Electronic supplementary information (ESI)

Effect of chain scission on flow characteristics of wormlike micellar solutions past a confined microfluidic cylinder: A numerical insight

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1. Choice of VCM model parameters

In this study, the VCM parameters that are chosen to carry out the present study are as follows: $\beta = 10^{-4}$, $\mu = 2.6$, $\varepsilon = 0.005$, $c_{aeq} = 1.6$, $c_{beq} = 0.8607$, $\delta_A = \delta_B = 10^{-2}$ and $\xi = 0.01-0.1$. The main reason behind the selection of this set of parameters is that the resulting rheological behaviour will follow the two properties of a wormlike micellar solution, namely, the shear-thinning behaviour and the extensional hardening and the subsequent thinning. These two trends can be evident in Fig.2 in the main manuscript. The main purpose of this article is to investigate the flow characteristics of a wormlike micellar solution with different level of chain scission energy which is controlled by the non-linear VCM model parameter $\xi$. In other words, how the flow phenomena would tend to change through this benchmark system as the micelles become progressively easier to break. Hence, we have selected the set of VCM parameters in such a way that it should not exhibit the shear-banding property. This property of a WLM solution has itself influence on the flow characteristics, for instance see refs 44-46 in the main manuscript. In fact, we have plan to investigate the influence of this shear-banding property on the flow characteristics, particularly on the onset and generation of this elastic instability in our future studies. This is also mentioned in the last paragraph of our conclusion section. Furthermore, the values which we have chosen are also not arbitrary. For instance, the values of the non-dimensional diffusivity and viscosity ratio are same or at least in the same order of magnitude with those obtained by fitting the experimental data$^{12}$. 

2. Numerical details

The diffusion terms in the momentum (Eq.2) and constitutive equations (Eqs.4-7) were discretized using the second-order accurate Gauss linear orthogonal interpolation scheme. The gradient terms were discretized using the Gauss linear interpolation scheme. While the linear systems of the pressure and velocity fields were solved using the Preconditioned Conjugate Solver (PCG) with DIC (Diagonal-based Incomplete Cholesky) preconditioner, the stress fields were solved using the Preconditioned Bi-conjugate Gradient Solver (PBiCG) solver with DILU (Diagonal-based Incomplete LU) preconditioner$^{3,4}$. The convective terms in the constitutive equations were discretized using the high-resolution CUBISTA (Convergent and Universally Bounded Interpolation Scheme for Treatment of Advection) scheme for its improved iterative convergence properties$^5$. In the present study, the pressure-velocity coupling was accomplished using the SIMPLE method, and the improved both side diffusion (ibSD) technique was used to stabilize the numerical solutions. The absolute tolerance level for the pressure, velocity and stress fields was set as $10^{-10}$. 
An excellent match can be seen between the present results and the results of Cromer et al.\textsuperscript{1}.

\textbf{Fig. S1.} Comparison between the present results (lines) and the results of Cromer et al.\textsuperscript{4} (symbols) for the flow of a wormlike micellar solution through a straight microchannel at a non-dimensional pressure gradient of $P = 1$. Non-dimensional streamwise velocity at (a) $\delta = 10^{-3}$ and (b) $\delta = 10^{-1}$. Magnitude of the non-dimensional conformational tensor component of the long chain $A$ at (c) $\delta = 10^{-3}$ and (d) $\delta = 10^{-1}$. Other non-dimensional VCM parameters are: $\beta = 7 \times 10^{-5}$, $\mu = 1.9$, $c_{A_{eq}} = 0.9$, $c_{B_{eq}} = 1.4$, $\varepsilon = 6.27 \times 10^{-4}$, $\xi = 0.3$. 
3. Results and discussion

Fig. S2. Representative of streamline profiles (a) Newtonian fluid (b) WLM solution, $\xi = 0$, $\text{Wi} = 0.445$ (c) WLM solution, $\xi = 0.01$, $\text{Wi} = 0.001$.

Fig. S3. Representative of streamline profiles (a) $\text{Wi} = 2.5$, $\xi = 0.01$, $t = 4$ (b) $\text{Wi} = 2.5$, $\xi = 0.01$, $t = 5$ (c) $\text{Wi} = 2.5$, $\xi = 0.1$, $t = 4$ (d) $\text{Wi} = 2.5$, $\xi = 0.1$, $t = 5$

Fig. S4: Velocity magnitude plots (a) Newtonian fluid (b) WLM solution, $\xi = 0.01$, $\text{Wi} = 0.001$. 
**Fig. S5:** Distribution of the long chain number density (a-d) and principal stress difference (e-h), (a, e) $\xi = 0.1, W\nu = 0.001$ (b, f) $\xi = 0.01, W\nu = 0.001$ (c, g) $\xi = 0.1, W\nu = 0.5$ (d, h) $\xi = 0.01, W\nu = 0.5$.

**References**