Electronic Supplementary Information

Nanowire networks: how does small-world character evolve with dimensionality?

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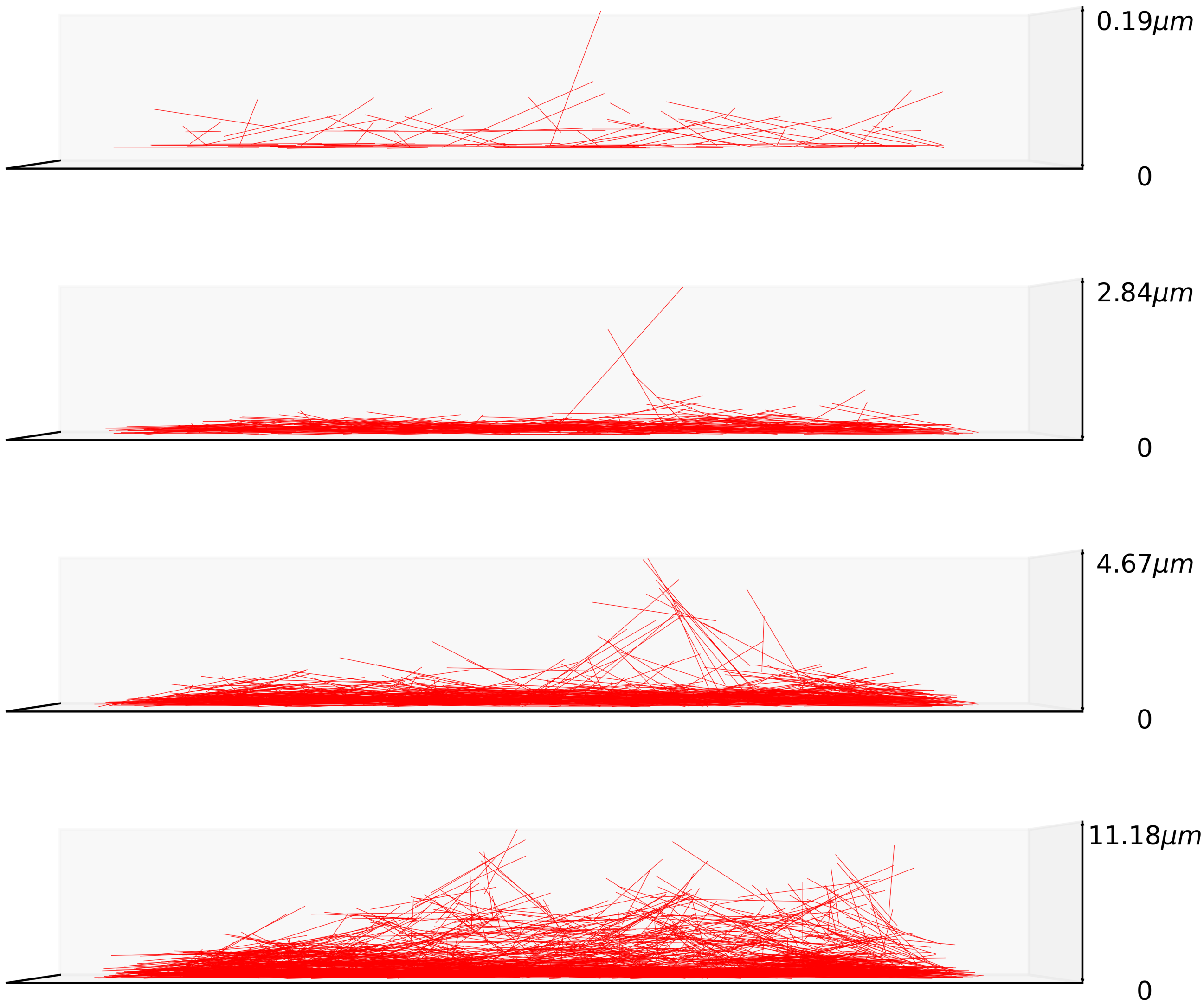

Fig. S1 The stacking of an example quasi-3D network as the number of wires N is increased. Values of N are 100, 500, 1000, 2000 (from top to bottom). The nanowires have a diameter of 20nm and $\Lambda = 30\mu m$, $\lambda = 6\mu m$, $d = 0.5$. The maximum height of the network is shown on the right ranging from $\sim 0.13\mu m$ for $N = 100$, to $\sim 11\mu m$ for $N = 2000$.

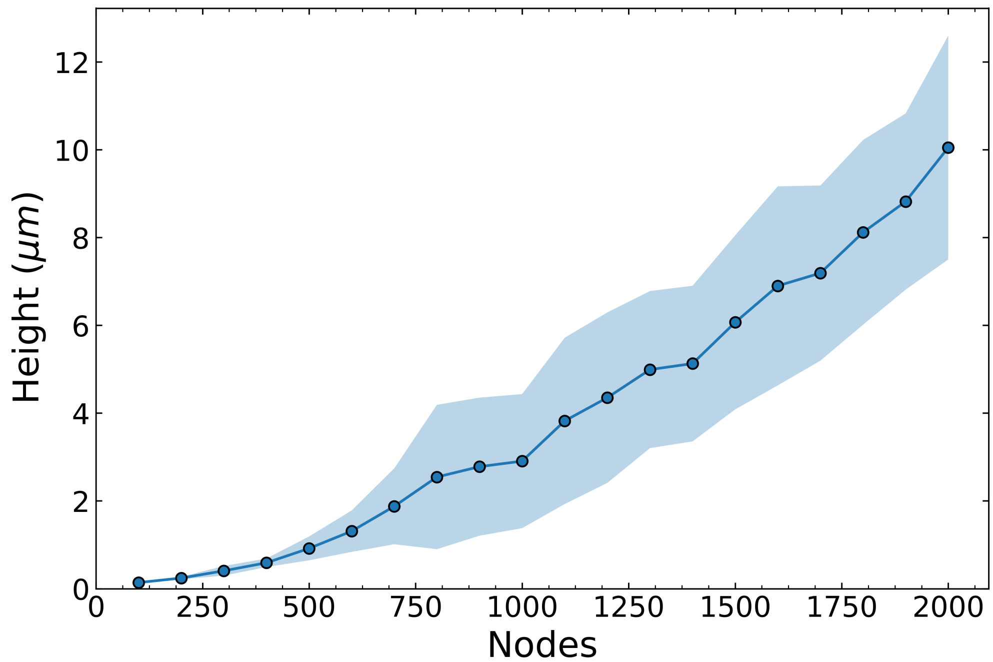

Fig. S2 The mean height of network with increasing N, averaged over 10 different networks. As N increases and stacking becomes more important and the maximum height increases. The shaded region is one standard deviation. The broadening of the distribution arises from the random stacking of the nanowires.

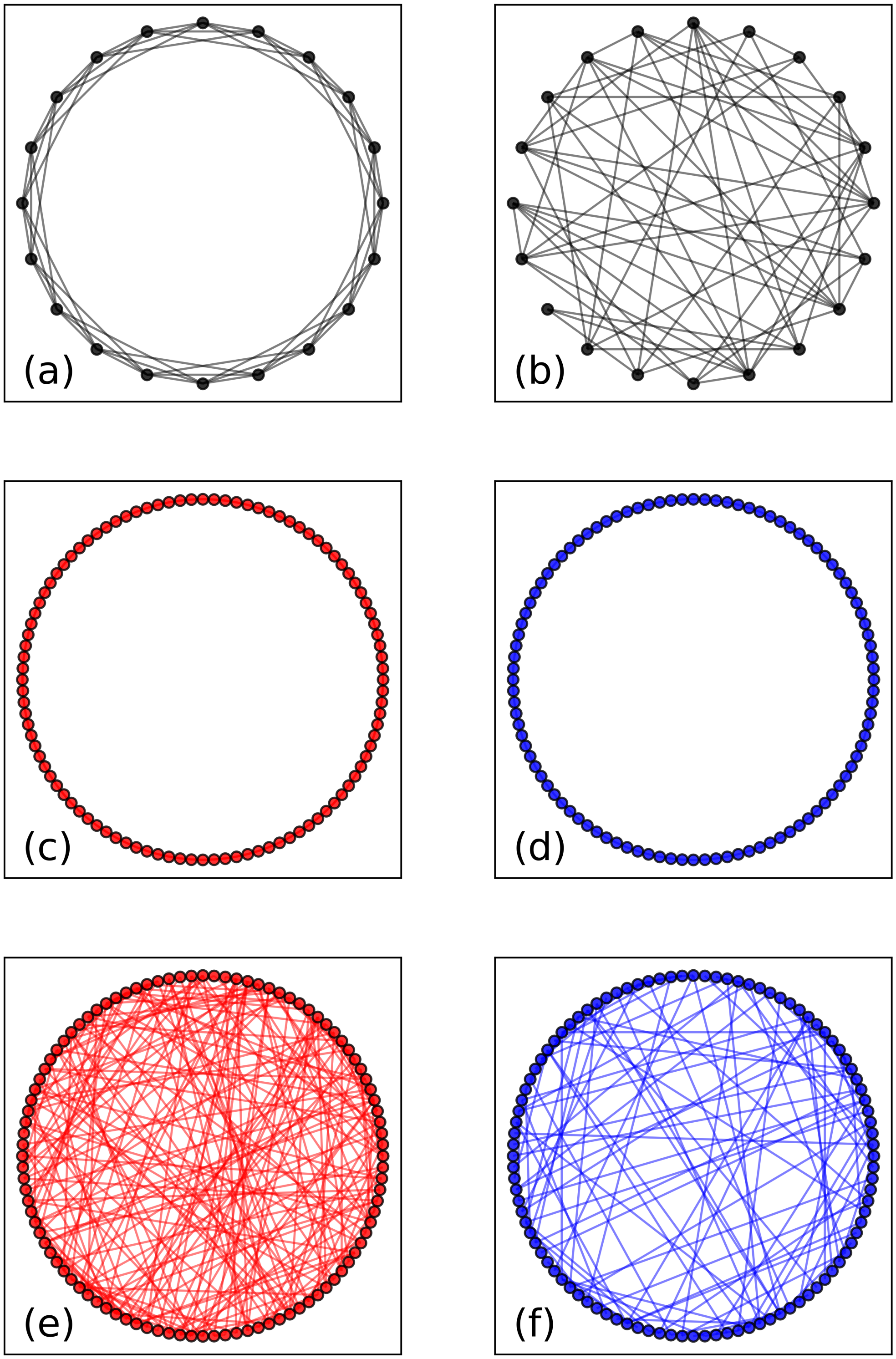

Fig. S3 Construction of Watts Strogatz networks. For each value of N, the $2D$ and $3D$ nanowire networks have a mean degree $\bar{k}_N^{2D}$ and $\bar{k}_N^{3D}$ respectively. A WS network is created first by defining the number of nodes N and the mean degree $\bar{k}$. A regular ring lattice is then constructed with N nodes with each of the nodes connected to their nearest $\bar{k}$ neighbours ($\bar{k}/2$ on each side). For each node, every edge is then rewired with a probability p. $p = 0$ is effectively the regular ring lattice unchanged, and $p = 1$ means that the wiring of the network is entirely random. (a) WS networks for a simple example with a small number of nodes $N = 20$ and $\bar{k} = 6$ with $p = 0$, i.e. a regular ring lattice, and (b) the same network re-wired with $p = 1$, i.e. a random network. (c) and (d) regular Watts-Strogatz networks ($p = 0$) corresponding to 2D and Q3D wire networks: $N = 100$ with $\bar{k}_{100}^{2D} = 4$ (red) and $\bar{k}_{100}^{3D} = 2$ (blue). Note that with these larger networks the edges are not discernible on this scale – see a for an example of a smaller network where the wiring is visible. (e) and (f) WS networks with all edges rewired with probability $p = 1$ to create random networks.

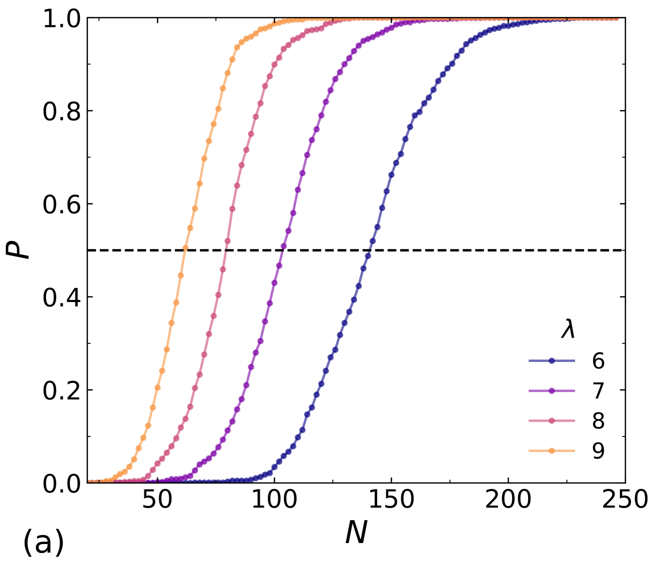

Fig. S4 (a) The probability of the formation of a network that spans the $30 \times 30 \mu m$ system as a function of N for different values of λ (increasing length from right to left). The percolation threshold is the value N_c for which the probability of spanning the system is 0.5 (horizontal dotted line). For smaller values of λ, it takes more wires to form a spanning network. As λ increases, the value of N_c decreases. (b) Data from (a) replotted as a function of the network density $\xi = N\frac{\lambda^2}{\Lambda^2}$. The scaled percolation threshold for all λ is $\xi_c = 5.63726$ (vertical dotted line). This scaling behaviour is in good agreement with the previous results of Li and Zhang (see Ref 23 in the main text).

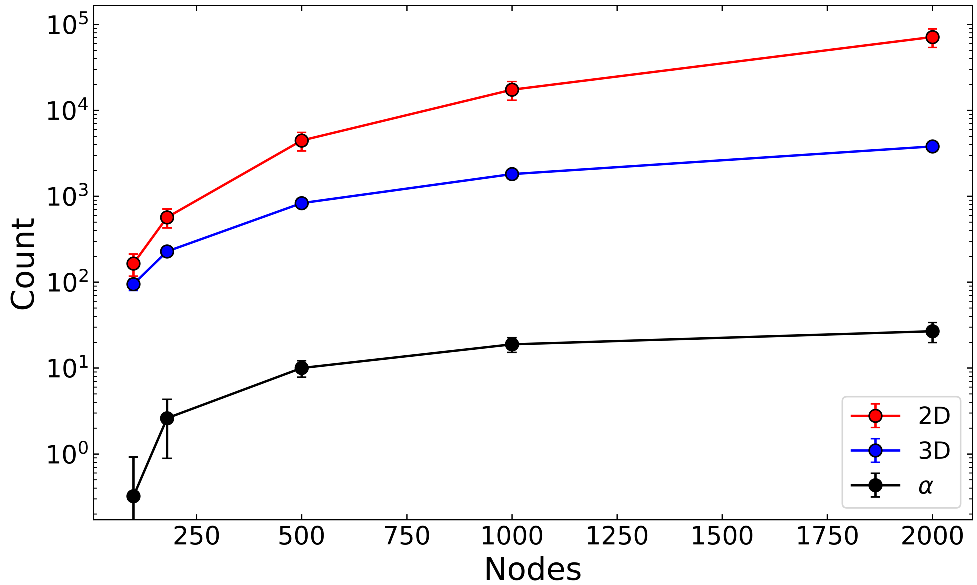

Fig. S5 For each wire in the network, we calculated the distance to all other wires. For a network with N wires, this means a total of $N(N-1)/2$ distances to be found. The literature shows that 1nm is a reasonable cut off distance for the range of tunnelling conduction (see Ref 22, 36-38 in the main text). The three curves show the number of junctions in the network (direct wire-to-wire contacts) for the 2D (red) and Q3D case (blue). Recall that in 2D there is an Ohmically conducting junction for every wire that is deposited on top of another wire but that in 3D the wires are often separated vertically as they become stacked during deposition, so that the number of conducting junctions is significantly lower in 3D. The black curve shows the number of wire-wire distances in the Q3D case that are smaller than 1nm, the range of tunnelling conduction. The number of distances within this cut-off is consistently less than 1% of the number of Ohmically conducting connections in the 2D network. This is consistent with previous results that demonstrates that tunnelling has a negligible effect on the conductance of networks of wires (see Ref 38 in the main text).

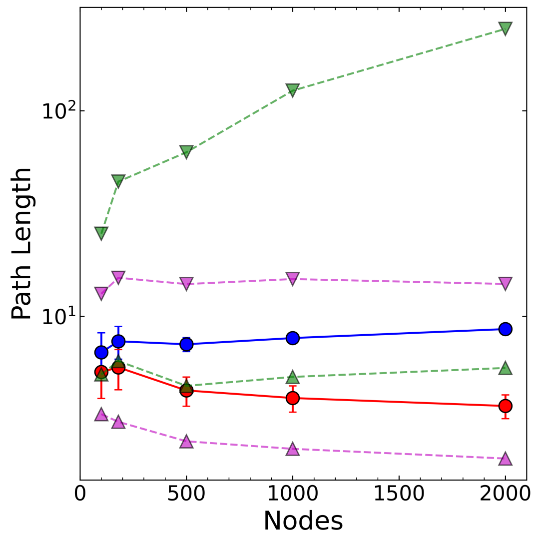

Fig. S6 An alternative of Fig. 3c with the y-axis on a logarithmic scale, showing the high values of path length for the regular WS network ($p = 0$ corresponsing to the 2D network.

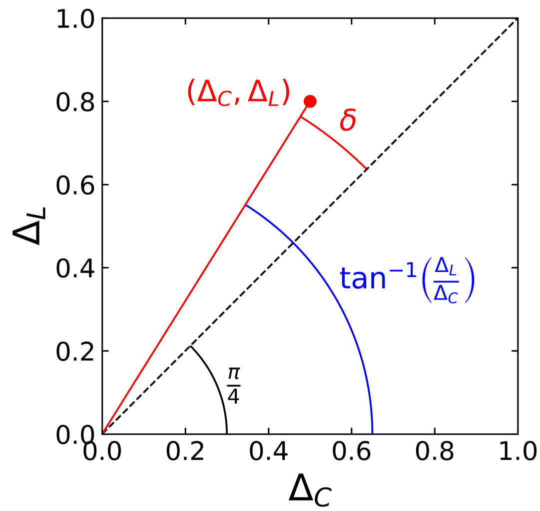

Fig. S7 Schematic illustration of the calculation of the deviation, δ (see methods for definition and Ref 39). When Δ_L and Δ_C contribute equally to the small-world propensity (dashed line) then $\delta = 0$. Geometrically, δ can be considered as the angular deviation from the dashed line. The numerical value is normalised (Eq. 8 in the main text) so that when $\Delta_C \to 1$ and $\Delta_L \to 0$, $\delta \to -1$, which indicates that a large deviation of the clustering (a low C) from that of the corresponding WS regular network is responsible for reduction in the small-world propensity, ϕ. When $\Delta_L \to 1$ and $\Delta_C \to 0$, then $\delta \to 1$. In this case it is the large deviation of the path length (a high L) from that of the random network that is responsible for a reduction in ϕ.