The error propagation can be expressed as:

$$S_N = \sqrt{\left(\frac{\partial f}{\partial x_1}\right)^2 S_{X1}^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 S_{X2}^2 + \left(\frac{\partial f}{\partial x_3}\right)^2 S_{X3}^2} \qquad (1)$$

$$\sigma = \sqrt{\sum_{1\leqslant i<j}^{n}\left(\left(\frac{\partial f}{\partial x_i}\right)\sigma_i\right)^2 + 2\sum_{1\leqslant i<j}^{n}\rho_{ij}\left(\frac{\partial f}{\partial x_i}\right)\left(\frac{\partial f}{\partial x_j}\right)\sigma_i\sigma_j} \qquad (1)$$

where ρ_{ij} is the error correlation between the parameters of i and j. If all of parameters x_1, x_2, … x_j are independent (i.e. $\rho_{(xi,xj)} \approx 0$), the eqn(1) can be simplified as: $\sigma = \sqrt{\sum_{1\leqslant i<j}^{n}\left(\left(\frac{\partial f}{\partial x_i}\right)\sigma_i\right)^2}$. In this study, the parameters includes $(^{207}Pb/^{206}Pb)_m$, $(^{206}Pb/^{238}U)_m$ and $f_{Th/U}$. Generally, the value and precision of $(^{207}Pb/^{206}Pb)_m$ here is mainly depended on ^{207}Pb in common lead; the value and precision of $(^{206}Pb/^{238}U)_m$ is mainly depended on ^{206}Pb in radiogenic lead; the uncertainty of $f_{Th/U}$ is mainly related to the possible Th/U difference between magma and whole rock. Therefore, it is reasonable to ignore the correlation between different parameters in this study.

As $f_{Th/U}$ represent the ratio of the D of U and Th, respectively between source and magma and zircon crystal, the f is defined as follows:

$$f_{Th/U} = \frac{D^{Th}{}_{Zircon/SourceMagma}}{D^{U}{}_{Zircon/SourceMagma}} \qquad (2)$$

$(D/^{207}Pb)_m$ can be calculated as following equation:

$$\left(\frac{^{207}Pb}{D}\right)_m = \left(\frac{^{207}Pb}{^{206}Pb}\right)_m^{*}\left(1\Big/\left(1+\left(\frac{^{238}U}{^{206}Pb_m}\right)\left(\frac{\lambda_{238}}{\lambda_{230}}\left(1-e^{-\lambda_{230}t}\right)\left(1-f_{Th/U}\right)e^{\lambda_{238}t}\right)\right)\right) \qquad (3)$$

$$-\left(\frac{^{207}Pb}{D}\right)_m = \left(\frac{^{207}Pb}{^{206}Pb}\right)_m^{*}\left(1\Big/\left(1+\left(\frac{^{206}Pb}{^{207}Pb}\right)_m\left(\frac{^{238}U}{^{206}Pb_m}\right)\left(\frac{\lambda_{238}}{\lambda_{230}}\left(1-e^{-\lambda_{230}t}\right)\left(1-f_{Th/U}\right)e^{\lambda_{238}t}\right)\right)\right) \qquad (3)$$

Where the subscript m represents the measured value and superscript * represents the ratio of radiogenic Pb. λ_{238} and λ_{230} are the decay constants of ^{238}U and ^{230}Th ($\lambda_{238} = 1.55125 \times 10^{-10}$ yr^{-1} and $\lambda_{230} = 9.195 \times 10^{-6}$ yr^{-1}).

The error of $(^{206}Pb/^{207}Pb)_m$, $(^{206}Pb/^{238}U)$ and $f_{Th/U}$ can be propagated into the error of $(D/^{207}Pb)_m$ ($\sigma_{(D/207Pb)m}$) as follow:

$$\frac{\partial ({}^{207}Pb/D)_m}{\partial ({}^{206}Pb/{}^{238}U)_m} = -\frac{e^{\lambda_{238}t} * \left(\frac{{}^{207}Pb}{{}^{206}Pb}\right)_m * (\lambda_{238}/\lambda_{230}) * (1 - f_{\frac{Th}{U}}) * (exp(-\lambda_{230}*t) - 1)}{\left(\left(\frac{{}^{206}Pb}{{}^{238}U}\right)_m\right)^2 * \left(\frac{e^{\lambda_{238}t}*(\lambda_{238}/\lambda_{230})*(1-f_{\frac{Th}{U}})*(exp(-\lambda_{230}*t)-1)}{\left(\frac{{}^{206}Pb}{{}^{238}U}\right)_m} - 1\right)^2} \quad (4)$$

$$\frac{\partial ({}^{207}Pb/D)_m}{\partial ({}^{207}Pb/{}^{206}Pb)_m} = -\frac{1}{\frac{e^{\lambda_{238}t}*(\lambda_{238}/\lambda_{230})*(1-f_{\frac{Th}{U}})*(exp(-\lambda_{230}*t)-1)}{\left(\frac{{}^{206}Pb}{{}^{238}U}\right)_m} - 1} \quad (5)$$

$$\frac{\partial ({}^{207}Pb/D)_m}{\partial f_{\frac{Th}{U}}} = \frac{e^{\lambda_{238}t}*(\lambda_{238}/\lambda_{230})*(1-f_{\frac{Th}{U}})*(exp(-\lambda_{230}*t)-1)}{\left(\frac{{}^{206}Pb}{{}^{238}U}\right)_m * \left(\frac{e^{\lambda_{238}t}*(\lambda_{238}/\lambda_{230})*(1-f_{\frac{Th}{U}})*(exp(-\lambda_{230}*t)-1)}{\left(\frac{{}^{206}Pb}{{}^{238}U}\right)_m} - 1\right)^2} \quad (6)$$

$$\frac{\partial ({}^{207}Pb/D)_m}{\partial ({}^{206}Pb/{}^{238}U)_m} = -\frac{e^{-\lambda_{238}t} * \left(\frac{{}^{207}Pb}{{}^{206}Pb}\right)_m * (\lambda_{238}/\lambda_{230}) * (1 - f_{\frac{Th}{U}}) * (exp(-\lambda_{230}*t) - 1)}{\left(\left(\frac{{}^{206}Pb}{{}^{238}U}\right)_m\right)^2 * \left(\frac{e^{-\lambda_{238}t}*(\lambda_{238}/\lambda_{230})*(1-f_{\frac{Th}{U}})*(exp(-\lambda_{230}*t)-1)}{\left(\frac{{}^{206}Pb}{{}^{238}U}\right)_m} - 1\right)^2} \quad ~~(4)~~$$

$$\frac{\partial ({}^{207}Pb/D)_m}{\partial ({}^{207}Pb/{}^{206}Pb)_m} = -\frac{1}{\frac{e^{-\lambda_{238}t}*(\lambda_{238}/\lambda_{230})*(1-f_{\frac{Th}{U}})*(exp(-\lambda_{230}*t)-1)}{\left(\frac{{}^{206}Pb}{{}^{238}U}\right)_m} - 1} \quad ~~(5)~~$$

$$\frac{\partial ({}^{207}Pb/D)_m}{\partial f_{\frac{Th}{U}}} = \frac{e^{-\lambda_{238}t}*(\lambda_{238}/\lambda_{230})*(1-f_{\frac{Th}{U}})*(exp(-\lambda_{230}*t)-1)}{\left(\frac{{}^{206}Pb}{{}^{238}U}\right)_m * \left(\frac{e^{-\lambda_{238}t}*(\lambda_{238}/\lambda_{230})*(1-f_{\frac{Th}{U}})*(exp(-\lambda_{230}*t)-1)}{\left(\frac{{}^{206}Pb}{{}^{238}U}\right)_m} - 1\right)^2} \quad ~~(6)~~$$

$$\sigma_{({}^{207}Pb/D)_m} = \sqrt{\left(\frac{\partial ({}^{207}Pb/D)_m}{\partial ({}^{206}Pb/{}^{238}U)_m}\right)^2 \sigma_{({}^{206}Pb/{}^{238}U)_m}{}^2 + \left(\frac{\partial ({}^{207}Pb/D)_m}{\partial ({}^{207}Pb/{}^{206}Pb)_m}\right)^2 \sigma_{({}^{207}Pb/{}^{206}Pb)_m}{}^2 + \left(\frac{\partial ({}^{207}Pb/D)_m}{\partial f_{\frac{Th}{U}}}\right)^2 \sigma_{f_{\frac{Th}{U}}}{}^2} \quad (7)$$

Similarly, $D/{}^{238}U$ can be expressed as:

$$\frac{D}{{}^{238}U} = \left(\frac{{}^{206}Pb}{{}^{238}U}\right)_m + \frac{\lambda_{238}}{\lambda_{230}}(1 - e^{-\lambda_{230}t})(1 - f_{Th/U})e^{\lambda_{238}t} \quad (8)$$

The error of $({}^{206}Pb/{}^{207}Pb)_m$ and $f_{Th/U}$ can be propagated into the error of $D/{}^{238}U$ ($\sigma_{D/238U}$) as follow:

$$\frac{\partial (D/{}^{238}U)}{\partial ({}^{206}Pb/{}^{238}U)_m} = 1 \quad (9)$$

$$\frac{\partial (D/{}^{238}U)}{\partial f_{\frac{Th}{U}}} = -e^{\lambda_{238}t}*\lambda_{238}/\lambda_{230}*(exp(-\lambda_{230}*t) - 1) \quad (10)$$

$$\sigma_{(D/^{238}U)} = \sqrt{\left(\frac{\partial_{(D/^{238}U)}}{\partial_{(^{206}Pb/^{238}U)_m}}\right)^2 \sigma_{(^{206}Pb/^{238}U)_m}{}^2 + \left(\frac{\partial_{(D/^{238}U)}}{\partial_{f_{\frac{Th}{U}}}}\right)^2 \sigma_{f_{\frac{Th}{U}}}{}^2} \tag{11}$$

The proportion of common Pb relative to total ^{206}Pb (f_{206})can be expressed as:

$$f_{206} = \frac{\left(\frac{^{207}Pb}{D}\right)_m - \left(\frac{^{207}Pb}{^{206}Pb}\right)^*}{\left(\frac{^{207}Pb}{^{206}Pb}\right)_c - \left(\frac{^{207}Pb}{^{206}Pb}\right)^*} \tag{12}$$

The error of $(^{206}Pb/D)_m$ and$(^{207}Pb/^{206}Pb)_c$ can be propagated into the error of f_{206} as follow:

$$\frac{\partial_{f_{206}}}{\partial_{(^{207}Pb/D)_m}} = \frac{1}{\left(\frac{^{207}Pb}{^{206}Pb}\right)_c - \left(\frac{^{207}Pb}{^{206}Pb}\right)^*} \tag{13}$$

$$\frac{\partial_{f_{206}}}{\partial_{(^{207}Pb/^{206}Pb)c}} = -\frac{\left(\frac{^{207}Pb}{D}\right)_m - \left(\frac{^{207}Pb}{^{206}Pb}\right)^*}{\left(\left(\frac{^{207}Pb}{^{206}Pb}\right)_c - \left(\frac{^{207}Pb}{^{206}Pb}\right)^*\right)^2} \tag{14}$$

$$\sigma_{f_{206}} = \sqrt{\left(\frac{\partial_{f_{206}}}{\partial_{(^{207}Pb/D)_m}}\right)^2 \sigma_{(^{207}Pb/D)_m}{}^2 + \left(\frac{\partial_{f_{206}}}{\partial_{(^{207}Pb/^{206}Pb)c}}\right)^2 \sigma_{(^{207}Pb/^{206}Pb)_c}{}^2} \tag{15}$$

The age (t) can be calculated as follow:

$$t = \ln\left(\left(D/^{238}U\right) * \left(1 - f_{206}\right) + 1\right) / \lambda_{238} \tag{16}$$

The uncertainty of age (t) (σ_t) can be propagated with the error of f_{206} and D/^{238}U:

$$\frac{\partial_t}{\partial_{(D/^{238}U)}} = \frac{f_{206} - 1}{\lambda_{238}\left(\frac{D}{^{238}U}\left(f_{206} - 1\right) - 1\right)} \tag{17}$$

$$\frac{\partial_t}{\partial_{f_{206}}} = \frac{\frac{D}{^{238}U}}{\lambda_{238}\left(\frac{D}{^{238}U}\left(f_{206} - 1\right) - 1\right)} \tag{18}$$

$$\sigma_t = \sqrt{\left(\frac{\partial_t}{\partial_{(D/^{238}U)}}\right)^2 \sigma_{(D/^{238}U)}{}^2 + \left(\frac{\partial_t}{\partial_{f_{206}}}\right)^2 \sigma_{f_{206}}{}^2} \tag{19}$$