# The $\mathrm{NV}^{-} \ldots . \mathrm{N}^{+}$charged pair in Diamond: a <br> Quantum-Mechanical investigation 

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## Supplementary Materials

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## 1. Computational Details

In this section we summarize some of the computational details that are not specific of the present manuscript, but are common to many of the recent publications by the present authors and collaborators, and describing the properties of defects in diamond and silicon.

The truncation criteria of the Coulomb and exchange infinite lattice series are controlled by five thresholds, $\mathrm{T}_{i}$, which have been set to $8\left(\mathrm{~T}_{1}-\mathrm{T}_{4}\right)$ and $16\left(\mathrm{~T}_{5}\right)$. The convergence tolerance on energy for the SCF step has been set to $10^{-8} \mathrm{E}_{h}$ for structural optimizations, and to $10^{-10} \mathrm{E}_{h}$ for frequency calculations.
The DFT exchange-correlation contribution and its gradient are evaluated by numerical integration over the unit cell volume. The generation of the integration grid points is based on an atomic partition method, originally proposed by Becke [1], in which the radial and angular points are obtained from Gauss-Legendre quadrature and Lebedev two-dimensional distributions respectively. The choice of a suitable grid is crucial both for numerical accuracy and cost consideration. In this study a pruned grid with 75 radial and 974 angular points has been used.
Reciprocal space sampling is based on a regular Pack-Monkhorst 2] sub-lattice grid centered at the $\Gamma$ point (i.e. at the center of the first Brillouin zone), that leads to 2 sample points along each of the reciprocal lattice vectors, which corresponds to 4 and 6 k-points for $\mathrm{C}_{3 v}$ and $\mathrm{C}_{S}$ symmetry, respectively, in the irreducible part of the Brillouin zone, after point symmetry has been taken into account.

Harmonic phonon frequencies, $\omega_{p}$ at the $\Gamma$ point were obtained by diagonalization of the mass-weighted Hessian matrix of second energy derivatives with respect to atomic


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\begin{equation*}
W_{a i, b j}^{\Gamma}=\frac{H_{a i, b j}^{\mathbf{0}}}{\sqrt{M_{a} M_{b}}} \quad \text { with } \quad H_{a i, b j}^{\mathbf{0}}=\left(\frac{\partial^{2} E}{\partial u_{a i}^{\mathbf{0}} \partial u_{b j}^{\mathbf{0}}}\right) \tag{1}
\end{equation*}
$$

where atoms $a$ and $b$ (with atomic masses $M_{a}$ and $M_{b}$ ) in the reference cell, $\mathbf{0}$, are displaced along the $i$-th and $j$-th Cartesian directions, respectively. First order derivatives are computed analytically, whereas second order derivatives are obtained numerically, using a two-point formula (as the difference of the gradient at the equilibrium position, and after a displacement of $0.003 \AA$ along each cartesian coordinate; note that as the optimization of the structure is a numerical process, the gradient at equilibrium is not exactly null, but simply lower than the threshold of the optimizer.
Integrated intensities for IR absorption $\mathcal{I}_{p}$ were computed for each normal mode $Q_{p}$ from the mass-weighted effective-mode Born-charge vector $\vec{Z}_{p}$ [8, 9] using a CPHF/KS analytical method: 10

$$
\begin{equation*}
\mathcal{I}_{p} \propto\left|\vec{Z}_{p}\right|^{2} \quad \text { with } \quad \vec{Z}_{p}=\frac{\partial \vec{\mu}}{\partial Q_{p}} \tag{2}
\end{equation*}
$$

## 2. Figures



Figure 1: Properties of the $\mathrm{NV}^{-}\left(C_{3 v}\right.$ symmetry) and $\mathrm{N}^{+}\left(T_{d}\right.$ symmetry) defects obtained with the $\mathrm{S}_{216}$ supercell at the B3LYP/6-21G level and adopting the CC model. Mulliken net charges and spin momenta (in $|e|$, bold and italic respectively) are reported for each symmetry irreducible atom. Bond distances (in $\AA$ ) and Mulliken bond populations (in $|e|$, italic) are reported below each pair of atoms.
A


Figure 2: IR spetra of diamond contanining $N$ defects. A, A defect (a couple of substitutional vicinal N atoms); B , the B defect (a vacancy V surrounded by 4 N atoms); C , the C defect (a single substitutional N atom, $\mathrm{N}^{+}$defect ).

## 3. Tables

Table 1: Bond distance (in $\AA$ ), Mulliken atomic charges (in $|e|$ ), and spin densities (in $|e|$ ) for the triplet state of $\mathrm{NV}^{-} \ldots \mathrm{N}^{+}$defect. The results of the $\mathrm{S}_{64}, \mathrm{~S}_{216}$, and $\mathrm{S}_{512}$ supercells are reported. The data refer to the negative part of the double defect. For comparison, the data of the isolated $\mathrm{NV}^{-}$and $\mathrm{NV}^{0}$ centers are also reported in the last two columns, obtained with the $\mathrm{S}_{216}$ supercell.

|  | $\mathrm{NV}^{-} \ldots \mathrm{N}^{+}$ |  |  | NV- | NV ${ }^{0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{S}_{64}$ | $\mathrm{S}_{216}$ | $\mathrm{S}_{512}$ |  |  |
| $\overline{\mathrm{d}\left(\mathrm{N}-\mathrm{N}^{+}\right)}$ | 6.228 | 7.956 | 12.334 | - | - |
| $\mathrm{d}\left(\mathrm{N}-\mathrm{C}_{(b)}\right)$ | 1.497 | 1.497 | 1.498 | 1.501 | 1.525 |
| $\mathrm{d}\left(\mathrm{C}_{(b)}-\mathrm{C}_{(f)}\right)$ | 1.553 | 1.554 | 1.554 | 1.557 | 1.560 |
| $\mathrm{d}\left(\mathrm{C}_{(b)} \mathrm{C}_{(g)} \mathrm{C}_{(g)}\right.$ | 1.565 | 1.566 | 1.566 | 1.569 | 1.564 |
| $\mathrm{d}\left(\mathrm{C}_{(c)}-\mathrm{C}_{(e)}\right)$ | 1.517 | 1.519 | 1.519 | 1.521 | 1.507 |
| $\mathrm{d}\left(\mathrm{C}_{(c)} \mathrm{C}_{(d)}\right)$ | 1.510 | 1.509 | 1.508 | 1.510 | 1.514 |
| $\mathrm{q}(\mathrm{N})$ | -0.40 | -0.40 | -0.40 | -0.40 | -0.38 |
| $\mathrm{q}\left(\mathrm{C}_{(b)}\right)$ | +0.14 | +0.14 | +0.14 | $+0.13$ | +0.12 |
| $\mathrm{q}\left(\mathrm{C}_{(c)}\right)$ | +0.08 | +0.08 | +0.08 | $+0.07$ | +0.09 |
| $\mathrm{q}\left(\mathrm{C}_{(d)}\right)$ | -0.03 | -0.03 | -0.03 | -0.04 | -0.04 |
| $\mathrm{q}\left(\mathrm{C}_{(e)}\right)$ | -0.03 | -0.03 | -0.03 | -0.04 | -0.04 |
| $\mathrm{q}\left(\mathrm{C}_{(f)}\right)$ | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 |
| $\mathrm{q}\left(\mathrm{C}_{(g)}\right)$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| $\mathrm{q}\left(\mathrm{C}_{(p)}\right)$ | +0.004 | +0.003 | $+0.003$ | +0.003 | +0.001 |
| $\mathrm{q}\left(\mathrm{C}_{(q)}\right)$ | -0.006 | +0.001 | +0.002 | +0.002 | +0.001 |
| $\mu(\mathrm{N})$ | 0.001 | -0.001 | -0.001 | 0.000 | 0.154 |
| $\mu\left(\mathrm{C}_{(b)}\right)$ | +0.001 | 0.000 | 0.000 | 0.000 | -0.007 |
| $\mu\left(\mathrm{C}_{(c)}\right)$ | $+0.632$ | +0.632 | +0.631 | +0.632 | +0.883 |
| $\mu\left(\mathrm{C}_{(d)}\right)$ | -0.048 | -0.048 | -0.048 | -0.048 | -0.058 |
| $\mu\left(\mathrm{C}_{(e)}\right)$ | -0.039 | -0.039 | -0.039 | -0.039 | -0.062 |
| $\mu\left(\mathrm{C}_{(f)}\right)$ | +0.002 | 0.000 | 0.000 | 0.000 | +0.010 |
| $\mu\left(\mathrm{C}_{(g)}\right)$ | +0.011 | +0.008 | +0.008 | +0.008 | $+0.012$ |
| $\mu\left(\mathrm{C}_{(p)}\right)$ | +0.034 | +0.034 | +0.034 | $+0.034$ | $+0.053$ |
| $\underline{\mu\left(\mathrm{C}_{(q)}\right)}$ | +0.041 | +0.038 | +0.038 | $+0.038$ | $+0.054$ |

Table 2: Bond distance (in $\AA$ ), Mulliken atomic charges (in $|e|$ ), and spin densities (in $|e|$ ) for the positively charged part of the $\mathrm{NV}^{-} \ldots \mathrm{N}^{+}$defect are reported iand compared with the isolated $\mathrm{N}^{+}\left(\mathrm{T}_{d}\right)$ center.

|  | $\mathrm{NV}^{-} \ldots \mathrm{N}^{+}$ |  |  | $\mathrm{N}^{+}\left(\mathrm{T}_{d}\right)$ |
| :--- | :---: | :---: | :---: | :---: |
|  | $\mathrm{S}_{64}$ | $\mathrm{~S}_{216}$ | $\mathrm{~S}_{512}$ |  |
| $\mathrm{~d}\left(\mathrm{~N}^{+}-\mathrm{C}_{(x)}\right)$ | 1.584 | 1.585 | 1.585 | 1.582 |
| $\mathrm{~d}\left(\mathrm{~N}^{+}-\mathrm{C}_{y)}\right)$ | 1.587 | 1.585 | 1.585 | 1.582 |
| $\left.\mathrm{~d}\left(\mathrm{C}_{(x)}\right)_{(z)}\right)$ | 1.559 | 1.555 | 1.555 | 1.553 |
| $\mathrm{q}\left(\mathrm{N}^{+}\right)$ | -0.42 | -0.42 | -0.42 | -0.42 |
| $\mathrm{q}\left(\mathrm{C}_{(x))}\right.$ | 0.12 | 0.12 | 0.12 | 0.12 |
| $\mathrm{q}\left(\mathrm{C}_{(y))}\right.$ | 0.12 | 0.12 | 0.12 | 0.12 |
| $\mu\left(\mathrm{~N}^{+}\right)$ | 0.011 | 0.000 | 0.000 | 0.000 |
| $\mu\left(\mathrm{C}_{(x))}\right.$ | -0.002 | 0.000 | 0.000 | 0.000 |
| $\mu\left(\mathrm{C}_{(y))}\right.$ | -0.006 | 0.000 | 0.000 | 0.000 |

Table 3: Net atomic charges (in $|e|$ ) of the defect atoms for the negatively charged part of the $\mathrm{NV}^{-}-\mathrm{N}^{+}$center. m refers to the multiplicity; R indicates the distance in $\AA$ between the given atom and the vacancy center. Values in parentheses refer to Hirshfield charges. The sum of the charges of a given shell is also reported. The comparison with the $\mathrm{NV}^{-}$and $\mathrm{NV}^{0}$ defects is also considered.


Table 4: Net atomic charges (in $|e|$ ) in the positively charged part of the $\mathrm{NV}^{-}-\mathrm{N}^{+}$center. m is the multiplicity, R indicates the distance in $\AA$ between the given atom and the vacancy center. Values in parentheses refer to Hirshfield charges. The sum of the charges in a given shell is also reported. Also the data for the pure $\mathrm{N}^{+}\left(\mathrm{T}_{d}\right)$ defect are shown.

| site | m | R | $\mathrm{NV}^{-} \ldots \mathrm{N}^{+}$ |  | $\mathrm{N}^{+}\left(\mathrm{T}_{d}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $S_{216}$ | $S_{512}$ |  |
| $\mathrm{N}^{+}$ | 1 | 0.00 | $\begin{gathered} -0.419 \\ (+0.061) \end{gathered}$ | $\begin{gathered} -0.419 \\ (+0.063) \end{gathered}$ | $\begin{gathered} -0.420 \\ (+0.071) \end{gathered}$ |
| $\mathrm{C}_{(x)}$ | 4 | $\sim 1.59$ | $\begin{gathered} +0.119 \\ (+0.102) \end{gathered}$ | $\begin{gathered} +0.118 \\ (+0.101) \end{gathered}$ | $\begin{gathered} +0.124 \\ (+0.101) \end{gathered}$ |
| $\mathrm{C}_{(y)}$ | 12 | $\sim 2.55$ | $\begin{gathered} -0.010 \\ (-0.032) \end{gathered}$ | $\begin{gathered} -0.009 \\ (-0.029) \end{gathered}$ | $\begin{gathered} -0.004 \\ (-0.022) \end{gathered}$ |
| $\mathrm{C}_{(z)}$ | 12 | $\sim 2.99$ | $\begin{gathered} 0.000 \\ (+0.005) \\ \hline \end{gathered}$ | $\begin{gathered} 0.000 \\ (+0.004) \\ \hline \end{gathered}$ | $\begin{gathered} +0.005 \\ (+0.005) \\ \hline \end{gathered}$ |
| Sum Shell 1 30-atoms |  |  | $\begin{gathered} -0.063 \\ (+0.145) \\ \hline \end{gathered}$ | $\begin{gathered} -0.055 \\ (+0.167) \\ \hline \end{gathered}$ | $\begin{gathered} +0.088 \\ (+0.271) \\ \hline \end{gathered}$ |
| Sum Shell 2 | 6 | $\sim 3.60$ |  | $\begin{gathered} -0.003 \\ (-0.021) \end{gathered}$ |  |
| Sum Shell 3 | 12 | $\sim 3.93$ |  | $\begin{gathered} +0.009 \\ (+0.108) \\ \hline \end{gathered}$ |  |
| Sum Shell 4 | 24 | $\sim 4.41$ |  | $\begin{gathered} -0.006 \\ (-0.045) \end{gathered}$ |  |
| Sum Shell 5 | 16 | $\sim 4.68$ |  | $\begin{gathered} +0.007 \\ (+0.005) \\ \hline \end{gathered}$ |  |
| Sum Shell 6 | 12 | $\sim 5.09$ |  | $\begin{gathered} +0.006 \\ (+0.003) \\ \hline \end{gathered}$ |  |
| Sum <br> Shells 1, 2, 3, 4, 5, 6 |  |  |  | $\begin{gathered} -0.042 \\ (+0.217) \\ \hline \end{gathered}$ |  |

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