

**Supplementary Information for**  
**‘Hierarchical glass transition of hard hemidisks with local assemblies’**

Wei Zheng,<sup>1,2,\*</sup> Qun-Li Lei,<sup>2,\*</sup> Yuqiang Ma,<sup>1,†</sup> and Ran Ni<sup>2,‡</sup>

<sup>1</sup>*National Laboratory of Solid State Microstructures and Department of Physics,  
Collaborative Innovation Center of Advanced Microstructures, Nanjing 210093, China*

<sup>2</sup>*School of Chemical and Biomedical Engineering,  
Nanyang Technological University, 62 Nanyang Drive, 637459, Singapore*

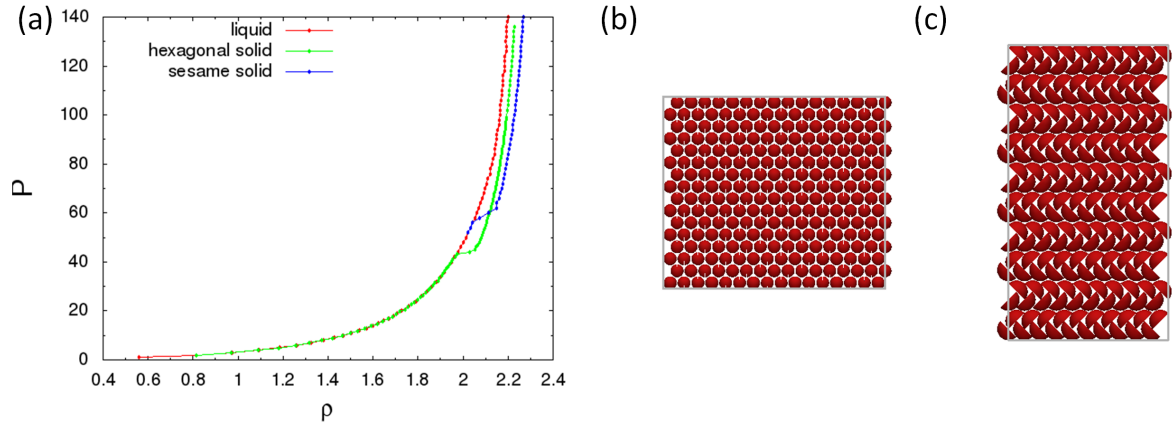


Fig. S1: (a) Equation of state of different phases for the hemi-disk systems. (b) hexagonal crystal phase. (c) closest-packed sesame crystal phase. The highest packing fraction of the closest-packed sesame crystal phase found in our simulation is 0.9331.

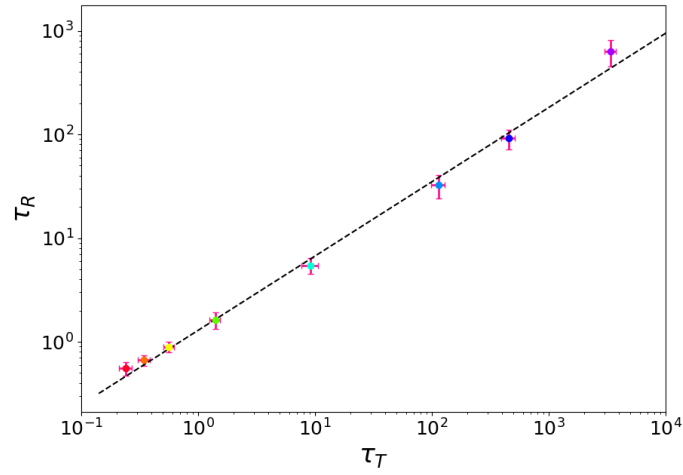


Fig. S2: Cross plot of  $\tau_t$  and  $\tau_r$  for translational and rotational freedom degree respectively using the same data points from Fig. 2 in the main text. Different color represent different density. From red to blue, the density increases.

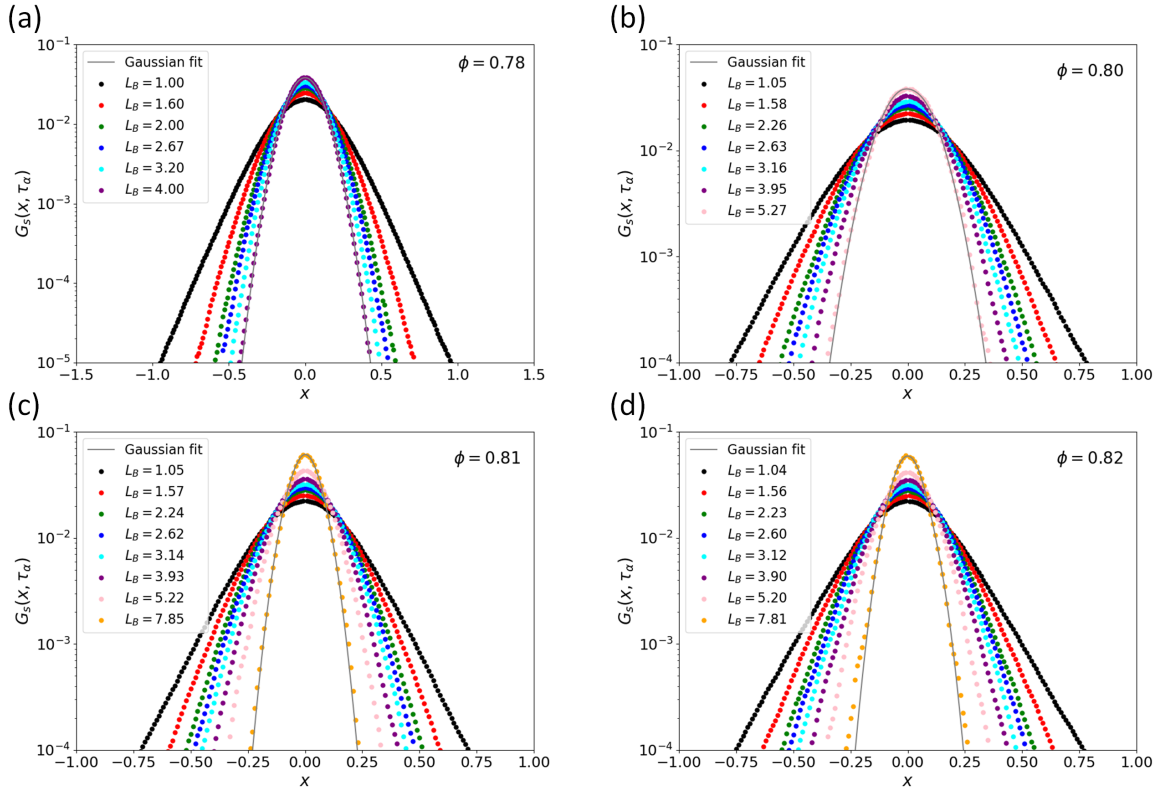


Fig. S3: (a-d) Coarse-graining van Hove function of translational displacement using different block lengths. Here  $x$  represents the sum of particles displacements in the block for both  $x, y$  directions for systems with different densities.

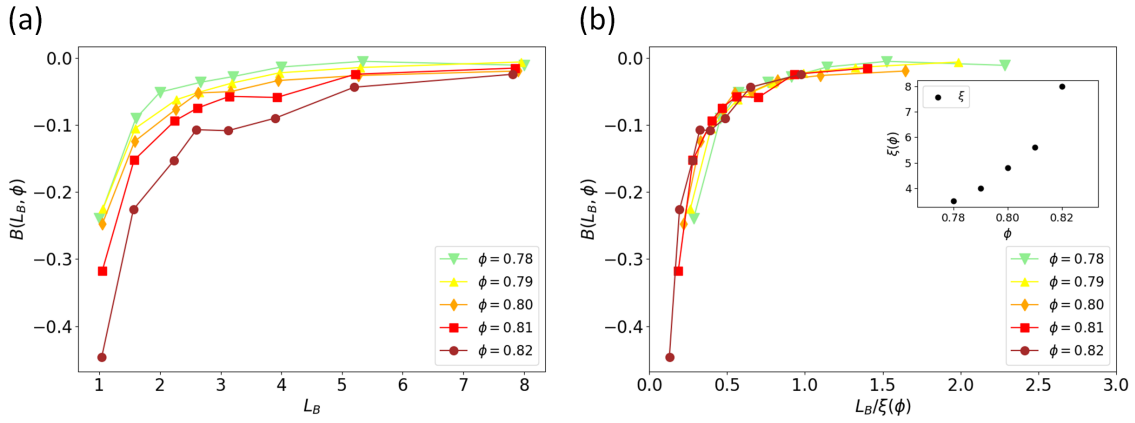


Fig. S4: (a) Binder cumulant of the distribution in Fig. S3 as a function of block lengths for systems with different densities. (b) Collapsing of Binder cumulant by introducing the parameter  $\xi$  (heterogeneity length scale). The inset shows the obtained values of  $\xi$ .

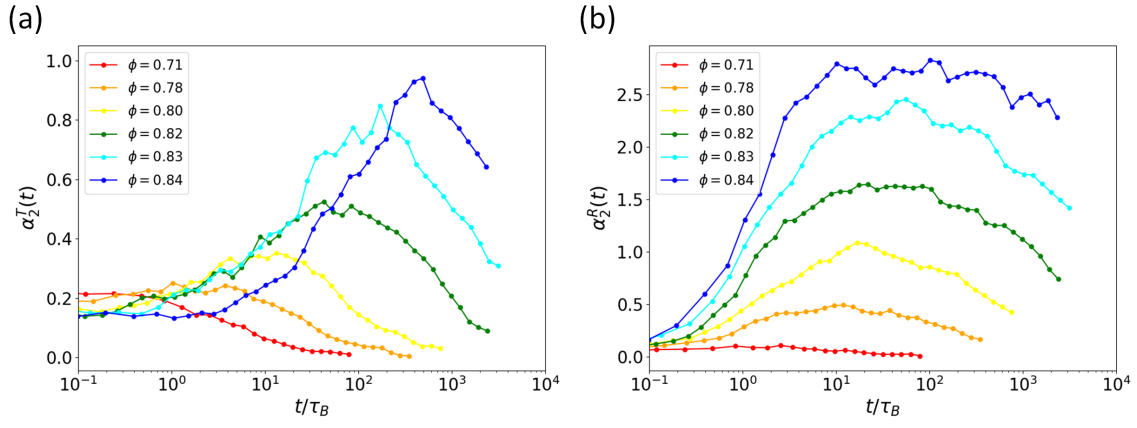


Fig. S5: (a-b) Non-Gaussian Parameter (NGP) for the translational freedom degree (a) and rotational freedom degree (b) for systems at different densities.

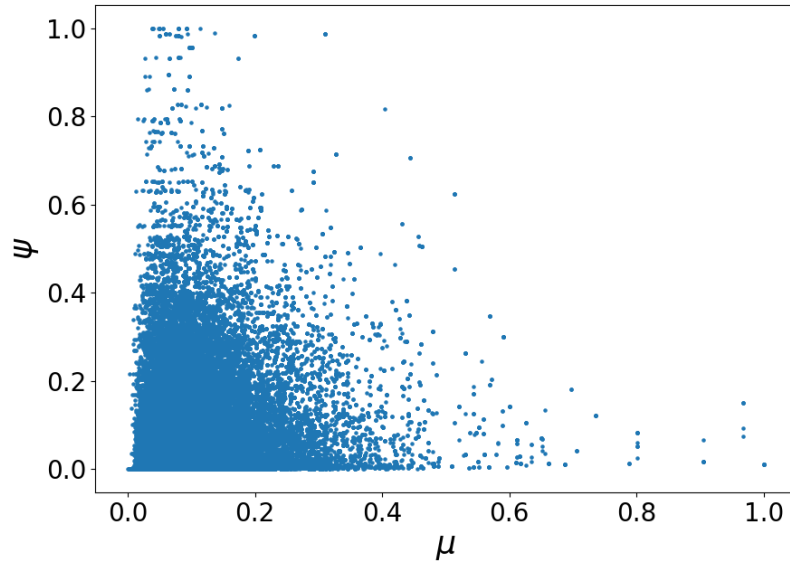


Fig. S6: Scattering plot of the distribution for the mobilities in translation freedom degree ( $\mu$ ) and rotational freedom degree ( $\psi$ ) for all particles in the system.