Supplement for "Polarization-dependent excitons and plasmon activity in nodal-line semimetal ZrSiS"

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Anisotropic effective masses

Let us consider a solid with a gap whose constant energy surfaces $\varepsilon(\boldsymbol{k})$ are ellipsoids, with effective masses $m_\parallel$ and $m_\perp$ for electrons and holes along the ellipsoid axis and in the plane perpendicular to it, respectively. In such a system, the binding energy of an exciton is given by

$$E(\lambda) = -\frac{3}{2}\left(\frac{e^2}{4\pi\varepsilon\hbar}\right)^2\left(\frac{2}{\mu_\perp} + \frac{1}{\lambda^2\mu_\parallel}\right)^{-1} I(\lambda)^2, \tag{S1}$$

where e is the electron charge, ε is the dielectric constant of the solid, $\hbar$ is the reduced Planck's constant and $\mu_\parallel$ and $\mu_\perp$ are the reduced masses parallel and perpendicular to the ellipsoid axis, respectively. The function $I(\lambda)$ is given by

$$I(\lambda) = \frac{1}{2}\int_0^\pi \frac{\sin\theta d\theta}{\sqrt{1+(\lambda^2-1)\cos^2\theta}} = \begin{cases} \dfrac{\operatorname{arcsinh}\sqrt{\lambda^2-1}}{\sqrt{\lambda^2-1}} & \text{for } \lambda > 1 \\ \dfrac{\arcsin\sqrt{1-\lambda^2}}{\sqrt{1-\lambda^2}} & \text{for } \lambda < 1 \end{cases} \tag{S2}$$

and λ is the solution of

$$\frac{\mu_\perp}{\mu_\parallel} = 2\lambda^2\frac{1-\lambda I(\lambda)}{I(\lambda)-\lambda} \tag{S3}$$

Full details about this formalism can be found in [1]. In our case, the effective masses for the first excitons are summarized in Table S1 below.

According to this table, the binding energies for $\boldsymbol{E} \parallel Ox$ and $\boldsymbol{E} \parallel Oz$ are $E_b = 0.02E_0$ and $E_b = 0.16E_0$, where $E_b = -\frac{3}{2}\left(\frac{e^2}{4\pi\varepsilon\hbar}\right)^2$. These values indeed differ by a factor 8, as the main text indicates.

[1] A. Schindlmayr. Excitons with anisotropic effective mass. *Eur. J. Phys.*, 18:374–376, 1997.

$\boldsymbol{E} \parallel Ox$							
	$m_{\perp,1}$	$m_{\perp,2}$	$m_{\perp} = \sqrt{m_{\perp,1} m_{\perp,2}}$	$m_{\parallel}$	$\mu_{\perp}$	$\mu_{\parallel}$	λ
Electrons	0.363	0.218	0.281	0.011	0.223	0.010	3.1417
Holes	4.611	0.257	1.089	0.136			
$\boldsymbol{E} \parallel Oz$							
	$m_{\perp,1}$	$m_{\perp,2}$	$m_{\perp} = \sqrt{m_{\perp,1} m_{\perp,2}}$	$m_{\parallel}$	$\mu_{\perp}$	$\mu_{\parallel}$	λ
Electrons	0.609	3.718	1.505	1.134	0.576	0.378	1.162
Holes	0.261	3.342	0.933	0.567			

Table S1. Estimation of the λ parameter appearing in Eq. (S1).