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The error propagation can be expressed as:

$$\sigma=\sqrt{\sum_{1\leqslant i<j}^{n}\left(\left(\frac{\partial f}{\partial x_i}\right)\sigma_i\right)^2+2\sum_{1\leqslant i<j}^{n}\rho_{ij}\left(\frac{\partial f}{\partial x_i}\right)\left(\frac{\partial f}{\partial x_j}\right)\sigma_i\sigma_j} \qquad (1)$$

where ρ_{ij} is the error correlation between the parameters of i and j. If all of parameters x_1, x_2, … x_j are independent (i.e. $\rho_{(xi,xj)}\approx 0$), the eqn(1) can be simplified as: $\sigma=\sqrt{\sum_{1\leqslant i<j}^{n}\left(\left(\frac{\partial f}{\partial x_i}\right)\sigma_i\right)^2}$. In this study, the parameters includes $({}^{207}Pb/{}^{206}Pb)_m$, $({}^{206}Pb/{}^{238}U)_m$ and $f_{Th/U.}$ Generally, the value and precision of $({}^{207}Pb/{}^{206}Pb)_m$ here is mainly depended on ${}^{207}Pb$ in common lead; the value and precision of $({}^{206}Pb/{}^{238}U)_m$ is mainly depended on ${}^{206}Pb$ in radiogenic lead; the uncertainty of $f_{Th/U}$ is mainly related to the possible Th/U difference between magma and whole rock. Therefore, it is reasonable to ignore the correlation between different parameters in this study.

As $f_{Th/U}$ represent the ratio of the D of U and Th, respectively between source and magma and zircon crystal, the f is defined as follows:

$$f_{Th/U}=\frac{D^{Th}{}_{Zircon/SourceMagma}}{D^{U}{}_{Zircon/SourceMagma}} \qquad (2)$$

$(D/{}^{207}Pb)_m$ can be calculated as following equation:

$$\left(\frac{{}^{207}Pb}{D}\right)_m=\left(\frac{{}^{207}Pb}{{}^{206}Pb}\right)_m{}^{*}\ (1/\left(1+\left(\frac{{}^{238}U}{{}^{206}Pb_m}\right)\left(\frac{\lambda_{238}}{\lambda_{230}}(1-e^{-\lambda_{230}t})(1-f_{Th/U})\right)e^{\lambda_{238}t}\right)) \qquad (3)$$

Where the subscript m represents the measured value and superscript * represents the ratio of radiogenic Pb. λ_{238} and λ_{230} are the decay constants of ${}^{238}U$ and ${}^{230}Th$ ($\lambda_{238}=1.55125\times10^{-10}\ yr^{-1}$ and $\lambda_{230}=9.195\times10^{-6}\ yr^{-1}$).

The error of $({}^{206}Pb/{}^{207}Pb)_m$, $({}^{206}Pb/{}^{238}U)$ and $f_{Th/U}$ can be propagated into the error of $(D/^{207}Pb)_m$ $(\sigma_{(D/207Pb)m})$ as follow:

$$\frac{\partial(^{207}Pb/D)_m}{\partial(^{206}Pb/^{238}U)_m} = -\frac{e^{\lambda_{238}t} * \left(\frac{^{207}Pb}{^{206}Pb}\right)_m * (\lambda_{238}/\lambda_{230}) * (1 - f_{\frac{Th}{U}}) * (exp(-\lambda_{230}*t) - 1)}{\left(\left(\frac{^{206}Pb}{^{238}U}\right)_m\right)^2 * \left(\frac{e^{\lambda_{238}t} * (\lambda_{238}/\lambda_{230}) * (1 - f_{\frac{Th}{U}}) * (exp(-\lambda_{230}*t) - 1)}{\left(\frac{^{206}Pb}{^{238}U}\right)_m} - 1\right)^2} \quad (4)$$

$$\frac{\partial(^{207}Pb/D)_m}{\partial(^{207}Pb/^{206}Pb)_m} = -\frac{1}{\frac{e^{\lambda_{238}t} * (\lambda_{238}/\lambda_{230}) * (1 - f_{\frac{Th}{U}}) * (exp(-\lambda_{230}*t) - 1)}{\left(\frac{^{206}Pb}{^{238}U}\right)_m} - 1} \quad (5)$$

$$\frac{\partial(^{207}Pb/D)_m}{\partial f_{\frac{Th}{U}}} = \frac{e^{\lambda_{238}t} * (\lambda_{238}/\lambda_{230}) * (1 - f_{\frac{Th}{U}}) * (exp(-\lambda_{230}*t) - 1)}{\left(\frac{^{206}Pb}{^{238}U}\right)_m * \left(\frac{e^{\lambda_{238}t} * (\lambda_{238}/\lambda_{230}) * (1 - f_{\frac{Th}{U}}) * (exp(-\lambda_{230}*t) - 1)}{\left(\frac{^{206}Pb}{^{238}U}\right)_m} - 1\right)^2} \quad (6)$$

$$\sigma_{(^{207}Pb/D)_m} = \sqrt{\left(\frac{\partial(^{207}Pb/D)_m}{\partial(^{206}Pb/^{238}U)_m}\right)^2 \sigma_{(^{206}Pb/^{238}U)_m}^2 + \left(\frac{\partial(^{207}Pb/D)_m}{\partial(^{207}Pb/^{206}Pb)_m}\right)^2 \sigma_{(^{207}Pb/^{206}Pb)_m}^2 + \left(\frac{\partial(^{207}Pb/D)_m}{\partial f_{\frac{Th}{U}}}\right)^2 \sigma_{f_{\frac{Th}{U}}}^2} \quad (7)$$

Similarly, $D/^{238}U$ can be expressed as:

$$\frac{D}{^{238}U} = \left(\frac{^{206}Pb}{^{238}U}\right)_m + \frac{\lambda_{238}}{\lambda_{230}}(1 - e^{-\lambda_{230}t})(1 - f_{Th/U})e^{\lambda_{238}t} \quad (8)$$

The error of $(^{207}Pb/^{206}Pb)_m$ and $f_{Th/U}$ can be propagated into the error of $D/^{238}U$ $(\sigma_{D/238U})$ as follow:

$$\frac{\partial(D/^{238}U)}{\partial(^{206}Pb/^{238}U)_m} = 1 \quad (9)$$

$$\frac{\partial(D/^{238}U)}{\partial f_{\frac{Th}{U}}} = -e^{\lambda_{238}t} * \lambda_{238}/\lambda_{230} * (exp(-\lambda_{230}*t) - 1) \quad (10)$$

$$\sigma_{(D/^{238}U)} = \sqrt{\left(\frac{\partial(D/^{238}U)}{\partial(^{206}Pb/^{238}U)_m}\right)^2 \sigma_{(^{206}Pb/^{238}U)_m}^2 + \left(\frac{\partial(D/^{238}U)}{\partial f_{\frac{Th}{U}}}\right)^2 \sigma_{f_{\frac{Th}{U}}}^2} \quad (11)$$

The proportion of common Pb relative to total ^{206}Pb (f_{206})can be expressed as:

$$f_{206} = \frac{\left(\frac{^{207}Pb}{D}\right)_m - \left(\frac{^{207}Pb}{^{206}Pb}\right)^*}{\left(\frac{^{207}Pb}{^{206}Pb}\right)_c - \left(\frac{^{207}Pb}{^{206}Pb}\right)^*} \quad (12)$$

The error of $(^{207}Pb/D)_m$ and$(^{207}Pb/^{206}Pb)_c$ can be propagated into the error of f_{206} as follow:

$$\frac{\partial f_{206}}{\partial (^{207}Pb/D)_m} = \frac{1}{\left(\frac{^{207}Pb}{^{206}Pb}\right)_c - \left(\frac{^{207}Pb}{^{206}Pb}\right)^*} \tag{13}$$

$$\frac{\partial f_{206}}{\partial (^{207}Pb/^{206}Pb)c} = -\frac{\left(\frac{^{207}Pb}{D}\right)_m - \left(\frac{^{207}Pb}{^{206}Pb}\right)^*}{\left(\left(\frac{^{207}Pb}{^{206}Pb}\right)_c - \left(\frac{^{207}Pb}{^{206}Pb}\right)^*\right)^2} \tag{14}$$

$$\sigma_{f_{206}} = \sqrt{\left(\frac{\partial f_{206}}{\partial (^{207}Pb/D)_m}\right)^2 \sigma_{(^{207}Pb/D)_m}{}^2 + \left(\frac{\partial f_{206}}{\partial (^{207}Pb/^{206}Pb)c}\right)^2 \sigma_{(^{207}Pb/^{206}Pb)_c}{}^2} \tag{15}$$

The age (t) can be calculated as follow:

$$t = \ln((D/^{238}U)*(1 - f_{206}) + 1)/\lambda_{238} \tag{16}$$

The uncertainty of age (t) (σ_t) can be propagated with the error of f_{206} and $(D/^{238}U)_m$:

$$\frac{\partial_t}{\partial (D/^{238}U)} = \frac{f_{206} - 1}{\lambda_{238}\left(\frac{D}{^{238}U}(f_{206} - 1) - 1\right)} \tag{17}$$

$$\frac{\partial_t}{\partial f_{206}} = \frac{\frac{D}{^{238}U}}{\lambda_{238}\left(\frac{D}{^{238}U}(f_{206} - 1) - 1\right)} \tag{18}$$

$$\sigma_t = \sqrt{\left(\frac{\partial_t}{\partial (D/^{238}U)}\right)^2 \sigma_{(D/^{238}U)}{}^2 + \left(\frac{\partial_t}{\partial f_{206}}\right)^2 \sigma_{f_{206}}{}^2} \tag{19}$$