

Supplemental Material

**High-temperature and multichannel quantum anomalous Hall effect
in pristine and alkali-metal-doped CrBr₃ monolayers**

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Supporting note 1. Topological states of electron-doped CrBr₃ monolayer

Our calculated results in the main text have demonstrated that the topologically nontrivial band gap of monolayered CrBr₃ locates far above the Fermi level, indicating that the topological states can not be directly observed in the experiment. Our further calculations have shown that the Fermi level can be tuned into the band gap via one electron doping per unit cell, the corresponding doping concentration is $\sim 2.8 \times 10^{14} \text{ cm}^{-2}$. Fig. S1 gives the band structure of electron-doped CrBr₃ monolayer with considering of spin-orbit coupling (SOC). Obviously, a global band gap of 12.3 meV is achieved in the electron-doped system. The calculated band structure obtained from Wannier interpretation is nearly the same as that obtained from DFT calculations. Similar to the pristine CrBr₃, both positive and negative Berry curvatures appear along the high symmetry path. By integrating these nontrivial Berry curvatures over the first BZ, a high Chern number of $C = 2$ is again obtained, demonstrating the topological properties of pristine CrBr₃ can be well preserved by means of electron-doping.

Supporting note 2. Hubbard U effects on the band gap of the pristine CrBr_3 and alkali-metal-doped CrBr_3

In order to better describe the Cr $3d$ electrons, the GGA + U method was employed. The effective Hubbard U effects on the SOC-induced band gap and topological properties of the pristine and alkali-metal-doped CrBr_3 are investigated for pristine and alkali-metal-doped CrBr_3 monolayer, as shown in Fig. S4. Although U affects the quantitative value of the band gap, it does not alter the central physics of this study, suggesting the robustness of the topological states in all systems, especially in the alkali-metal-doped CrBr_3 systems.

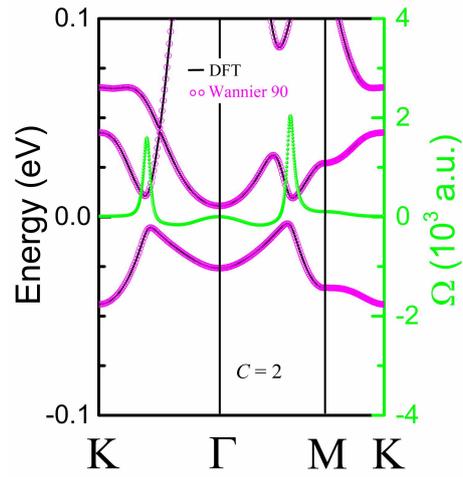


Fig. S1 The band structure of one-electron-doped CrBr₃ monolayer with SOC using the DFT calculations (solid black curves) and Wannier interpretation (open magenta circles). The green dots denote the Berry curvature (in atomic units (a.u.)).

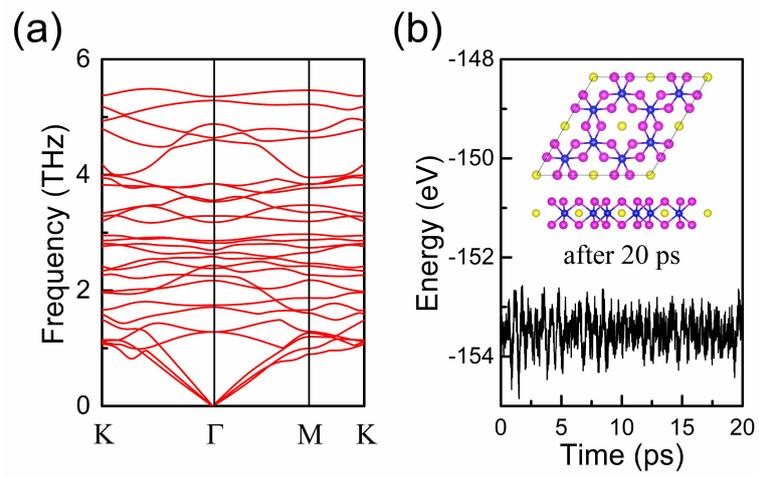


Fig. S2 (a) Calculated phonon spectra of a Na-doped CrBr₃ monolayer. (b) AIMD simulations of a Na-doped CrBr₃ monolayer at 300 K. The insets in (b) display the final structure after 20 ps.

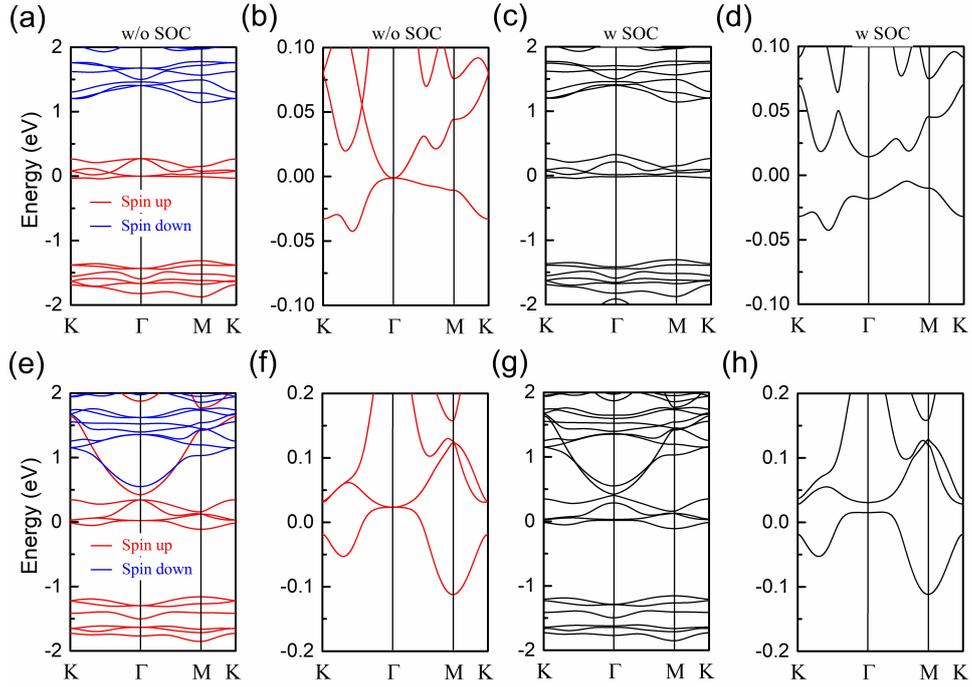


Fig. S3 Band structures of a Li-doped CrBr_3 monolayer without (a) and with SOC (c). The red and blue lines in (a) represent spin-up and spin-down bands, respectively. (b) and (d) are the magnified bands near Fermi level in (a) and (c). (e)-(h) The same as (a)-(d) but for a K-doped CrBr_3 monolayer.

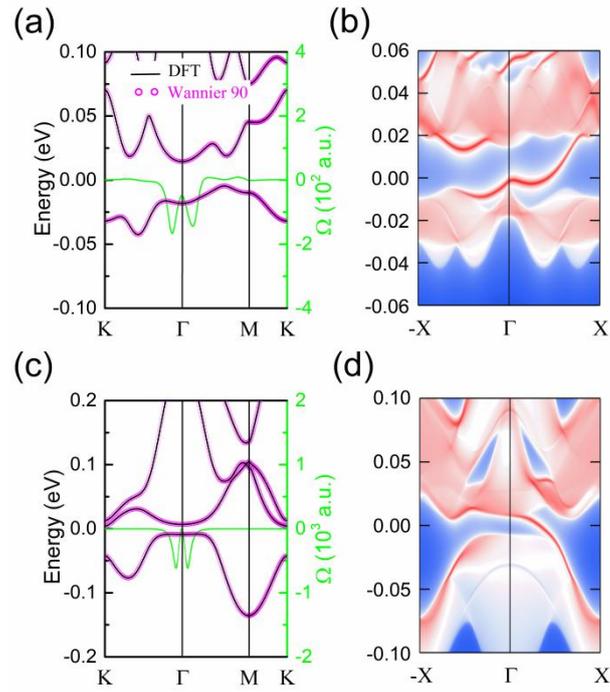


Fig. S4 (a) Calculated band structures of a Li-doped CrBr_3 monolayer with SOC using the DFT calculations (solid black curves) and Wannier interpretation (open magenta circles). The green dots denote the Berry curvature (in atomic units (a.u.)). (b) Calculated edge states of a half-infinite monolayer. (c) and (d) The same as (a) and (b) but for a K-doped CrBr_3 monolayer.

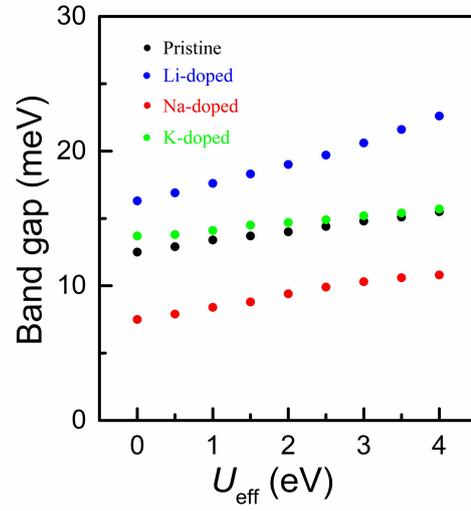


FIG. S5 Hubbard U effects on the nontrivial band gaps of pristine and alkali-metal-doped CrBr_3 monolayer.

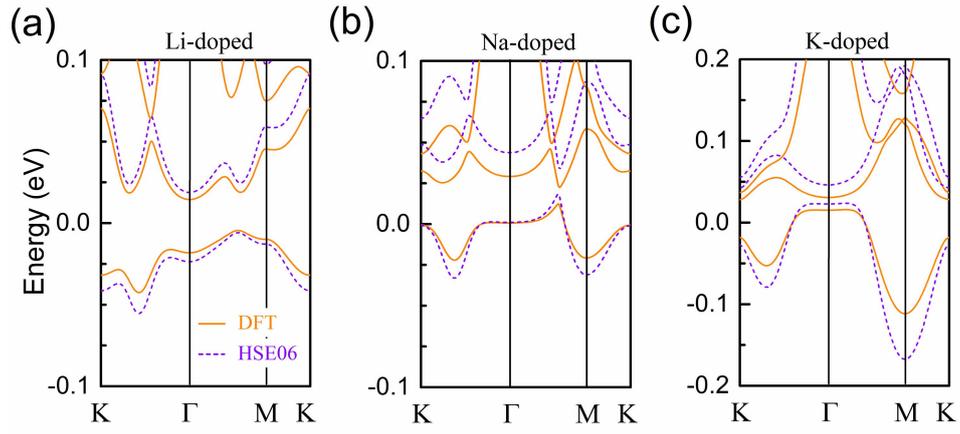


FIG. S6 Calculate band structures of Li (a), Na (b), and K (c)-doped CrBr₃ monolayers using DFT+*U* and HSE06 method.