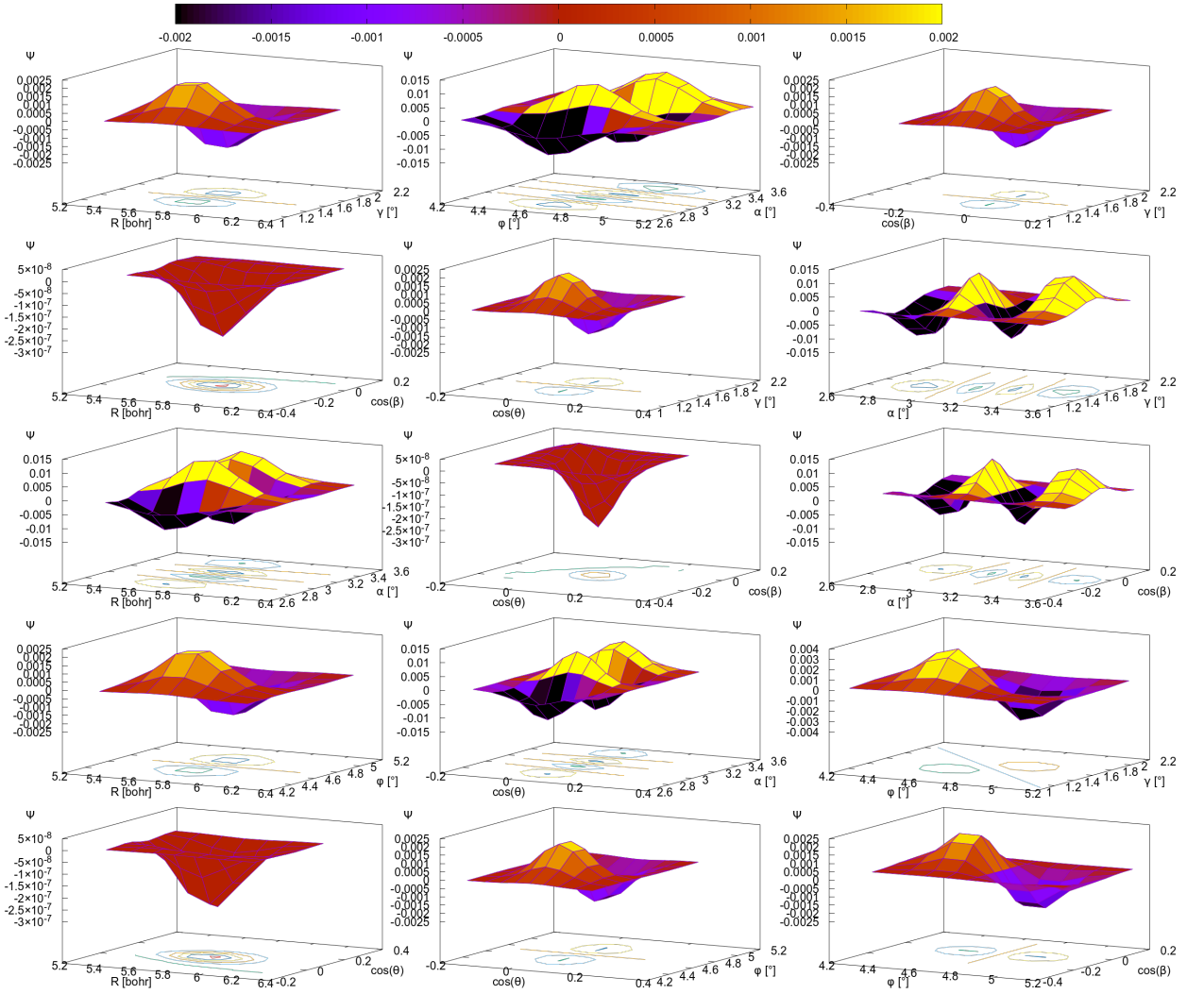


FIG. 7.  $\Psi_6$  with  $\tilde{\nu}_6 - \tilde{\nu}_0 = 211.1 \text{ cm}^{-1}$  (see also caption to Figure 1).

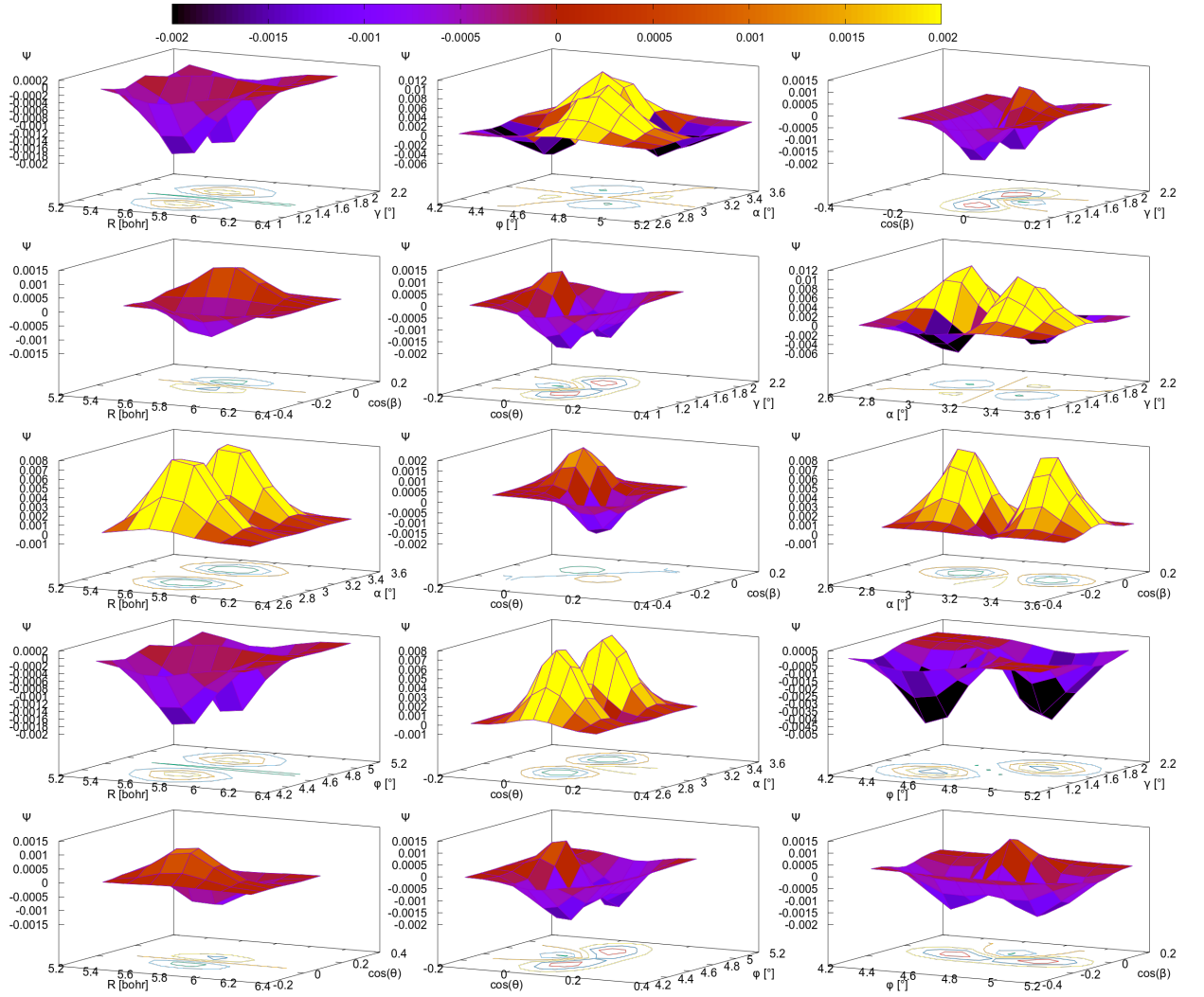


FIG. 8.  $\Psi_7$  with  $\tilde{\nu}_7 - \tilde{\nu}_0 = 232.6 \text{ cm}^{-1}$  (see also caption to Figure 1).

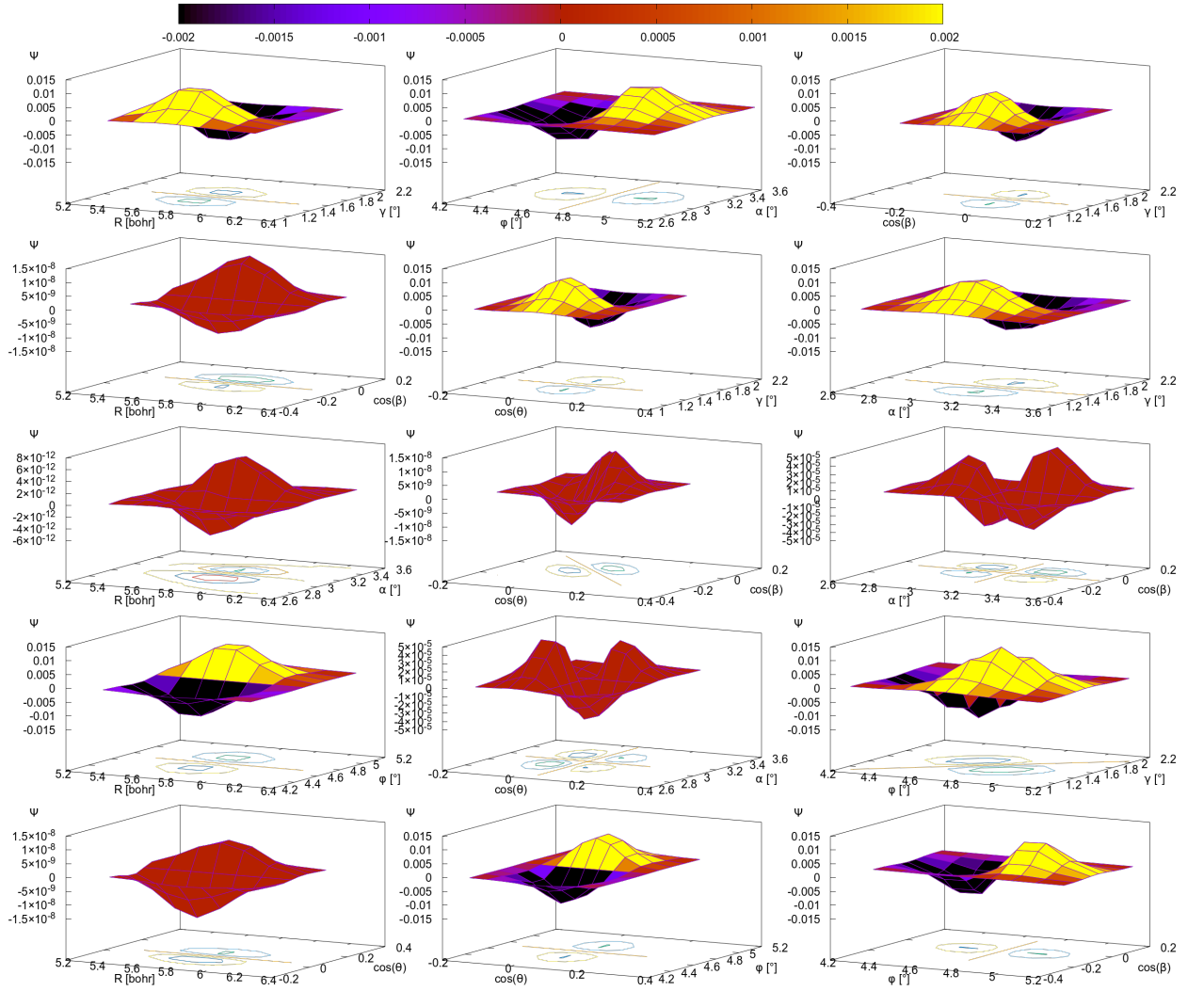


FIG. 9.  $\Psi_8$  with  $\tilde{\nu}_8 - \tilde{\nu}_0 = 239.4 \text{ cm}^{-1}$  (see also caption to Figure 1).

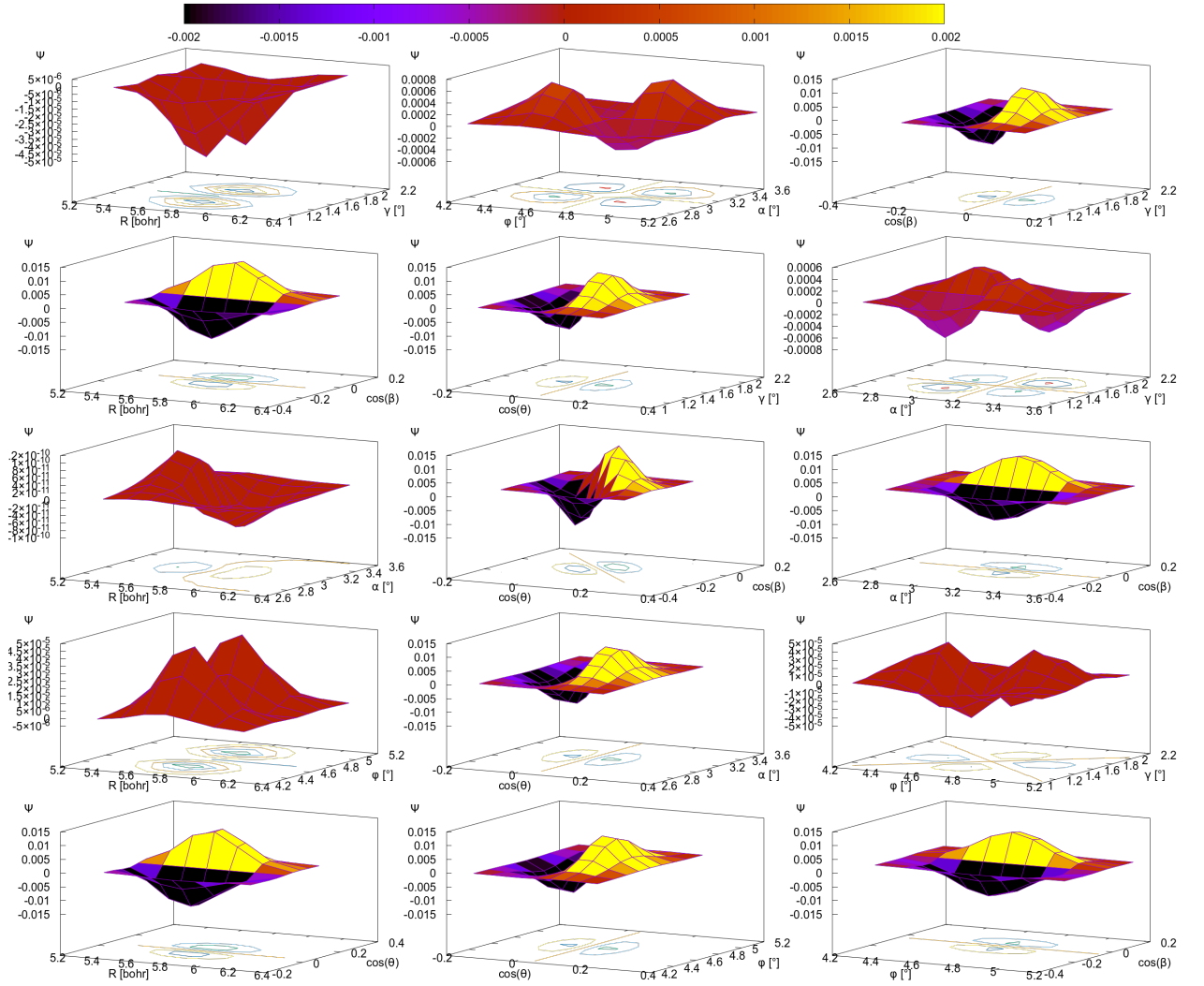


FIG. 10.  $\Psi_9$  with  $\tilde{\nu}_9 - \tilde{\nu}_0 = 253.5 \text{ cm}^{-1}$  (see also caption to Figure 1).

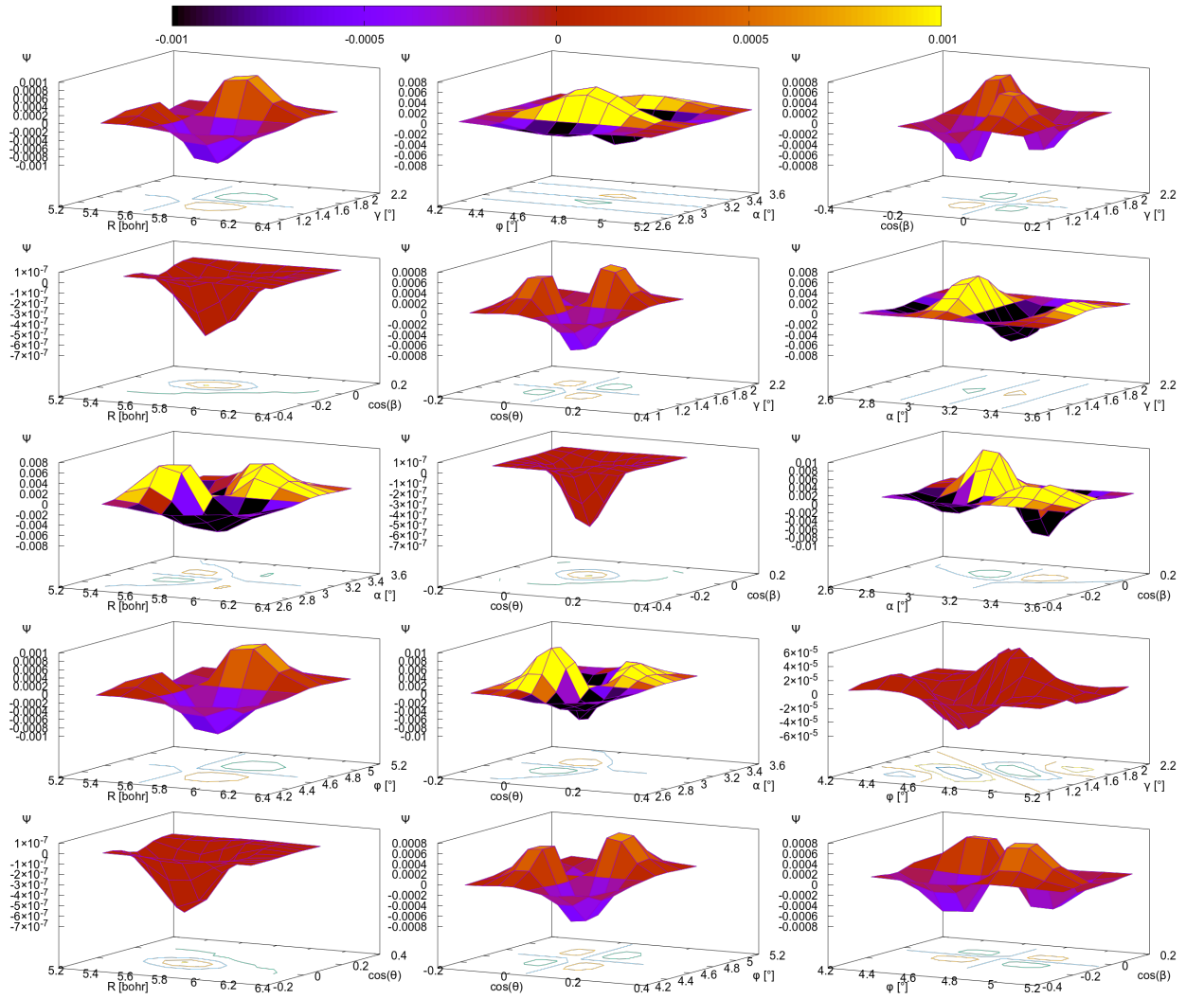


FIG. 11.  $\Psi_{10}$  with  $\tilde{\nu}_{10} - \tilde{\nu}_0 = 262.0 \text{ cm}^{-1}$  (see also caption to Figure 1).



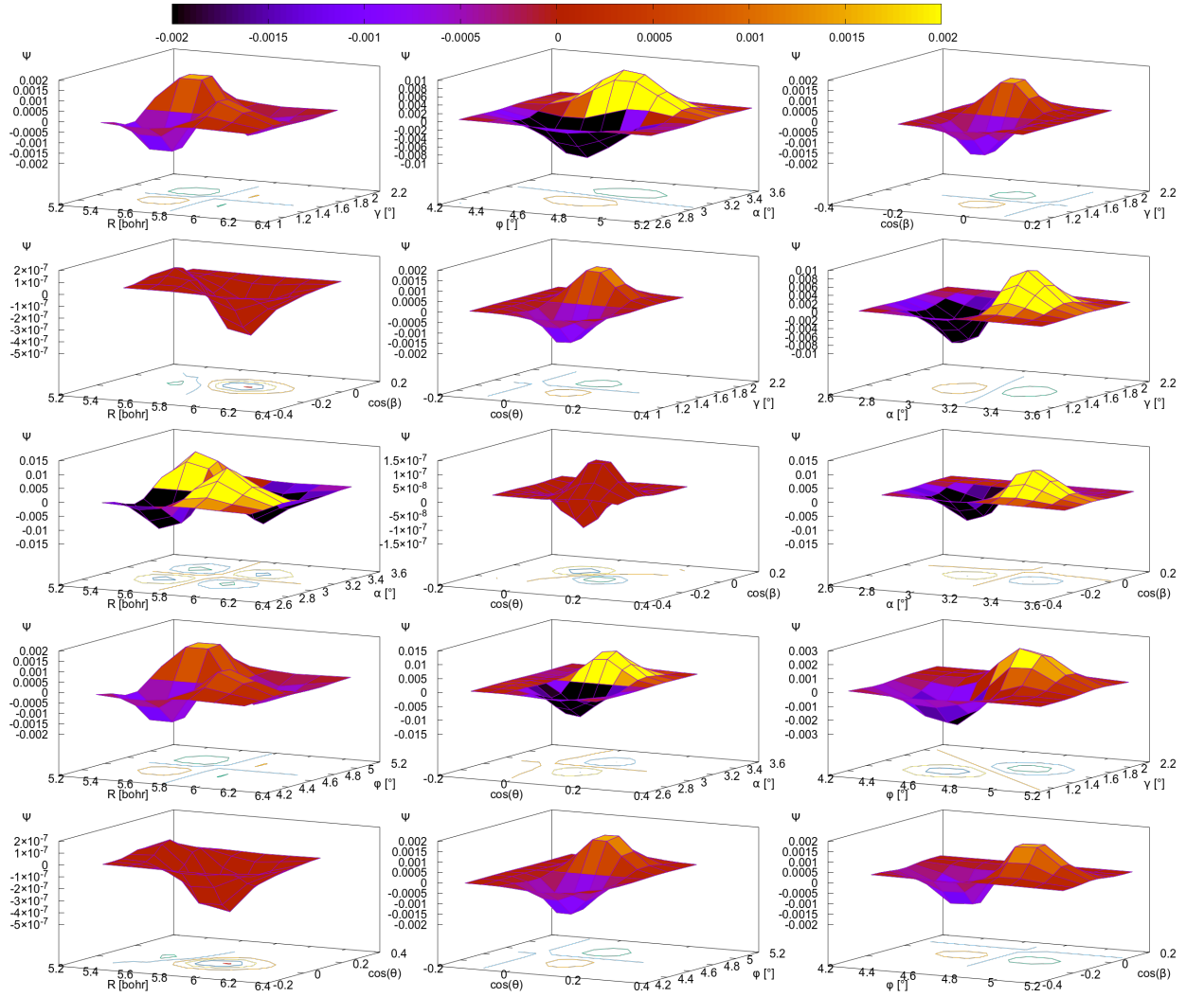


FIG. 12.  $\Psi_{11}$  with  $\tilde{\nu}_{11} - \tilde{\nu}_0 = 277.3 \text{ cm}^{-1}$  (see also caption to Figure 1).

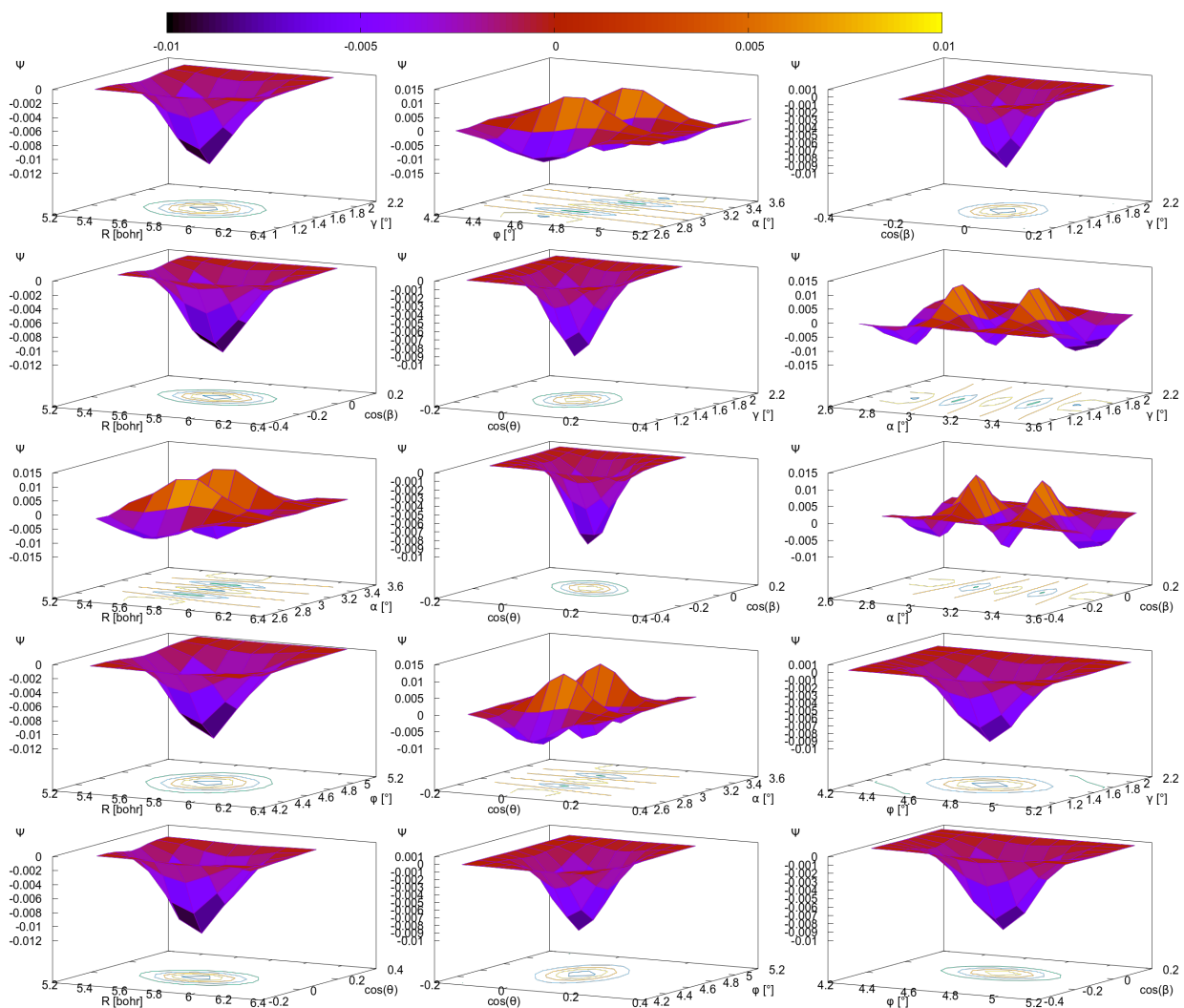


FIG. 13.  $\Psi_{12}$  with  $\tilde{\nu}_{12} - \tilde{\nu}_0 = 280.4 \text{ cm}^{-1}$  (see also caption to Figure 1).

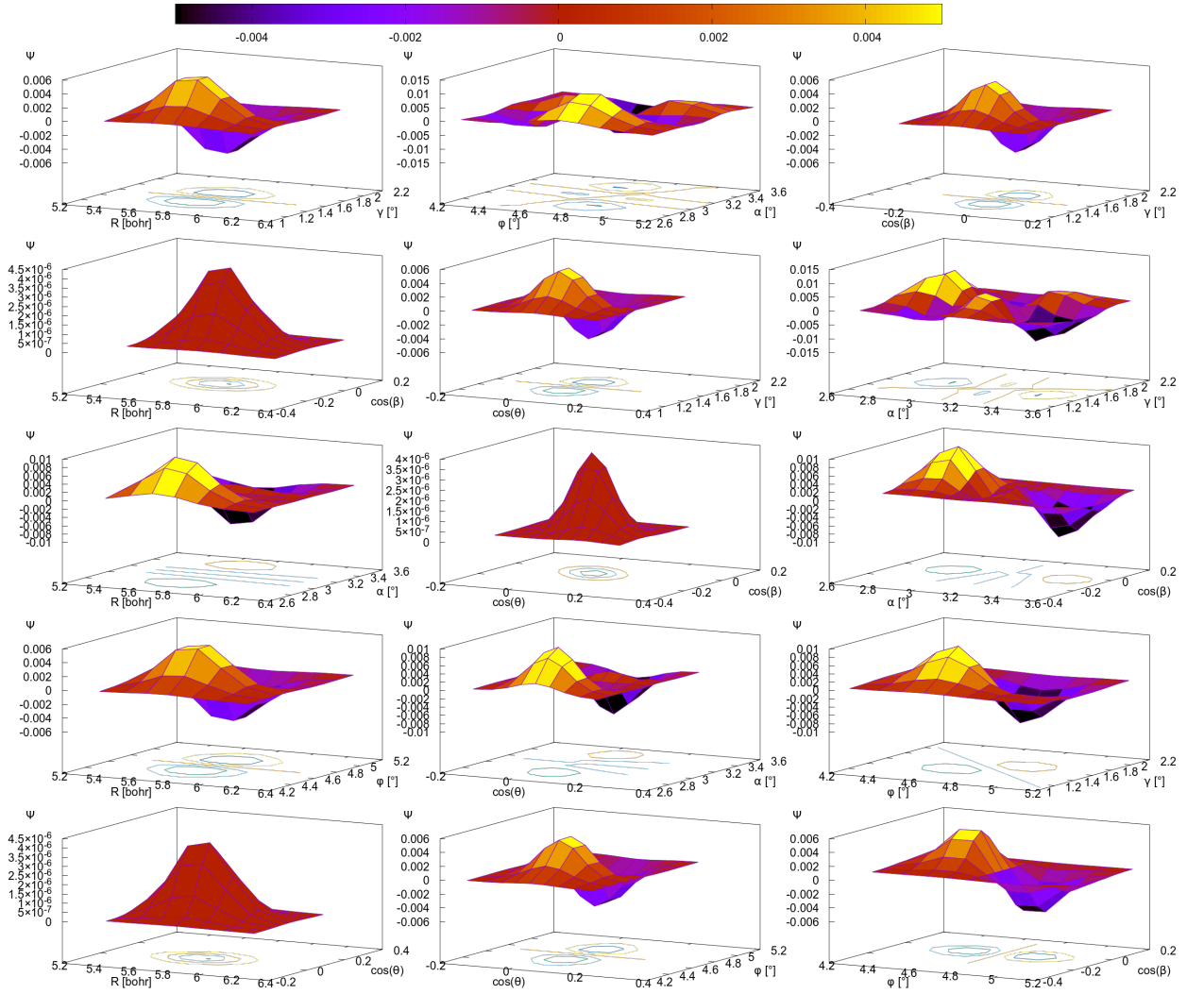


FIG. 14.  $\Psi_{13}$  with  $\tilde{\nu}_{13} - \tilde{\nu}_0 = 302.9 \text{ cm}^{-1}$  (see also caption to Figure 1).

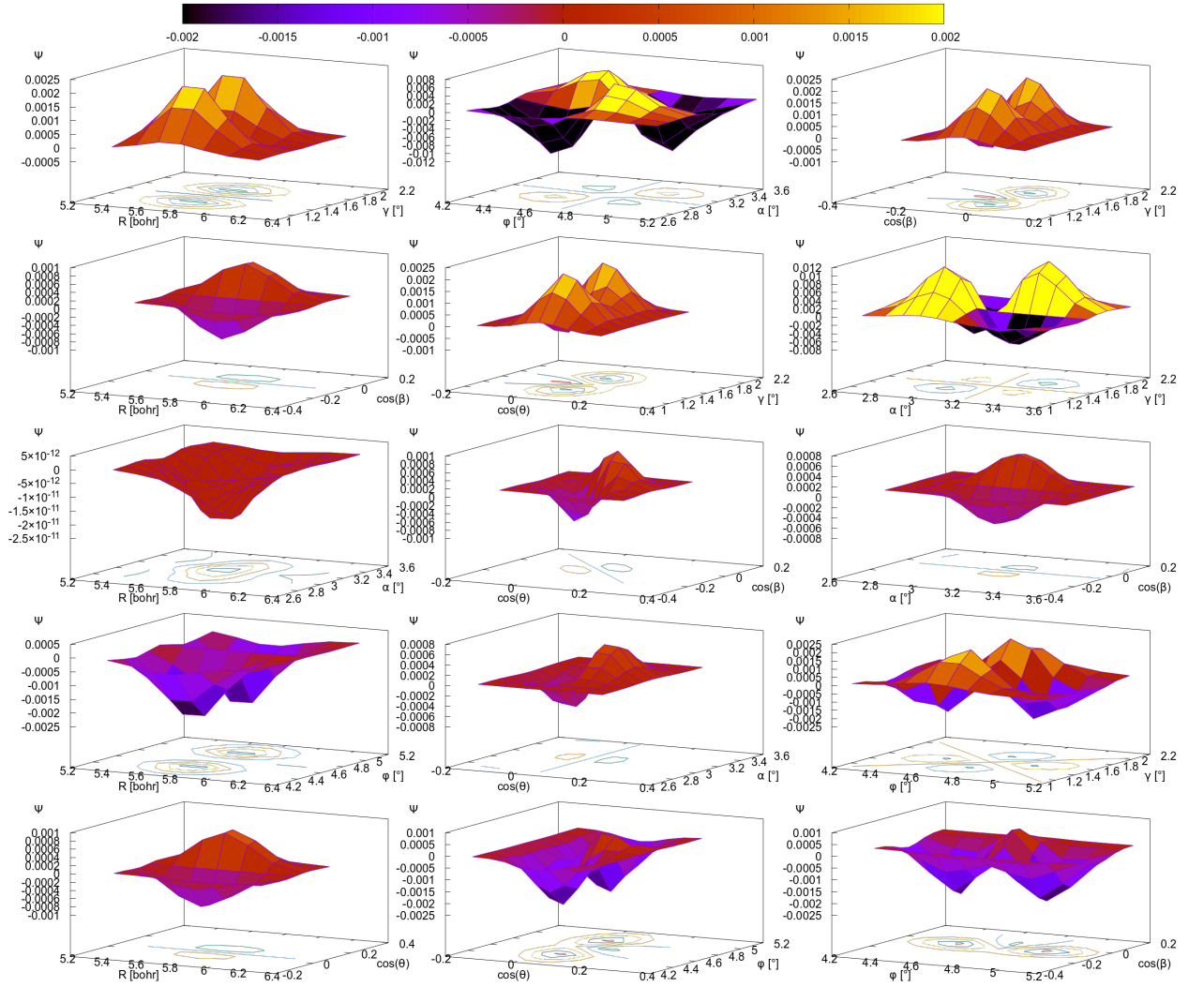


FIG. 15.  $\Psi_{14}$  with  $\tilde{\nu}_{14} - \tilde{\nu}_0 = 309.8 \text{ cm}^{-1}$  (see also caption to Figure 1).