Viscous Bouncing

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Supplementary Information

Contact Time

We show in Figure S1 the measured contact time for water ($\eta = 1$ mPa.s) and water/glycerol mixture ($\eta = 80$ mPa.s) for drops with R = 1 mm. Results are plotted as a function of the Weber number $We = \rho V^2 R/\gamma$. In the regime of moderate velocity, the contact time tends to remain independent of impact velocity, a property expected in the spring-mass model of Richard *et al.* (2002). In addition, the contact time is slightly higher for the larger viscosity, as stressed by the dotted lines, in agreement with the discussion in the accompanying paper. We complete this observation by showing two additional regimes not discussed in the paper.

1) In the low-impact velocity regime, the contact time τ increases with decreasing impact velocity, an effect shown by *Chevy et al.* (2012) to arise from the existence of a logarithmic correction for the spring stiffness. The effective softness becoming smaller, the contact time increases. The small shift between the two viscosities discussed in the accompanying paper is found to persist in this regime, in agreement with our arguments: a change in the effective stiffness of the spring affects the period of the spring, but not the damping effect of viscosity.

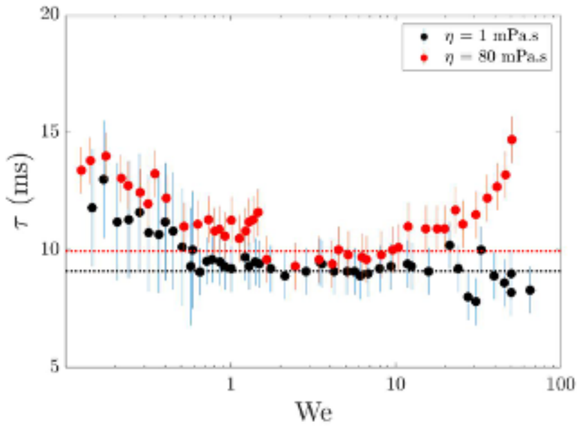

Figure S1. Contact time τ as a function of the Weber number $We = \rho V^2 R/\gamma$ for $\eta = 1$ mPa.s (black dots) and $\eta = 80$ mPa.s (red dots). The dotted lines represent the mean contact time for each viscosity in the moderate velocity range ($1 < We < 10$) where the contact time tends to remain independent of impact velocity.

2) In the regime of high impact velocity, the behaviors are much highly contrasted. Data with water becomes more scattered and contact time slightly diminishes, which is due to splashing. In contrast, using a more viscous liquid (red data) does not lead to splashing (highly viscous drops are more stable against deformation), and this regime is found to be accompanied by a significant increase of contact time. In this highly non-linear regime of deformation, the increase in contact time could be attributed to the slow recoil of viscous drops (*Bartolo et. al.* 2005 [ref. 13 in the accompanying paper]): rather than describing the rebound as an oscillation of the drop, we must describe the recoiling stage as a dewetting process sensistive to the viscosity of the liquid.

Coefficient of Restitution

Figure S2 shows the variation of coeffcient of restitution ε as a function of the Weber number *We* for the same viscosities as in figure S1. The behavior for water is documented in the literature: data below $We \approx 1$ plateau: in this regime of small deformation, the coefficient of restitution reaches its maximum fixed by the intrinsic adhesion of water on the material. Fluctuations are seen, coming from the experimental procedure: at such small impact velocity, the rebounds correspond to successive rebounds for which an incoming drop is still vibrating – and ε is known to depend on the phase of the vibration (in other words, the vibration energy is not negligible compared to the traslational kinetic energy of the drop). Above $We \approx 1$, ε decreases with the impact velocity, as discussed and modeled by Biance *et al.* (2012) [ref. 14 in the accompanying paper] and is shown to be due to the transfer of kinetic energy into vibrational energy at impact – a phenomenon all the more relevant since *V* is large.

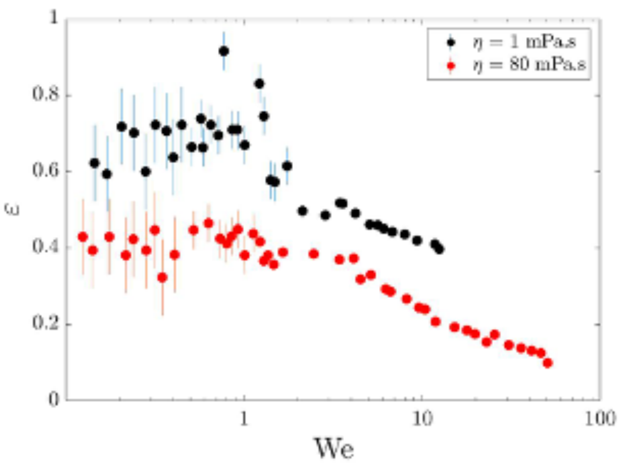

Figure S2. Variation of coefficient of restitution ε against increasing Weber number, *We* for two different viscosities of $\eta = 1$ mPa.s (black dots) and $\eta = 80$ mPa.s (red dots).

All these phenomena remain observed with the larger viscosity, with two differences: 1) Values are shifted to smaller ε, as discussed in the accompanying paper: a more viscous solution induces a larger dissipation and thus generates a softer shock. 2) Data can be extended by a factor 5 in Weber number, owing to the absence of fragmentation. The general trend is observed to be the same, with a strong decrease of ε as the velocity increases – in non-linear regimes of deformation, as discussed above, a non-negligible dissipation takes place during the recoiling phase of the rebound, causing both an increase of contact time and a decrease of coefficient of restitution.

Derivation of equations

As discussed in the paper, the dynamic equation for the deformation of a viscous drop impacting on a solid surface can be written down as:

$$m\ddot{r} + \eta R\dot{r} + \gamma x = 0 \quad (1)$$

where *r* denotes the general deformation of the drop, and η, γ, *m* and *R* the drop viscosity, surface tension, mass and radius, respectively. After normalizing time by the inertio-capillary scaling $\tau_0 = \sqrt{\rho R^3/\gamma}$ (where ρ is the liquid density) and *r* by the drop radius *R,* we get:

$$\ddot{x} + Oh\dot{x} + x = 0 \tag{2}$$

where *x* denotes the dimensionless deformation. This equation shows that the damping of the oscillations is decided by the Ohnesorge number $Oh = \eta/(\gamma\rho R)^{1/2}$. For an underdamped scenario, a solution to the equation exists when,

$$Oh < 2$$

which puts a limit to the phenomena of bouncing. Under this condition, the general solution of equation 2 can be written down as:

$$x(t) = C_1 \exp\left(-Oh.\frac{t}{2}\right)\cos\left(\frac{\sqrt{4-Oh^2}t}{2}\right) + C_2 \exp\left(-Oh.\frac{t}{2}\right)\sin\left(\frac{\sqrt{4-Oh^2}t}{2}\right) \tag{3}$$

for the deformation of the drop and:

$$v(t) = -\frac{Oh}{2}\left[C_1 \exp\left(-Oh.\frac{t}{2}\right)\cos\left(\frac{\sqrt{4-Oh^2}}{2}t\right) + C_2 \exp\left(-Oh.\frac{t}{2}\right)\sin\left(\frac{\sqrt{4-Oh^2}}{2}t\right)\right] + \left(\frac{\sqrt{4-Oh^2}}{2}\right)\left[-C_1 \exp\left(-Oh.\frac{t}{2}\right)\sin\left(\frac{\sqrt{4-Oh^2}}{2}t\right) + C_2 \exp\left(-Oh.\frac{t}{2}\right)\cos\left(\frac{\sqrt{4-Oh^2}}{2}t\right)\right] \tag{4}$$

for its velocity. The constants of integration are calculated after expressing the initial conditions, namely $x = 0$ and $V = V_0\tau_0/R$, which gives:

$$x(t) = \frac{2V_0\tau_0}{R\sqrt{4-Oh^2}}\exp\left(-Oh.\frac{t}{2}\right)\sin\left(\frac{\sqrt{4-Oh^2}}{2}t\right) \tag{5}$$

$$v(t) = -\frac{V_0\tau_0 Oh}{R\sqrt{4-Oh^2}}\exp\left(-Oh.\frac{t}{2}\right)\sin\left(\frac{\sqrt{4-Oh^2}}{2}t\right) + \frac{V_0\tau_0}{R}\exp\left(-Oh.\frac{t}{2}\right)\cos\left(\frac{\sqrt{4-Oh^2}}{2}t\right) \tag{6}$$

The drop takes off after half a cycle when we recover the value $x = 0$, which provides an estimate of the contact time:

$$\tau = \frac{2\pi}{\sqrt{4 - Oh^2}} \tag{7}$$

The velocity of the drop at this time is its take-off velocity:

$$v(\tau) = -\frac{V_0 \tau_0}{R} \exp\left(\frac{-\pi . Oh}{\sqrt{4 - Oh^2}}\right) \tag{8}$$

which can be rescaled with respect to the impact velocity to give the coefficient of restitution:

$$\varepsilon = \exp\left(\frac{-\pi . Oh}{\sqrt{4 - Oh^2}}\right) \tag{9}$$

Captions of the movies

Movie 1. Side view of a water drop of radius *R* = 1 mm impacting on a super-hydrophobic surface with velocity *V* = 0.3 m/s. The drop takes-off 10 ms after its impact.

Movie 2. Same experiment with a water/glycerol mixture 80 times more viscous than water. The drop is able to bounce off in the same 10 ms, but the rebound is weaker.

Movie 2. Same experiment with a water/glycerol mixture 200 times more viscous than water. The drop detaches from the surface after a significantly higher time (16 ms instead of 10 ms), but it barely rises above the substrate (viscous bouncing limit).