

# **Isotopic separation of helium through graphyne membranes: a ring polymer molecular dynamics study (Electronic Supplementary Information)**

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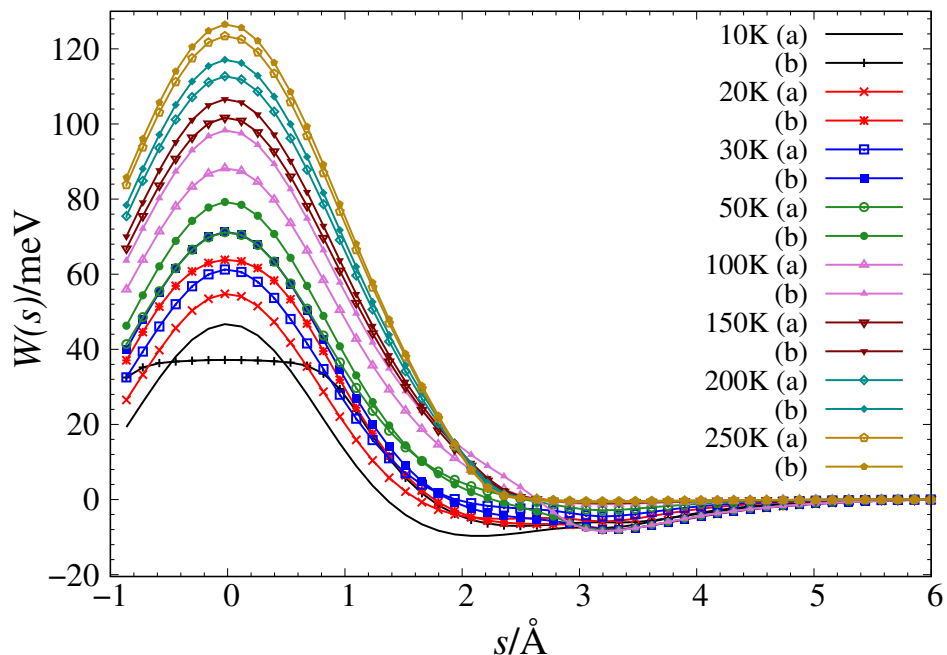
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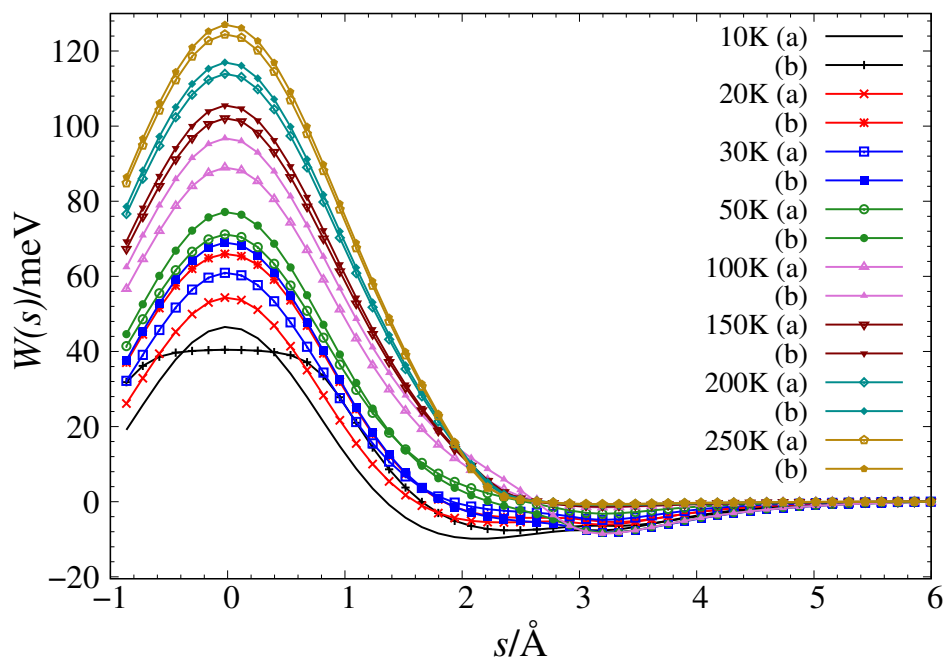
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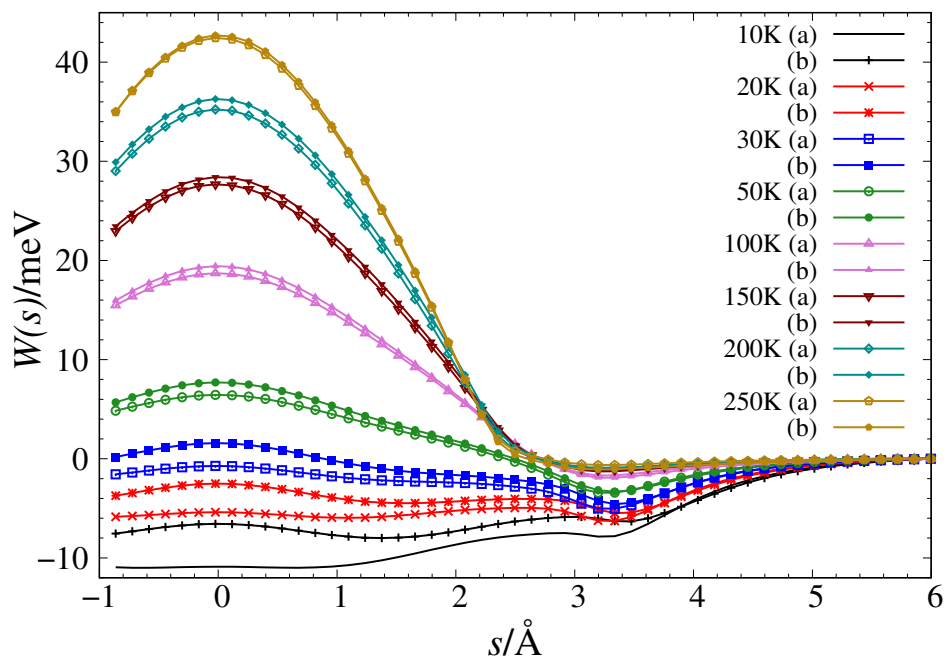
# 1. Potential of mean force



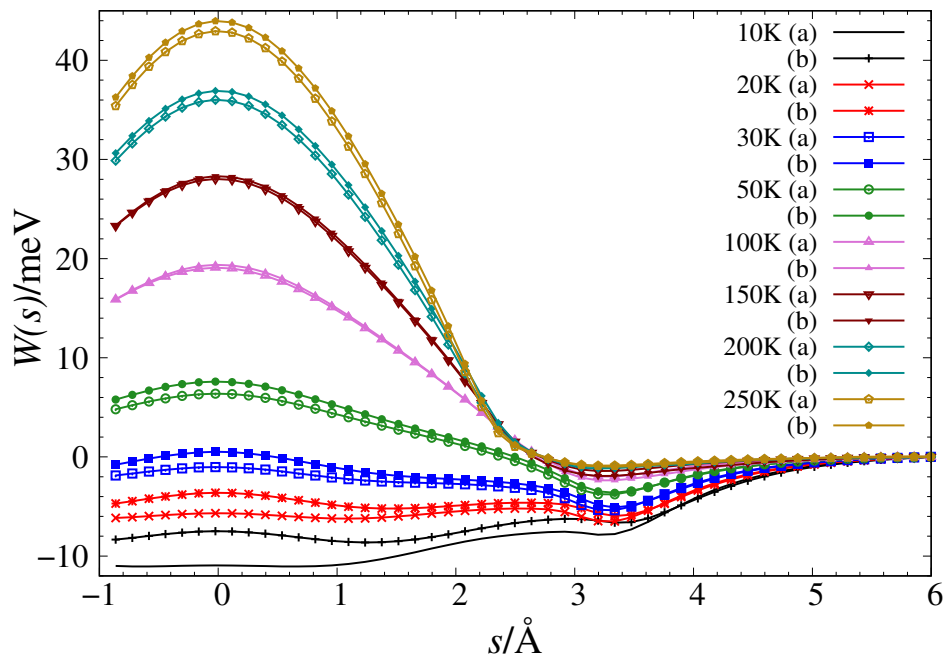
**Fig. S1:** Variation of the (a) classical and (b) RPMD potential of mean force,  $W(s)$ , (in meV) for  $^3\text{He}$  atom along the reaction coordinate  $s$  (in Å) perpendicular to the graphdiyne membrane within the temperature range 10–250 K.



**Fig. S2:** Variation of the (a) classical and (b) RPMD potential of mean force,  $W(s)$ , (in meV) for  $^4\text{He}$  atom along the reaction coordinate  $s$  (in Å) perpendicular to the graphdiyne membrane within the temperature range 10–250 K.

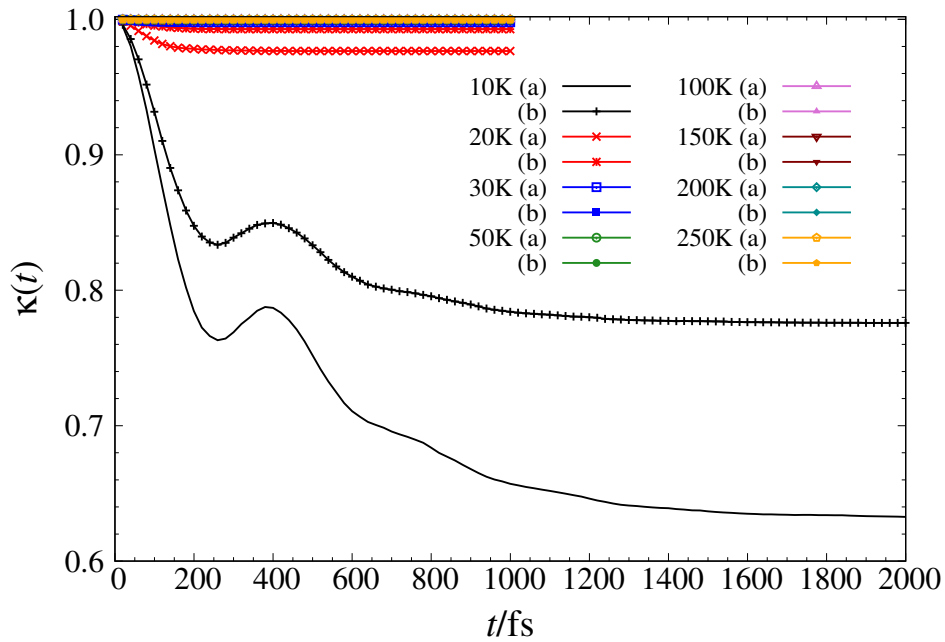


**Fig. S3:** Variation of the (a) classical and (b) RPMD potential of mean force,  $W(s)$ , (in meV) for  $^3\text{He}$  atom along the reaction coordinate  $s$  (in Å) perpendicular to the graphtriyne membrane within the temperature range 10–250 K.

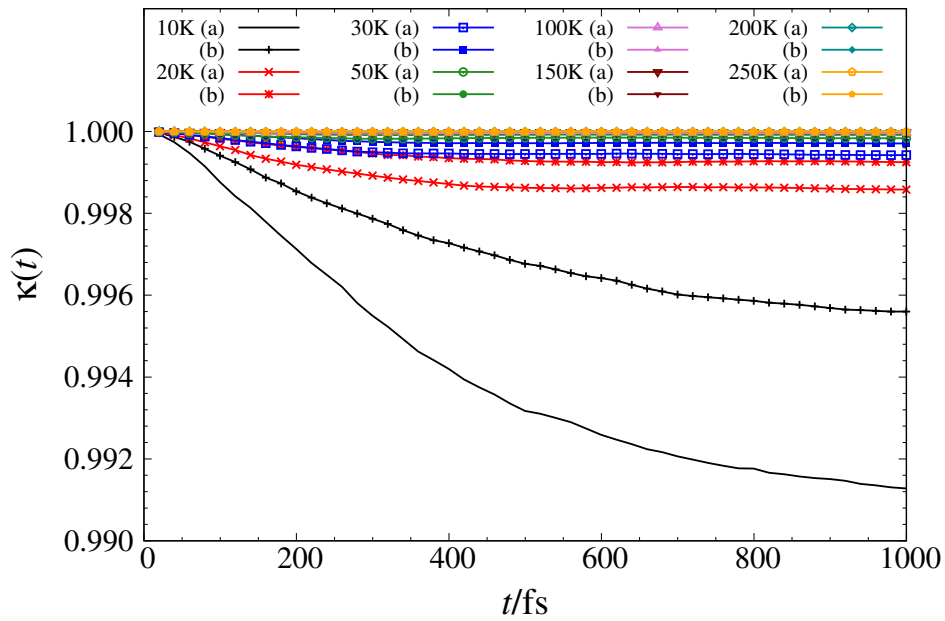


**Fig. S4:** Variation of the (a) classical and (b) RPMD potential of mean force,  $W(s)$ , (in meV) for  $^4\text{He}$  atom along the reaction coordinate  $s$  (in Å) perpendicular to the graphtriyne membrane within the temperature range 10–250 K.

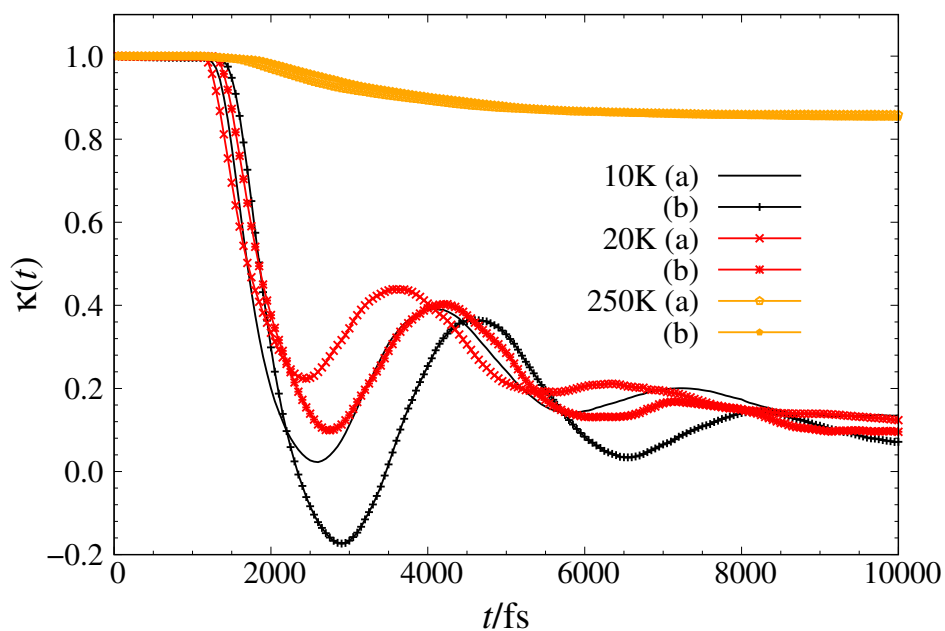
## 2. Ring polymer transmission coefficient



**Fig. S5:** Ring polymer time dependent transmission coefficient,  $\kappa(t)$ , in the temperature range 10–250 K for (a)  $^3\text{He}$  and (b)  $^4\text{He}$  atom transmission through the graphdiyne membrane.

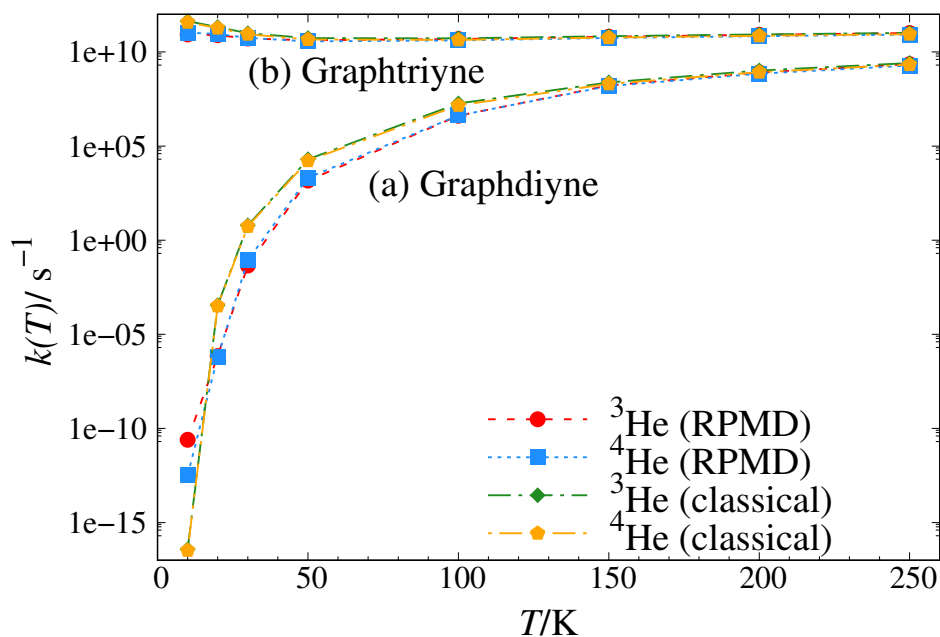


**Fig. S6:** Ring polymer time dependent transmission coefficient,  $\kappa(t)$ , in the temperature range 10–250 K for (a)  $^3\text{He}$  and (b)  $^4\text{He}$  atom transmission through the graphdiyne membrane.

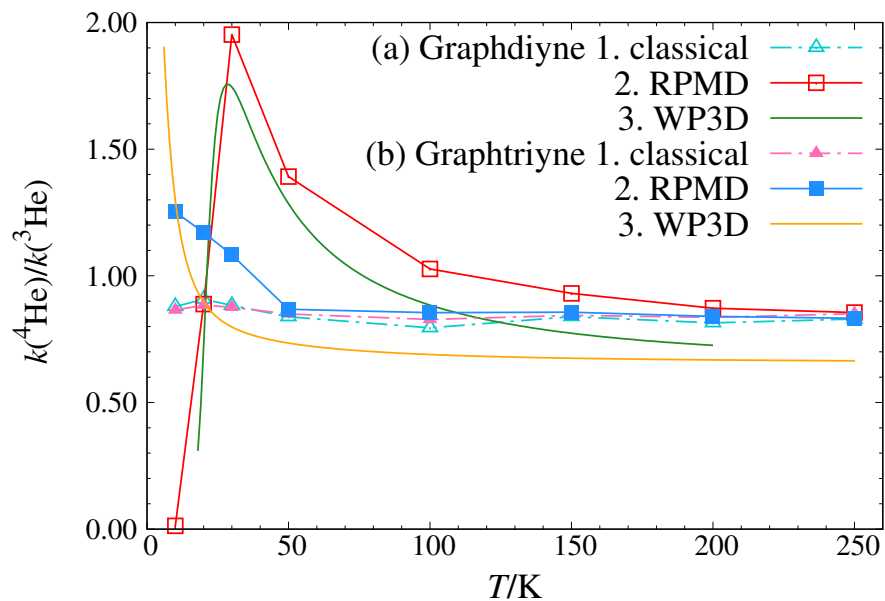


**Fig. S7:** Ring polymer time dependent transmission coefficient,  $\kappa(t)$ , at temperatures 10 K, 20 K, and 250 K for (a)  $^3\text{He}$  and (b)  $^4\text{He}$  atom transmission through the graphtriyne membrane.

### 3. Rate coefficient and selectivity



**Fig. S8:** Variation of the ring polymer molecular dynamics, RPMD ( $k_{\text{RPMD}}$ ) and classical ( $k_{\text{cl}}$ ) rate coefficients (in  $\text{s}^{-1}$ ) for the transmission of  $^3\text{He}$  [red circle (RPMD) and green diamond (classical)] and  $^4\text{He}$  [blue square (RPMD) and orange pentagon (classical)] through (a) graphdiyne and (b) graphtriyne membranes with temperature  $T$  (in K).



**Fig. S9:** Variation of the  $^4\text{He}/^3\text{He}$  rate coefficient ratio,  $k(^4\text{He})/k(^3\text{He})$ , calculated using 1. classical (triangle) 2. ring polymer molecular dynamics, RPMD (square) and 3. three-dimensional wave packet propagation method, WP3D (circle) for the He transmission through (a) graphdiyne and (b) graphtriyne membranes with temperature  $T$  (in K).

**Table S1:** Summary of the rate calculations for  $^3\text{He}$  and  $^4\text{He}$  atom transmission through the graphdiyne (Gr2) and graphtriyne (Gr3) membranes at temperatures ( $T$ ) 10, 20, 30, 50, 100, 150, 200, and 250 K:  $k_{\text{QTST}}$  ( $k_{\text{TST}}$ ) – centroid-density quantum (classical) transition state theory rate coefficient;<sup>a</sup>  $\kappa_{\text{RPMD}}$  ( $\kappa_{\text{cl}}$ ) – ring polymer (classical) transmission coefficient;  $k_{\text{RPMD}}$  ( $k_{\text{cl}}$ ) – ring polymer (classical) molecular dynamics rate coefficient;<sup>a</sup>  $^4\text{He}/^3\text{He}$  – ratio between the  $^4\text{He}$  and  $^3\text{He}$  rate coefficient.

$T/\text{K}$	Isotope	classical				RPMD					
		$k_{\text{TST}}$	$\kappa_{\text{cl}}$	$k_{\text{cl}}$	$^4\text{He}/^3\text{He}$	$k_{\text{QTST}}$	$\kappa_{\text{RPMD}}$	$k_{\text{RPMD}}$	$^4\text{He}/^3\text{He}$		
Gr2	10	$^3\text{He}$	3.80(-17)	1.00	3.80(-17)	0.88	3.97(-11)	0.63	2.51(-11)	0.01	
		$^4\text{He}$	3.34(-17)	1.00	3.34(-17)		4.20(-13)	0.78	3.26(-13)		
	20	$^3\text{He}$	3.54(-4)	1.00	3.54(-4)	0.91	7.15(-7)	0.98	6.99(-7)	0.89	
		$^4\text{He}$	3.21(-4)	1.00	3.21(-4)		6.25(-7)	0.99	6.21(-7)		
	30	$^3\text{He}$	6.09(0)	1.00	6.09(0)	0.88	4.53(-2)	0.99	4.52(-2)	1.95	
		$^4\text{He}$	5.38(0)	1.00	5.38(0)		8.84(-2)	0.99	8.83(-2)		
	50	$^3\text{He}$	2.00(4)	1.00	2.00(4)	0.84	1.48(3)	1.00	1.48(3)	1.39	
		$^4\text{He}$	1.68(4)	1.00	1.68(4)		2.06(3)	1.00	2.06(3)		
	100	$^3\text{He}$	1.84(7)	1.00	1.84(7)	0.80	4.10(6)	1.00	4.10(6)	1.03	
		$^4\text{He}$	1.46(7)	1.00	1.46(7)		4.21(6)	1.00	4.21(6)		
	150	$^3\text{He}$	2.41(8)	1.00	2.41(8)	0.84	1.65(8)	1.00	1.65(8)	0.93	
		$^4\text{He}$	2.02(8)	1.00	2.02(8)		1.54(8)	1.00	1.54(8)		
	200	$^3\text{He}$	1.03(9)	1.00	1.03(9)	0.81	8.07(8)	1.00	8.07(8)	0.87	
		$^4\text{He}$	8.40(8)	1.00	8.40(8)		7.04(8)	1.00	7.04(8)		
	250	$^3\text{He}$	2.57(9)	1.00	2.57(9)	0.83	2.22(9)	1.00	2.22(9)	0.85	
		$^4\text{He}$	2.14(9)	1.00	2.14(9)		1.90(9)	1.00	1.90(9)		
	Gr3	10	$^3\text{He}$	4.36(11)	1.00	4.36(11)	0.86	8.59(10)	0.99	8.51(10)	1.25
			$^4\text{He}$	3.77(11)	1.00	3.77(11)		1.07(11)	0.99	1.06(11)	
20		$^3\text{He}$	2.10(11)	1.00	2.10(11)	0.88	7.80(10)	0.99	7.79(10)	1.17	
		$^4\text{He}$	1.86(11)	1.00	1.86(11)		9.13(10)	0.99	9.12(10)		
30		$^3\text{He}$	1.00(11)	1.00	1.00(11)	0.88	5.19(10)	1.00	5.19(10)	1.08	
		$^4\text{He}$	8.77(10)	1.00	8.77(10)		5.62(10)	1.00	5.62(10)		
50		$^3\text{He}$	5.41(10)	1.00	5.41(10)	0.85	4.18(10)	1.00	4.18(10)	0.87	
		$^4\text{He}$	4.60(10)	1.00	4.60(10)		3.63(10)	1.00	3.63(10)		
100		$^3\text{He}$	5.29(10)	1.00	5.29(10)	0.83	4.84(10)	1.00	4.84(10)	0.86	
		$^4\text{He}$	4.38(10)	1.00	4.38(10)		4.14(10)	1.00	4.14(10)		
150		$^3\text{He}$	6.81(10)	1.00	6.81(10)	0.84	6.46(10)	1.00	6.46(10)	0.86	
		$^4\text{He}$	5.74(10)	1.00	5.74(10)		5.53(10)	1.00	5.53(10)		
200		$^3\text{He}$	8.58(10)	1.00	8.58(10)	0.84	8.16(10)	1.00	8.16(10)	0.84	
		$^4\text{He}$	7.17(10)	1.00	7.17(10)		6.85(10)	1.00	6.85(10)		
250		$^3\text{He}$	1.03(11)	1.00	1.03(11)	0.85	1.01(11)	1.00	1.01(11)	0.83	
		$^4\text{He}$	8.74(10)	1.00	8.74(10)		8.38(10)	1.00	8.38(10)		

<sup>a</sup> The thermal coefficients are given in  $\text{s}^{-1}$  and the numbers in the parentheses denote powers of ten.