Model for the Electro-mechanical Behavior of Elastic Organic Transistors

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I. Derivation of Electrical Behavior of Elastic Thin Film Transistors with Deformation

To examine the changes in the electrical characteristics of thin film transistors (TFTs) upon mechanical deformation, we use the extension ratio, λ. The extension ratio is defined as the ratio of the deformed dimension (x_i) to the initial dimension ($x_{i,0}$) in a particular direction, $\lambda_i = x_i / x_{i,0}$. This metric is preferable to strain for the description of large deformations of polymers. For most amorphous polymers Poisson's ratio is essentially equal to 0.5 and the product of the extension ratios is unity, i.e. $\lambda_1\lambda_2\lambda_3=1$, because the material is incompressible.

We present here the full derivation for uniaxial deformation along the channel length of an elastic TFT where all of the materials, semiconductor, dielectric, and electrodes, are deformed to the same extent and behave elastically. The following derivation follows the conventional model for polymer elasticity at small deformation, i.e. $\lambda <$ ~2.[1] For a TFT, we define the extension ratios along the channel width (W), length (L) and dielectric thickness (t) such that:

$$\lambda_W\lambda_L\lambda_t=1 \qquad (1)$$

The TFT as a whole is deformed by λ_L by a force along the channel direction. The other directions are unconstrained because no force is applied and their extension ratios are equivalent.

$$\lambda_W = \lambda_t \qquad (2)$$

This results in a simple relationship between the extension ratios given by Eq. 3 and 4.

$$\lambda_W \lambda_t = \frac{1}{\lambda_L} \qquad (3)$$

$$\lambda_W = \lambda_t = \frac{1}{\sqrt{\lambda_L}} \qquad (4)$$

These relationships can be used to derive the change in the gate capacitance per area C_G (F/cm^2) with deformation given by Eq. 5 and 6 where ε_0 is the vacuum permittivity, ε_r is the dielectric constant of the gate dielectric, and t is the thickness of the gate dielectric.

$$C_G = \frac{\varepsilon_0 \varepsilon_r}{t} \qquad (5)$$

$$C_{G,\lambda} = \frac{\varepsilon_0 \varepsilon_r}{\lambda_t t} = \sqrt{\lambda_L}\frac{\varepsilon_0 \varepsilon_r}{t} = \sqrt{\lambda_L} C_G \qquad (6)$$

The change in W/L of the TFT under uniaxial deformation is given by

$$\frac{\lambda_W W}{\lambda_L L} = \frac{W}{\lambda_L^{3/2} L} \qquad (7)$$

We assume that the total amount of trapped charge in the semiconductor is the origin of the threshold voltage V_T and is constant, $q_{trap} = V_T C_G$. This results in the following expressions:

$$C_G V_T = C_{G,\lambda} V_{T,\lambda} \qquad (8)$$

$$V_{T,\lambda} = \frac{1}{\sqrt{\lambda_L}} V_T \qquad (9)$$

With all of the geometric dependences in hand from Eq. 6, 7, and 9, we can substitute into the gradual channel model for the current-voltage behavior of TFTs to obtain the behavior in the linear and the saturation regimes as a function of uniaxial deformation resulting in Eq. 10 & 11 respectively.

$$I_{SD} = \frac{1}{\lambda_L}\frac{W}{L}C_G\mu\left[\left(V_G - \frac{1}{\sqrt{\lambda_L}}V_T\right)V_{SD} - \frac{V_{SD}^2}{2}\right] \qquad (10)$$

$$I_{SD} = \frac{1}{\lambda_L}\frac{W}{L}C_G\mu\left[\frac{1}{2}\left(V_G - \frac{1}{\sqrt{\lambda_L}}V_T\right)^2\right] \qquad (11)$$

The forms for uniaxial deformation along the channel width, W, and biaxial deformation along W and L can be derived similarly.

II. Derivation of Stability Criterion for Complementary Inverters with TFTs

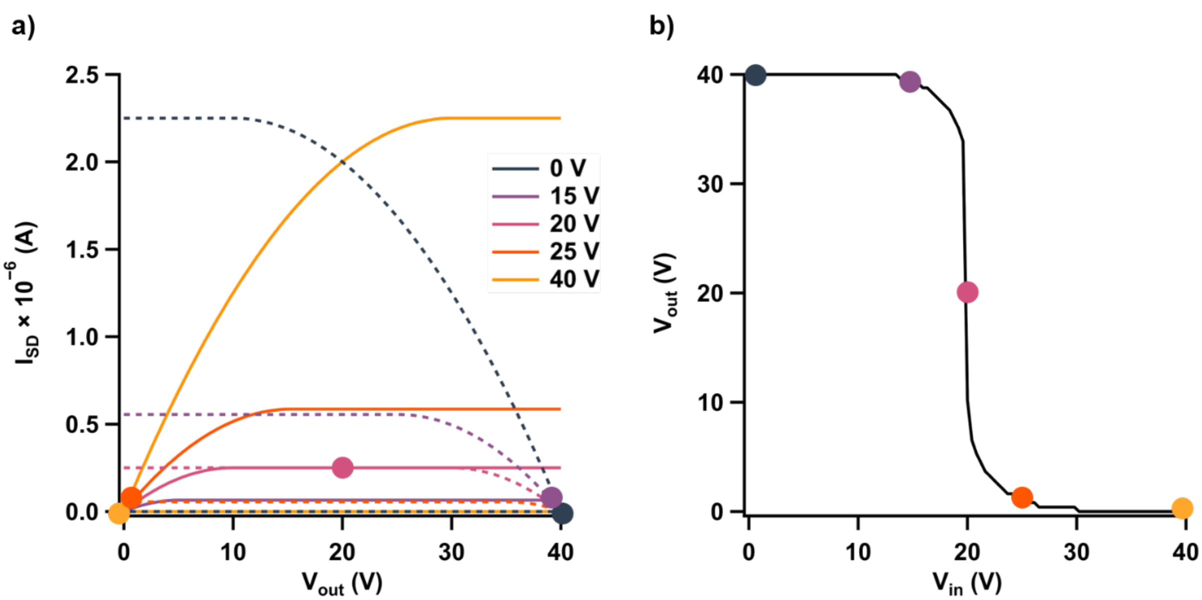

Figure S1. a) Load curves of the n- (solid) and p-type (dashed) TFTs over varying input voltages (V_{in} = 0, 15, 20, 25, and 40 V). **b)** The voltage transfer curve is created by finding the crossover points of the n- and p-type load curves for a sweep of input voltages.

The value of the inverting voltage, V_{inv}, for a complementary inverter where both the *p*- and *n*-type transistors are in the saturation regime is given by the standard expression in Eq. 12.[2] The device parameters of the two TFTs are given by Eq. 13.

$$V_{inv} = \frac{V_{DD} - \left|V_{T,p}\right| + V_{T,n}\sqrt{\beta_n / \beta_p}}{1 + \sqrt{\beta_n / \beta_p}} \qquad (12)$$

$$\beta_n / \beta_p = \frac{W_n / L_n \, \mu_n C_G}{W_p / L_p \, \mu_p C_G} \qquad (13)$$

For uniaxial deformation of an inverter with both TFTs oriented in the same direction, V_{inv} is given by Eq. 14. Here the value of V_{T} with deformation is assumed to be given by Eq. 9.

$$V_{inv} = \frac{V_{DD} - \frac{1}{\sqrt{\lambda_L}}\left|V_{T,p}\right| + \frac{1}{\sqrt{\lambda_L}} V_{T,n}\sqrt{\beta_n / \beta_p}}{1 + \sqrt{\beta_n / \beta_p}} \qquad (14)$$

The condition for stability of V_{inv} with deformation can be found by determining when:

$$\frac{dV_{inv}}{d\lambda_L} = 0 \qquad (15)$$

The resulting stability criterion is given by Eq. 16 that sets the design parameters for the two TFTs.

$$\sqrt{\beta_n / \beta_p} = \frac{\left|V_{T,p}\right|}{V_{T,n}} \qquad (16)$$

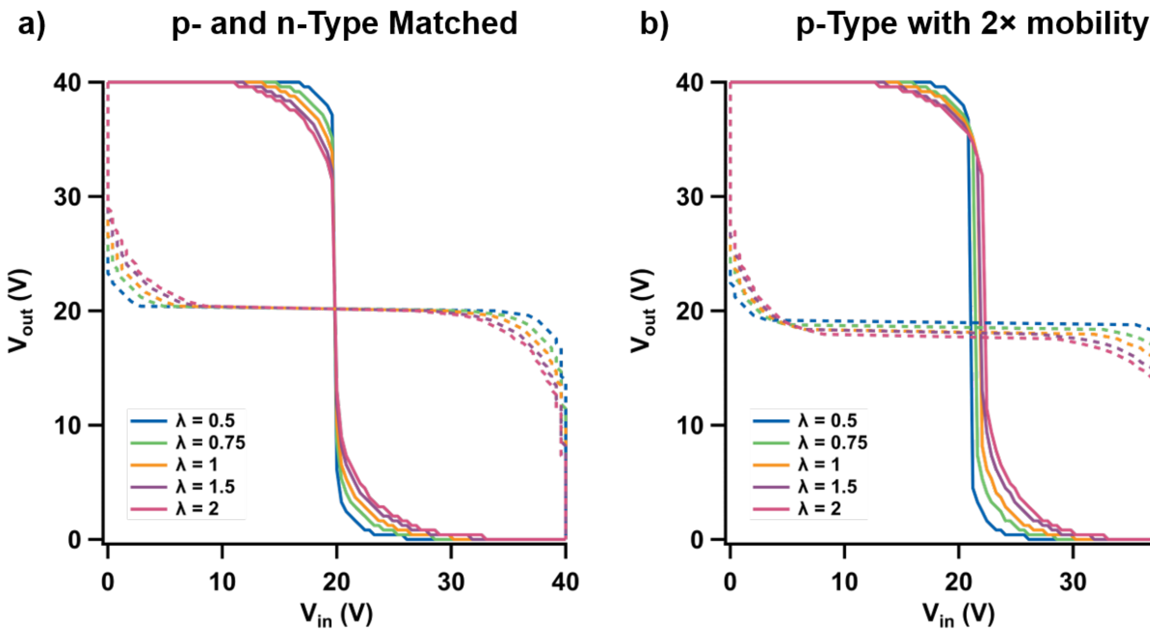

Figure 2 TFT inverter characteristics for a Case I (parallel layout) device deformed uniaxially along L with a) matched n- and p-type TFT specifications, satisfying the above defined stability criterion, and b) mis-matched n- and p-type TFT specifications (with the p-type TFT having double the mobility of the n-type). Note that the mis-matched device (not satisfying the stability criterion) suffers from drift in V_T with deformation.

If the TFTs are laid out into a circuit such that the p-type device is deformed along L and the n-type device is deformed by the same amount along W given by λ, then the resulting expression for V_{inv} is given by:

$$V_{inv} = \frac{V_{DD} - \frac{1}{\sqrt{\lambda}}\left|V_{T,p}\right| + \frac{1}{\sqrt{\lambda}} V_{T,n} \lambda^{3/2} \sqrt{\beta_n / \beta_p}}{1 + \lambda^{3/2} \sqrt{\beta_n / \beta_p}} \tag{17}$$

The derivative of this function has terms with varying powers of λ and a solution to satisfy Eq. 15 that is independent of extension is not possible.

III. Inverter with Organic Electrochemical Transistors (OECTs).

In the case of organic electrochemical transistors, we assume that V_T is not dependent on deformation, unlike the TFT case. Here the only term in Eq. 12 that will vary with deformation is the ratio of device parameters given in Eq. 13. For a circuit with OECTs laid out with their channels in the same direction (Case I in the main text), V_{inv} is always stable to deformation because both devices deform in the same way.

In the case that they are laid out in different directions (Case II in main text), V_{inv} can be given by the following:

$$V_{inv} = \frac{V_{DD} - \left|V_{T,p}\right| + V_{T,n}\lambda^{3/2}\sqrt{\beta_n/\beta_p}}{1 + \lambda^{3/2}\sqrt{\beta_n/\beta_p}}$$

The condition for stability given by Eq. 15 is given by:

$$V_{DD} = \left|V_{T,p}\right| + V_{T,n}$$

This condition limits the value of V_{DD}, which in turn affects the possible gain of the inverter but does lead to stable digital inverting operation.

References

(1) Treloar, L. R. G. *The Physics of Rubber Elasticity: By L.R.G. Treloar*, 3rd ed.; Oxford classic texts in the physical sciences; Clarendon Press ; Oxford University Press: Oxford : New York, 2005.

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