Supplementary Information

Unraveling Driving Regimes for Destabilizing Concentrated Emulsions within Microchannels

T. Porto Santos,*,†,‡ C. M. Cejas,*,‡ R. Lopes Cunha,† and P. Tabeling‡

†*Department of Food Engineering, Faculty of Food Engineering, University of Campinas, Rua Monteiro Lobato, 80-CEP 13083-862 Campinas, Brazil*

‡*Microfluidics, MEMS, Nanostructures Laboratory, CNRS Chimie Biologie Innovation (CBI) UMR 8231, Institut Pierre Gilles de Gennes (IPGG), ESPCI Paris, PSL Research University, 6 rue Jean Calvin 75005, Paris, France*

E-mail: tatiana.porto90@gmail.com; cesare.cejas@gmail.com

A: Resistance and flow rate calculations

A schematic of the microchamber is shown in Fig. 1. This microchamber houses the packed emulsion droplets. Pressure-induced stress occurs by injecting fluid (deionized water/saline solution) from the input. The fluid interacts with the emulsion packing and then exits at the output.

The resistance, R, of microchannels is calculated from the following equations.[1,2]

$$R = \frac{12\mu L}{WH^3}\left\{1 - \left[\left(\frac{192W}{W\pi^5}\right)\tan\left(\frac{\pi W}{2H}\right)\right]\right\}^{-1} \tag{1}$$

where $\mu = 10^{-3}$ Pa·s, L is the length, W is the width and H is the height of the microchannel chamber. The parameters L, H, and W refer to different parts of the location in the chamber as explained below.

For the triangular part of the chamber:

$L_{tri} = 2800\mu$m; $W_{tri} = 1650\mu$m; $H_{tri} = 55\mu$m

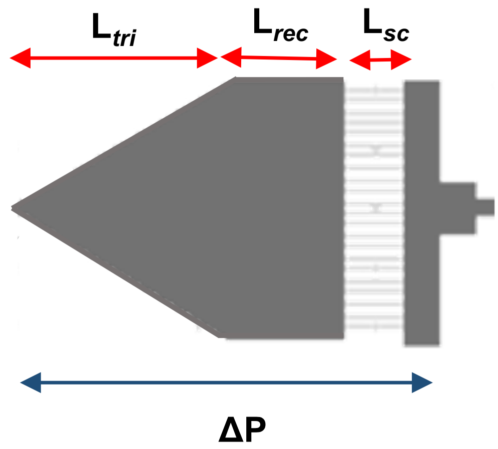

Figure 1: Sketch of the principal microchamber with its component parts. The entire microchamber can be subdivided into a triangular section (where L_{tri} is the length of this subsection), a rectangular section (where L_{rec} is the length of this subsection), and the section with the small channels at the outlet (where L_{sc} is the length of the small outlet channels). The pressure difference, ΔP, is calculated across the entire total length of the microchamber.

Using Eq.1, we can calculate the resistance of the triangular chamber, R_{tri}.

For the rectangular part of the chamber:

$L_{rec} = 1400\mu$m; $W_{rec} = 3300\mu$m; $H_{rec} = 55\mu$m

Using Eq.1, we can calculate the resistance of the rectangular chamber, R_{rec}.

For the small channels in the outlet:

$L_{sc} = 1020\mu$m; $W_{sc} = 20\mu$m; $H_{sc} = 55\mu$m or $H_{sc} = 20\mu$m (for P=20mbar and P=200mbar respectively).

Because the small channels are in parallel:

$$\frac{1}{R_{sc-total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... \frac{1}{R_N} \tag{2}$$

where N is the number of channels in the outlet, $N = 22$.

Since all resistances are equal ($R_1 = R_2 = ... = R_{sc}$), we can calculate the resistance of each individual outlet channel using the general Eq.1. The total resistance of all the small outlet channels combined is then:

$$R_{sc-total} = \frac{R_{sc}}{N} \tag{3}$$

Total resistance of the entire microfluidic chip:

From the aforementioned parameters, the total resistance of the entire chip is a series of its individual components:

$$R_T = R_{tri} + R_{rec} + R_{sc-total} \tag{4}$$

And the flow rate can be calculated from:

$$Q = \frac{\Delta P}{R_T} \tag{5}$$

where ΔP is either 20 or 200 mbar (converted to Pa) and R_T is the total resistance from Eq.4.

B: Permeability and flow rate calculations

When the interior of the microchannel chamber is packed with emulsion droplets, the system can be considered as a porous medium. Since we are injecting an external phase (saline solution/water) into the chamber, then the permeability of both emulsion droplet packing as well as the small channels should also be considered.

Permeability of the small channels (k_{sc}):

$$\frac{dP_{sc}}{dL} = \frac{\Delta P}{L_{sc}} \tag{6}$$

where $\frac{dP_{sc}}{dL}$ is the change in pressure per unit length of the small channels, L_{sc} is the length of the small channels and ΔP is the pressure difference applied in the system. Thus, the permeability of the small channels, k_{sc}, can be calculated according to Eq.7.

$$k_{sc} = \frac{\mu Q}{WHN} \frac{dL}{dP_{sc}} \tag{7}$$

where W, H and N are the width, height and number of the small channels, respectively: $H = 55\mu$m or 20μm and $N = 22$; Q is the flow rate obtained from Eq.5 and $\mu = 10^{-3}$ Pa·s.

Permeability of the the emulsion droplet packing (k_{drop}):

$$\frac{dP_{total}}{dL} = \frac{\Delta P}{L_{tri} + L_{rec} + L_{sc}} \tag{8}$$

where $\frac{dP_{total}}{dL}$ is the change in pressure per unit length of the entire system, L_{sc}, L_{tri}, and L_{rec} are the length of the small channels, the triangle chamber, and the rectangle chamber respectively. ΔP is the pressure applied in the system.

$$\frac{\sigma}{\sigma_0} = \left[\frac{\varphi - \varphi_c}{1 - \varphi_c}\right]^u \tag{9}$$

where $\frac{\sigma}{\sigma_0}$ is the conductivity of the fluid phase within the porous material with respect to the fluid's bulk conductivity.[3] $\varphi = 0.64$ is the porosity of the emulsion packing in quasi-2D cases assuming random close packing, $\varphi_c = 0.32$ is the percolation threshold of overlapping droplets in quasi-2D and $u = 1.3$ is the critical exponent.

The permeability of the porous medium (emulsion droplet packing), k_{drop}, is:

$$k_{drop} = C(l_c)^2 \frac{\sigma}{\sigma_0} \tag{10}$$

where C=1/12 is a constant for 2D systems; $l_c = 40\mu$m is the local pore geometry (usually the same order of magnitude of droplet size).

Therefore, total permeability is:

$$k_{total} = k_{sc} + k_{drop} \tag{11}$$

And the actual flow rate is:

$$Q_{final} = \left(\frac{k_{total}}{\mu}\right) \cdot P_{total} \cdot W \cdot H \cdot N \tag{12}$$

where W, H, and N are the width, height, and number of small channels respectively. In Eq.12, the area considered is $W \cdot H \cdot N$ and $\mu = 10^{-3}$Pa·s.

References

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(3) Scholz, C.; Wirner, F.; Gotz, J.; Rude, U.; Schroder-Turk, G. E.; Mecke, K.; Bechinger, C. Permeability of Porous Materials Determined from the Euler Characteristic. *Phys. Rev. Lett.* **2012**, *109*, 264504.