## Supplementary material

Figure S1 shows the outcome of some of the cycling tests we have performed to ensure that in the most demanding case for the numerical method  $\eta = 0.05$  and H = 50, the brushes have reached the equilibrium state and that the systems are not caught into a frozen or mestastable state: brushes react quickly, within few  $\Delta t_{frames}$ , to a sudden change in the value of the field, and similar steady states are reached during the different cycles performed (5 cycles). The average values at the steady states coincide with those obtained in the simulations we have reported in the manuscript. As expected  $N_m = 2$  brushes do react faster to sudden changes than brushes with a higher content of magnetic particles  $N_m = 4$  and  $N_m = 10$ .



**Fig. S1** Averaged temporal behaviour of the expected value of the centre of mass of the brush obtained from a set of 5 consecutive cycles that start from the equilibrated high field state H = 50, then suddenly the field is dropped to zero. Subsequently the field is suddenly reestablished at  $t = 125\Delta t_{frame}$  for  $N_m = 2$ , and at  $t = 1650\Delta t_{frame}$  for  $N_m = 4$ , 10. Time axis is given in number of  $\Delta t_{frames}$ . The plot shows the cases for  $N_m = 2$ , 4, 10 at  $\eta = 0.05$  which is the most demanding case for the numerical method. Grey shadowed areas in the background depict the range of fluctuations that non-averaged series typically display.

Figures S2 and S3 portray for the zero magnetic field case (H = 0), the average normalized number density profiles  $\rho(z)$  and  $\rho_{end}(z)$ , see eq. 14 respectively. For the sake of an easier comparison, in these figures we have plot separately the cases  $N_m = 1, 2, 4, 10$ . Figures S4 and S5 depict the limit case of high magnetic field (H = 50) and the average normalized number density profiles  $\rho(z)$  and  $\rho_{end}(z)$ , respectively. Again, in these figures we have plotted separately the cases  $N_m = 1, 2, 4, 10$ . Figures Normalized number density profiles  $\rho(z)$  and  $\rho_{end}(z)$ , respectively.

Figures S6 and S7 show for ( $N_m = 4$ ,  $\eta = 0$ ) and ( $N_m = 4$ ,  $\eta = 0.05$ ), respectively, a comparison of the density profiles obtained for brushes using the standard arrangement of magnetic beads in the filaments (as shown in figure 1), and brushes in which each filament uses a random positioning sequence of the magnetic monomers in the filament. Figures show both H = 0 and H = 10 cases.

Figures S8 and S9 show for  $N_m = 4$  and  $N_m = 10$ , respectively, the behaviour of both SN and NSB brushes under negative fields, i.e.



**Fig. S2** Average normalized density profiles  $\rho(z)$ , as defined in equation 14, for brushes with zero external field, H = 0, in a quiescent fluid for  $N_m = 1, 2, 4, 10$ . Length of the filaments is N = 20 colloidal particles. Error bars depict 90% confidence intervals.

 $\vec{H} = -H\hat{z}$ , and compare them to the H = 0 case.

Figure S10 shows *BEP* and *BEP*<sub>end</sub> brush behaviour for  $N_m = 4$ , 10 in the case of negative fields.



Fig. S3 Same than figure S2 but for the normalized density of free ends of the filaments (end beads),  $\rho_{end}(z)$ .



**Fig. S4** Average normalized density profiles  $\rho(z)$ , as defined in equation 14, for brushes with zero external field, H = 50, in a quiescent fluid for  $N_m = 1, 2, 4, 10$ . Length of the filaments is N = 20 colloidal particles. Ripples toward the free end of the brush for  $N_m = 10$  and H = 50 point out the very strong correlations that exist for this particular case. These correlations are due to the very stretched conformations that filaments adopt in a high strength field.



Fig. S5 Same than figure S4 but for the normalized density of free ends of the filaments (end beads),  $\rho_{end}(z)$ .



**Fig. S6** Density profile comparison between brushes using standard sequence for the position of magnetic particles (see figure 1) and brushes in which each filament of the brush uses a random magnetic positioning sequence. Profiles for H = 0 and H = 10 are shown for  $\eta = 0$  cases. The label "random" denotes the brushes having filaments with random positioning for their magnetic particles.



Fig. S7 Same as figure S6 for  $\eta = 0.05$ .



**Fig.** S8 Density profiles obtained for both NSB ( $\eta = 0$ ), and SB ( $\eta = 0.05$ ) brushes with  $N_m = 4$  in the case of negative files, that we denote as H = -1, -10, -50 to avoid confusions. These profiles are compared with the corresponding case H = 0.



Fig. S9 Same as figure S8 for  $N_m = 10$ .



**Fig. S10** *BEP* and *BEP*<sub>end</sub> behaviour for  $N_m = 4$  and  $N_m = 10$  brushes in the case of negative fields, fields along z-axis pointing inwards the grafting surface. We portray such fields using a minus sign.