Electronic Supporting Information (ESI) for the paper

## Singlet-triplet conversion in molecular hydrogen and its role in parahydrogen induced polarization

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This file contains the following information:

1. Model of singlet-triplet conversion in H <sub>2</sub>	S1
2. Optimization of experimental parameters	S3

## 1. Model of singlet-triplet conversion in H<sub>2</sub>

To implement the model for singlet-triplet conversion we numerically solved the equations for  $\sigma_f$  and  $\sigma_b$  given in the main article.

In this model, we first specify the Hamiltonians

$$\hat{H}_f = 0, \qquad \hat{H}_b = \frac{1}{2} \delta \omega \{ \hat{I}_{1z} - \hat{I}_{2z} \}$$

Thus, we neglect any spin evolution in the free form of H<sub>2</sub> and consider only a single term in  $\hat{H}_b$ , which originates from the frequency difference of the two protons in the bound form. After that, we go to the Liouville space and specify  $\sigma_f$  and  $\sigma_b$  as column vectors with 16 elements (corresponding to a two-spin system). In this representation,  $\hat{H}_{f,b}$  are written as superopeartors, which are  $16 \times 16$  matrices, whereas the relaxation superoperators are also matrices  $\hat{T}_{f,b}$ . To specify all elements of  $\hat{T}_{f,b}$  we use the model of partly correlated fluctuating local fields,  $B^{(1)}(t)$  and  $B^{(2)}(t)$ , experienced by the two spins. This means, that we use the same method as before<sup>1,2</sup> but averaging of the local fields is performed using the following expressions (here  $g(\tau)$  is the auto-correlation function of the process):

$$\begin{split} &\left\langle B^{(1)}_{i}(t)B^{(1)}_{j}(t+\tau)\right\rangle = \langle B^{2}\rangle\delta_{ij}g(\tau),\\ &\left\langle B^{(2)}_{i}(t)B^{(2)}_{j}(t+\tau)\right\rangle = \langle B^{2}\rangle\delta_{ij}g(\tau),\\ &\left\langle B^{(1)}_{i}(t)B^{(2)}_{j}(t+\tau)\right\rangle = C\times\langle B^{2}\rangle\delta_{ij}g(\tau) \end{split}$$

Hence, different components are non-correlated while the amplitudes and auto-correlation functions of the two fields are the same. The parameter  $\langle B^2 \rangle$  determines the effective relaxation rate in the system in the free or bound form,  $T_1^f$  and  $T_1^b$ . Specifically, in the limiting narrowing case, each T<sub>1</sub>-value is related to  $\langle B^2 \rangle$  as follows:  $T_1^{-1} = 2\tau_c \langle B^2 \rangle$ , where  $\tau_c$  is the motional correlation time. Hereafter, all products of the field components are expressed via  $1/T_1$ , omitting all unnecessary coefficients. The coefficient *C* defines the degree, to which the two fields are correlated: C = 0 means independent correlations and C = 1 means complete correlation. Here, we use *C* approaching unity, meaning that the singlet-triplet relaxation transitions are much slower than the relaxation transitions in the triplet manifold. We take C = 0.9 in order to keep singlet-triplet transitions operative (for C = 1 singlet-triplet relaxation is turned off).

To solve numerically the set of equations

$$\begin{cases} \frac{d}{dt}\sigma_f = -i[\hat{H}_f,\sigma_f] - \hat{\Gamma}_f\sigma_f - k_{as}\sigma_f + k_{dis}\sigma_b \\ \frac{d}{dt}\sigma_b = -i[\hat{H}_b,\sigma_b] - \hat{\Gamma}_b\sigma_b - k_{dis}\sigma_b + k_{as}\sigma_f \end{cases}$$

we proceed as follows. First, we compose a vector made out of  $\sigma_f$  and  $\sigma_b$ :

$$\sigma = \begin{pmatrix} \sigma_f \\ \sigma_b \end{pmatrix}$$

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and, second, rewrite the system in one equation:

$$\frac{d}{dt}\sigma = \hat{A}\sigma, \quad where \quad \hat{A} = \begin{pmatrix} -i\hat{H}_f - \hat{\Gamma}_f - k_{as}\hat{1} & k_{dis}\hat{1} \\ k_{as}\hat{1} & -i\hat{H}_b - \hat{\Gamma}_b - k_{dis}\hat{1} \end{pmatrix}$$

Here  $\hat{1}$  is the unity  $16 \times 16$  matrix. Using this equation, we can compute the  $\sigma$ -vector at any instant of time:

 $\sigma(t) = \exp\left[\hat{A}t\right]\sigma(0)$ 

Knowing  $\sigma$ , we can calculate any parameter of interest, e.g., the populations of the spin states of H<sub>2</sub>.

Parameters used in the calculations are:

- ✓ exchange reaction rates:  $k_{as} = 6 {}_{s^{-1}}, k_{dis} = 60 {}_{s^{-1}}$
- ✓ relaxation times  $T_1^f = 3$  s,  $T_1^b = 1$  s, C = 0.99✓ the value of  $\delta \omega / 2\pi$  was varied

## 2. Optimization of experimental parameters

The dependence of the signal enhancement in the SLIC-SABRE scheme is shown in **Figure 1S**. The optimal timing is given by  $t_d = 500$  ms,  $t_{cw} = 39$  ms, which was used in all experiments.



Figure 1S. Dependence of the <sup>15</sup>N signal enhancement (a) on the duration  $t_{cw}$  of the CW pulse (with  $t_d = 250$  ms) and (b) on the delay between polarization transfer cycles  $t_d$  (with  $t_{cw} = 39$  ms.). The experiments were performed with [S] = 60 mM and [C] = 9.5 mM. SLIC-SABRE parameters were set as follows:  $\omega_1/2\pi = 10$  Hz, offset  $\Delta/2\pi = -14$  Hz, number of repetitions n = 50.



**Figure 2S.** Dependence of the <sup>15</sup>N signal enhancement on the number of polarization cycles *n*. The experiments were performed using the SLIC-SABRE method with 192 mM of the substrate (containing 10% of <sup>15</sup>N nuclei). SLIC-SABRE parameters were set as follows:  $\omega_1/2\pi = 10$  Hz, offset  $\Delta/2\pi = -14$  Hz,  $t_d = 500$  ms,  $t_{cw} = 39$  ms.

The resulting NMR enhancement also strongly depends on the number of repetitions of the polarization transfer cycle. The *n*-dependence of the signal enhancement in the SLIC-SABRE scheme is shown in **Figure 2S**. The signal increases with *n*, showing that it is usually sufficient to have n = 50 cycles. All experiments reported in the main article have been done with 50 repetitions.

## References

- 1. S. E. Korchak, K. L. Ivanov, A. V. Yurkovskaya and H.-M. Vieth, J. Chem. Phys., 2010, 133, 194502.
- 2. K. Ivanov, A. Yurkovskaya and H.-M. Vieth, J. Chem. Phys., 2008, 129, 234513.