## **Supporting Information**

## Two-dimensional topological insulators exfoliated from

## Na<sub>3</sub>Bi-like Dirac semimetals

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Fig. S1 Electronic structures of bulk  $P\overline{3}c1$  phase of (a) Na<sub>3</sub>Bi, (b) K<sub>3</sub>Bi and (c) Rb<sub>3</sub>Bi from PBE + SOC. The dotted lines with different colors denote the band projection onto different atomic orbitals. The yellow squares indicate the positions of Dirac cones.



Fig. S2 Bonding/antibonding characters of the states of interlayer (a) Na-Bi, (b) Na-Na pairs in Na<sub>3</sub>Bi and interlayer

(d) Mo-S, (e) S-S pairs in  $MoS_2$ , as indicated by the calculated Crystal Orbital Hamiltonian Populations (COHPs). Electron localization function of (c) Na<sub>3</sub>Bi and (f) MoS<sub>2</sub> with isosurfaces of 0.2. –COHP > 0, –COHP = 0 and –COHP < 0 mean the bonding, non-bonding and anti-bonding characters of each electronic state over the chemical bonds in the system, respectively.



Fig. S3 Bonding/antibonding characters of the states of interlayer (a) K-Bi, (b) K-K pairs in K<sub>3</sub>Bi and interlayer (d) Rb-Bi, (e) Rb-Rb pairs in Rb<sub>3</sub>Bi, as indicated by the calculated COHPs. Electron localization function of (c) K<sub>3</sub>Bi and (f) Rb<sub>3</sub>Bi with isosurfaces of 0.2. –COOP > 0, –COOP = 0 and COOP < 0 mean the bonding, non-bonding and antibonding characters of each electronic state over the chemical bonds in the system, respectively.



Fig. S4 Optimized structures of n-layer (n=1-4) Na<sub>3</sub>Bi. The space group of monolayer, trilayer is P321, and that of bilayer, tetralayer is  $P\overline{3}$ .

Table S1 Topological invariant $(Z_2)$ of r	n-layer (n = 1-4) Na <sub>3</sub> Bi derived from the	P6 <sub>3</sub> /mmc phase of Na <sub>3</sub> Bi

$Z_2$	Na <sub>3</sub> Bi
monolayer	1
bilayer	1
trilayer	1
tetralayer	1



Fig. S5 Electronic structures of (a) monolayer, (b) bilayer, (c) trilayer and (d) tetralayer Na<sub>3</sub>Bi derived from the  $P6_3/mmc$  phase of Na<sub>3</sub>Bi, from PBE + SOC.



Fig. S6 Electronic structures of n-layer (n=1-4) of (a-d) K<sub>3</sub>Bi and (e-h) Rb<sub>3</sub>Bi derived from the  $P\overline{3}c1$  phase, from PBE + SOC. The dotted lines with different colors denote the band projection onto different atomic orbitals.



Fig. S7 Electronic structures of Na<sub>3</sub>Bi ( $P\overline{3}c1$ ) with the maximum band gap under strain. The maximum band gaps of bilayer and tetralayer Na<sub>3</sub>Bi under strain are 9.4% and 84% higher than the unstrained case. (a) Maximum band gap is achieved in bilayer at 14% expansion, (b) in tetralayer at 10% expansion.