

## Electronic Supplementary Information

### Spin Hall effect of transmitted light through $\alpha$ -Li<sub>3</sub>N-type topological semimetals

Guang-Yi Jia,<sup>1,\*</sup> Rui-Xia Zhang,<sup>1</sup> Ting Tang,<sup>2</sup> Qian Li,<sup>1</sup> Ali Ebrahimian,<sup>3</sup> Zahra Torbatian,<sup>4</sup> Reza Asgari<sup>3,†</sup>

<sup>1</sup>*School of Science, Tianjin University of Commerce, Tianjin 300134, P.R. China*

<sup>2</sup>*Tianjin Key Laboratory of Refrigeration Technology, Tianjin University of Commerce, Tianjin 300134, P.R. China*

<sup>3</sup>*School of Physics, Institute for Research in Fundamental Sciences, IPM, Tehran 19395-5531, Iran*

<sup>4</sup>*School of Nano Science, Institute for Research in Fundamental Sciences, IPM, Tehran 19395-5531, Iran*

E-mails: \*gyjia87@163.com; †asgari@ipm.ir

## I. Ab Initio Calculations

### A. Calculation Methods

In this study, for the sake of accurateness, structural optimization and electronic structure calculations are carried out using the all-electron full potential linearized augmented plane wave (FP-LAPW) method, as implemented in Exciting code [1]. The exchange-correlation potential is treated within the generalized gradient approximation of Perdew-Burke-Ernzerhof (PBE) [2]. In order to check the possible underestimation of the band gap within the PBE functional, hybrid density functional (HSE06) [3,4] is also employed. A  $20 \times 20 \times 20$  Monkhorst-pack k-point mesh is properly used in the computations. All the structures are fully relaxed until the maximum residual force on each atom is less than  $10^{-4}$  eVÅ<sup>-1</sup>. The optimized lattice constants of Li<sub>3</sub>N, Li<sub>2</sub>KN, and Na<sub>3</sub>N are  $a = b = 3.648$  Å and  $c = 3.885$  Å,  $a = b = 3.713$  Å and  $c = 5.701$  Å,  $a = b = 4.488$  Å and  $c = 4.660$  Å, respectively. The interstitial plane wave vector cut off  $K_{\text{Max}}$  is chosen in a way that  $R_{\text{MT}}K_{\text{Max}}$  equals 8. The  $G_{\text{Max}}$  parameter is taken to be 14 Bohr<sup>-1</sup>. The hole doping induced in  $\alpha$ -Li<sub>3</sub>N is simulated by shifting the Fermi level downward according to a standard rigid-band model. The influences of excitonic effects are important in order to correctly account for quantitative as well as qualitative features of optical spectra of Li<sub>3</sub>N. Therefore, to include the electron-hole interaction, being absent in the random phase approximation, we apply many-body perturbation theory on top of the DFT calculations. The Bethe-Salpeter Equation for two-particles Green's function [5,6] is solved using the Exciting code. The matrix eigenvalue form of BSE is thus given by [7-9]

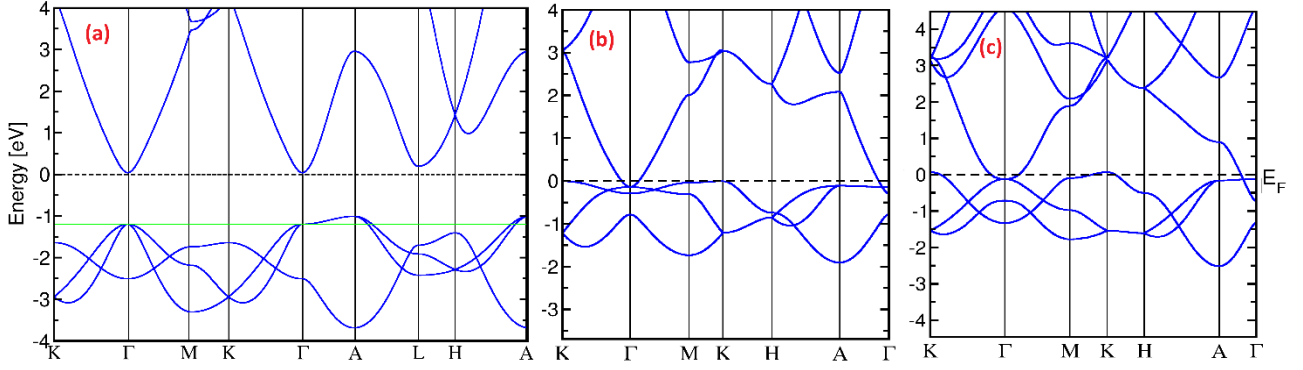
$$\sum_{v'c'k'} H_{vck, v'c'k'} A_{v'c'k'}^\lambda = E^\lambda A_{vck}^\lambda$$

where  $v$ ,  $c$ , and  $k$  indicate the valence band, conduction band, and k-points in the reciprocal space, respectively. Eigenvalues  $E^\lambda$  and eigenvectors  $A_{vck}^\lambda$  represent the excitation energy of the  $j$ th-correlated electron-hole pair and the coupling coefficients used to construct the exciton wave function, respectively. The optical properties of semimetal structures are considered via using time-dependent density-functional theory (TDDFT) [10]. The inverse dielectric matrix is related to the susceptibility  $\chi$  by the relation  $\epsilon^{-1}(q, \omega) = 1 + v(q)\chi(q, \omega)$ , where  $v(q)$  is the bare Coulomb potential. In TDDFT, the susceptibility  $\chi(q, \omega)$  is obtained by making use of the linear response to TDDFT through the solution of Dyson equation

$$\chi(q, \omega) = \chi_0(q, \omega) + \chi_0(q, \omega)[v(q) + f_{xc}(q)]\chi(q, \omega)$$

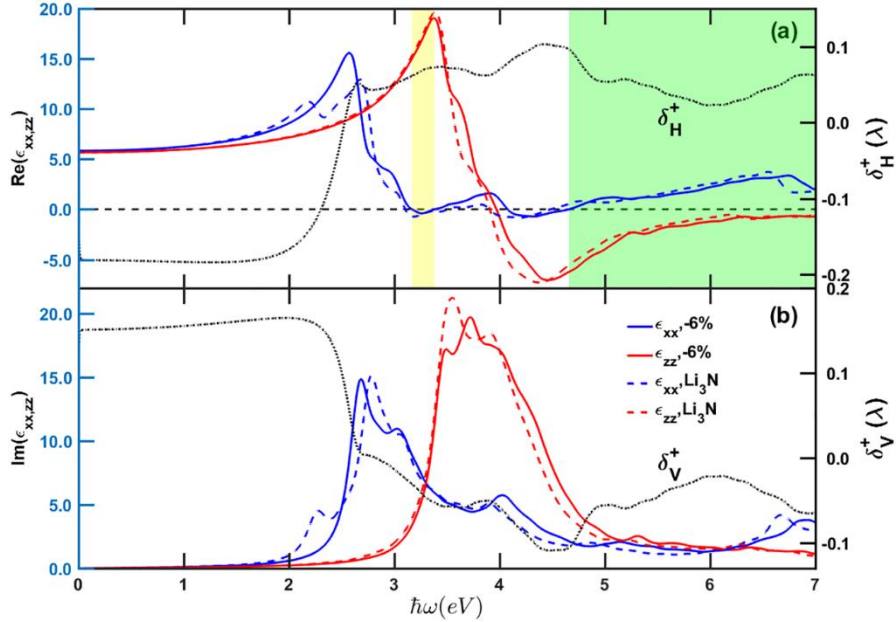
where  $\chi_0(q, \omega)$  is the KS susceptibility expressed in terms of the KS eigenvalues and eigenfunctions,  $f_{xc}(q)$  is the exchange-correlation kernel, for which we make use of the adiabatic local-density approximation (ALDA). To study the convergence of the excitonic features in the optical properties, for all structures, the dielectric function has been calculated for different numbers of k-point. The momentum matrix elements for the optical properties have been converged using  $30 \times 30 \times 30$  k-points. In fact, the excitation energies converged within 70 meV.

## B. Electronic Band Structures of Li<sub>3</sub>N, Na<sub>3</sub>N and Li<sub>2</sub>KN

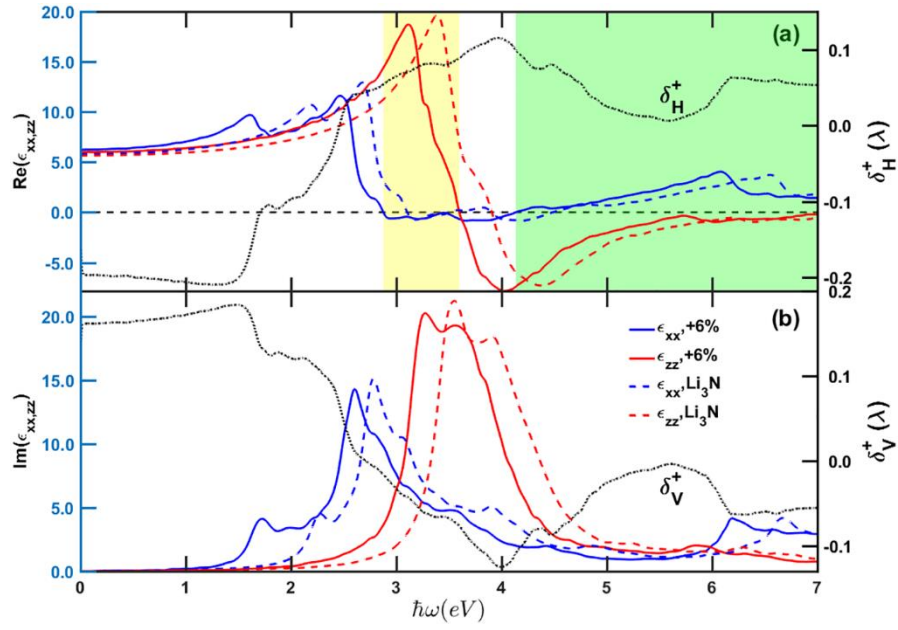


**Fig. S1** The calculated bulk electronic band structure of (a) Li<sub>3</sub>N, (b) Na<sub>3</sub>N and (c) Li<sub>2</sub>KN in PBE. The value of Fermi energy for hole-doped (0.1 holes/u.c.) Li<sub>3</sub>N is shown by a green solid line in (a).

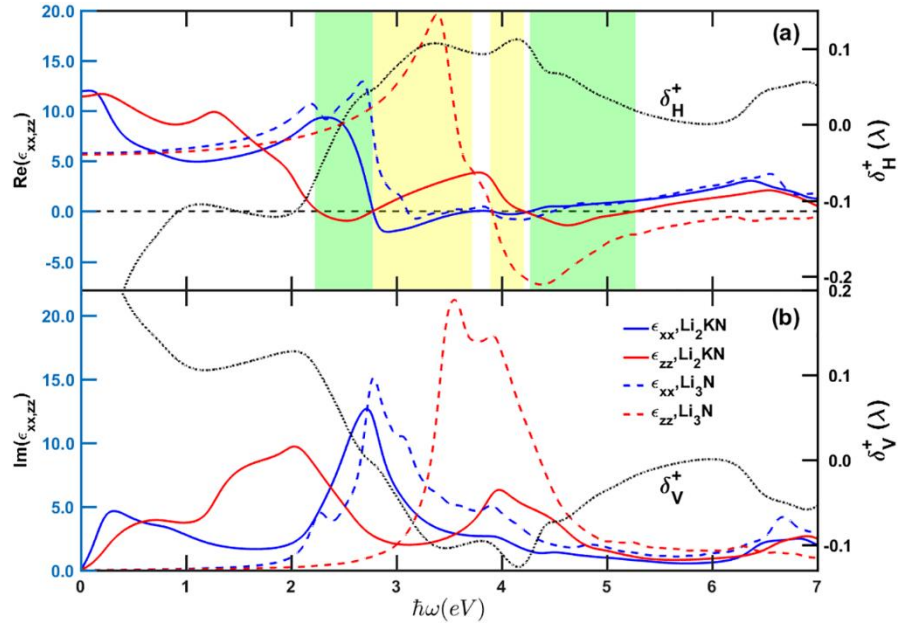
## II. Complex Permittivities and Related Optical Spectra



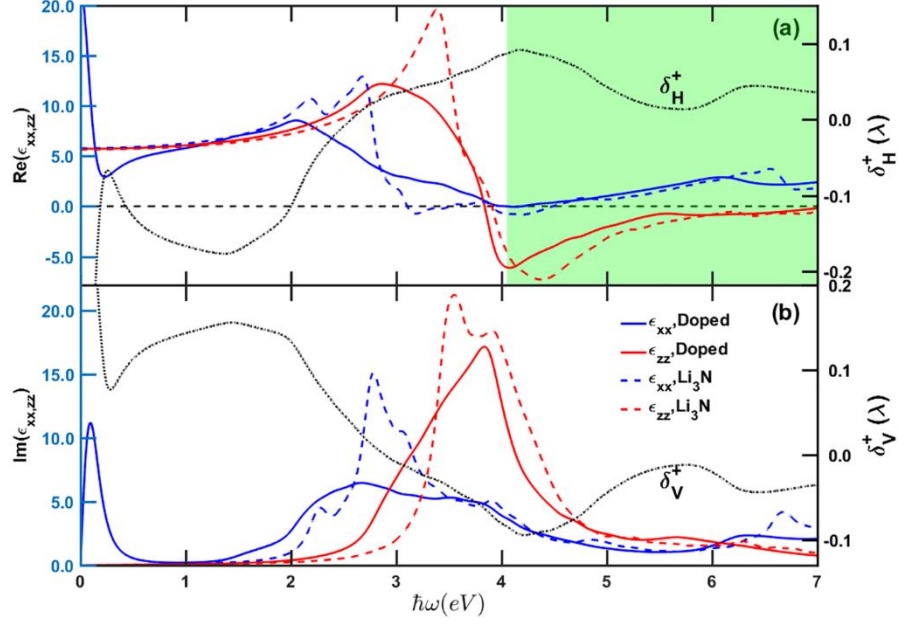
**Fig. S2** (a) Real and (b) imaginary parts of the in-plane ( $\epsilon_{xx}$ ) and the out-of-plane ( $\epsilon_{zz}$ ) components of the relative permittivities for Li<sub>3</sub>N and -6% strained Li<sub>3</sub>N. The green and yellow areas indicate the photon energy ranges of type-I and type-II hyperbolic properties for -6% strained Li<sub>3</sub>N, respectively. For comparison, the spectra of spin Hall shifts  $\delta_H^+$  and  $\delta_V^+$  at  $\theta_i = 10^\circ$  for -6% strained Li<sub>3</sub>N are shown by dot lines.



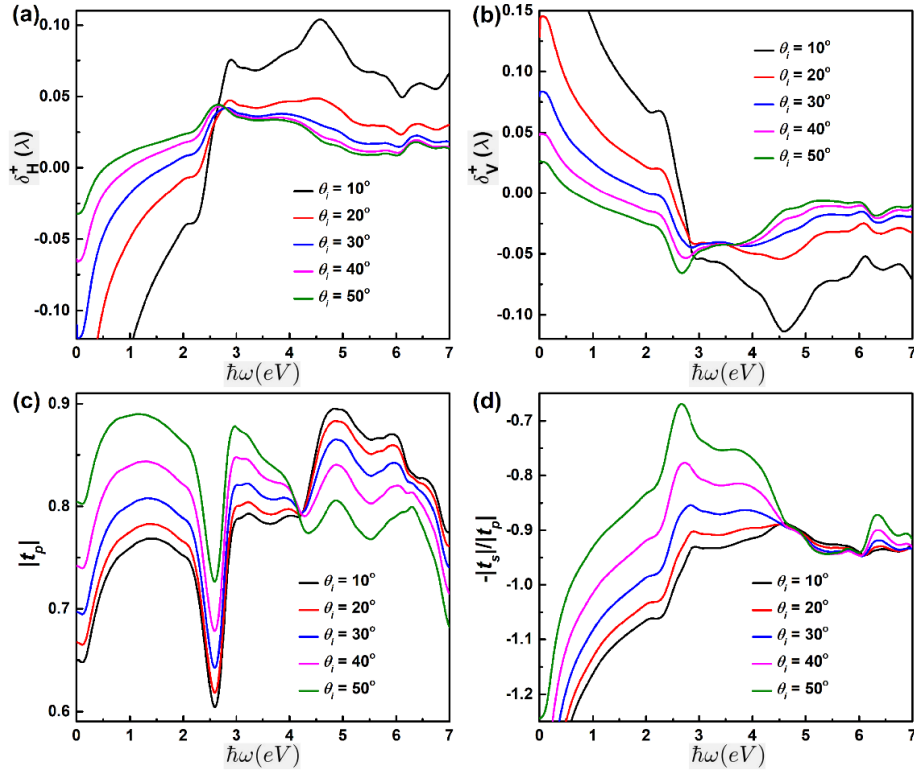
**Fig. S3** (a) Real and (b) imaginary parts of the in-plane ( $\epsilon_{xx}$ ) and the out-of-plane ( $\epsilon_{zz}$ ) components of the relative permittivities for  $\text{Li}_3\text{N}$  and +6% strained  $\text{Li}_3\text{N}$ . The green and yellow areas indicate the photon energy ranges of type-I and type-II hyperbolic properties for +6% strained  $\text{Li}_3\text{N}$ , respectively. For comparison, the spectra of spin Hall shifts  $\delta_H^+$  and  $\delta_V^+$  at  $\theta_i = 10^\circ$  for +6% strained  $\text{Li}_3\text{N}$  are shown by dot lines.



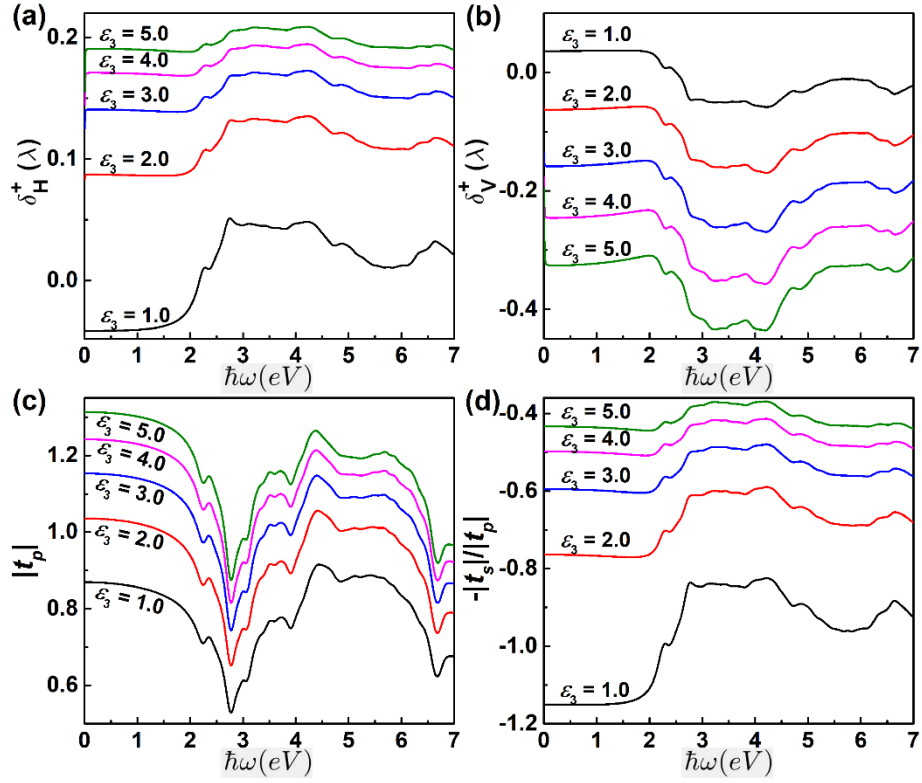
**Fig. S4** (a) Real and (b) imaginary parts of the in-plane ( $\epsilon_{xx}$ ) and the out-of-plane ( $\epsilon_{zz}$ ) components of the relative permittivities for  $\text{Li}_3\text{N}$  and  $\text{Li}_2\text{KN}$ . The green and yellow areas indicate the photon energy ranges of type-I and type-II hyperbolic properties for  $\text{Li}_2\text{KN}$ , respectively. The dielectric function of  $\text{Li}_2\text{KN}$  incorporates only the interband contribution and the intraband contribution is ignored here. For comparison, the spectra of spin Hall shifts  $\delta_H^+$  and  $\delta_V^+$  at  $\theta_i = 10^\circ$  for  $\text{Li}_2\text{KN}$  are shown by dot lines.



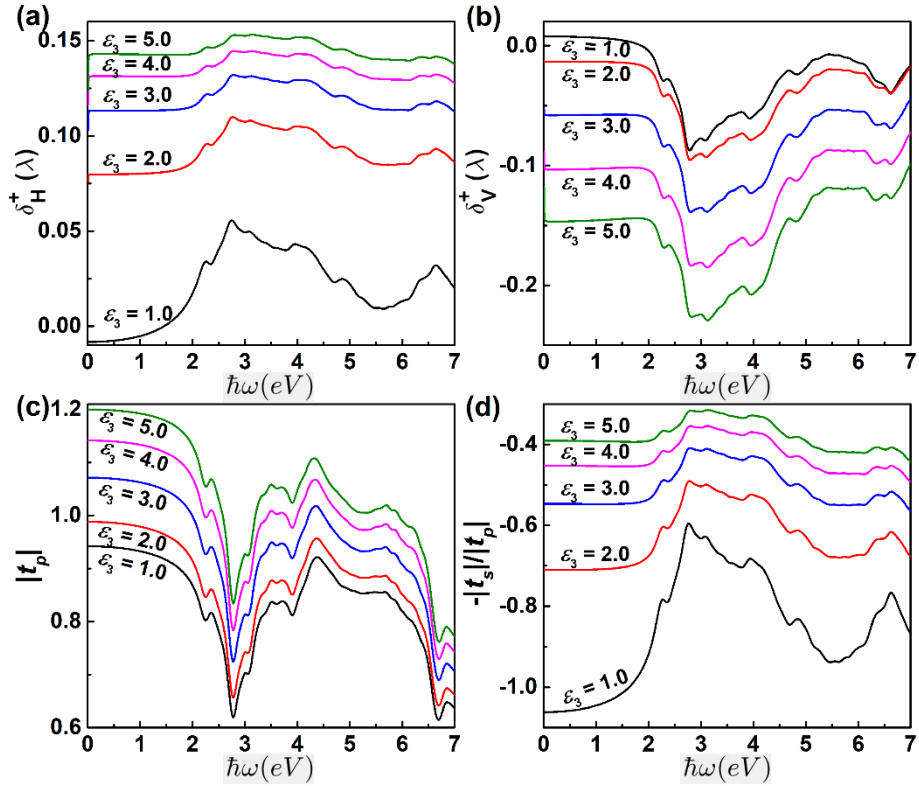
**Fig. S5** (a) Real and (b) imaginary parts of the in-plane ( $\epsilon_{xx}$ ) and the out-of-plane ( $\epsilon_{zz}$ ) components of the relative permittivities for  $\text{Li}_3\text{N}$  and hole-doped  $\text{Li}_3\text{N}$ . The green area indicates the photon energy range of type-I hyperbolic property for hole-doped  $\text{Li}_3\text{N}$ . For comparison, the spectra of spin Hall shifts  $\delta_H^+$  and  $\delta_V^+$  at  $\theta_i = 10^\circ$  for hole-doped  $\text{Li}_3\text{N}$  are shown by dot lines.



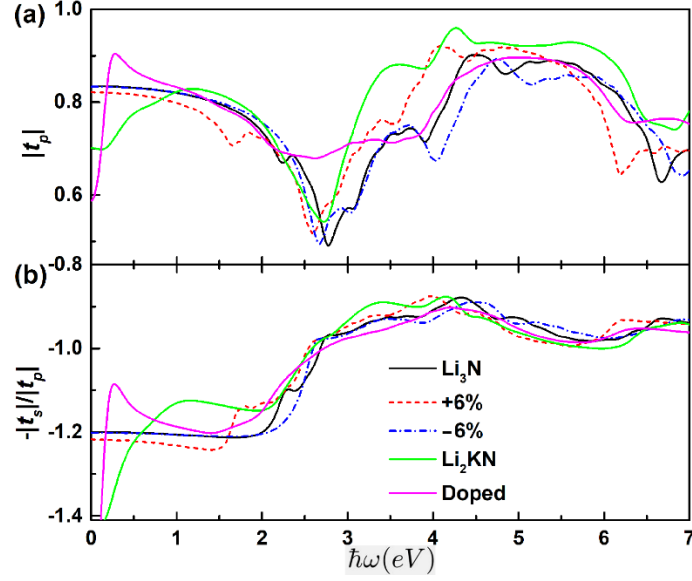
**Fig. S6** Spin Hall shifts of transmitted light for (a)  $H$ -polarized and (b)  $V$ -polarized light incident on  $\text{Na}_3\text{N}$  thin film. Optical spectra of (c) transmission coefficient  $|t_p|$  and (d)  $-|t_s|/|t_p|$  at different incident angles. The permittivity of exit medium is set as  $\epsilon_3 = 1.0$ .



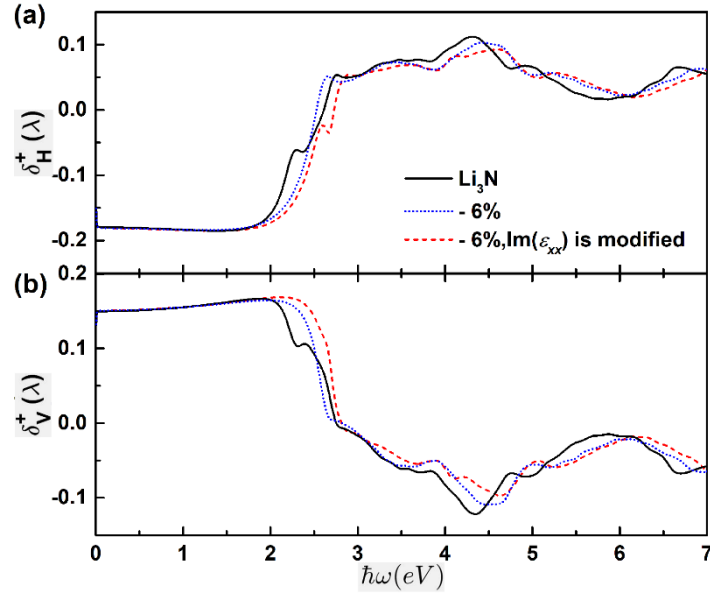
**Fig. S7** Spin Hall shifts of transmitted light for (a)  $H$ -polarized and (b)  $V$ -polarized light incident on  $\text{Li}_3\text{N}$  thin film. Optical spectra of (c) transmission coefficient  $|t_p|$  and (d)  $-|t_s|/|t_p|$  at different permittivities of exit medium. The incident angle is set as  $\theta_i = 30^\circ$ .



**Fig. S8** Spin Hall shifts of transmitted light for (a)  $H$ -polarized and (b)  $V$ -polarized light incident on  $\text{Li}_3\text{N}$  thin film. Optical spectra of (c) transmission coefficient  $|t_p|$  and (d)  $-|t_s|/|t_p|$  at different permittivities of exit medium. The incident angle is set as  $\theta_i = 50^\circ$ .



**Fig. S9** Optical spectra of (a)  $|t_p|$  and (b)  $-|t_s|/|t_p|$  for the samples of  $\text{Li}_3\text{N}$ , +6% and -6% strained  $\text{Li}_3\text{N}$  under lattice strain along the  $z$  axis,  $\text{Li}_2\text{KN}$ , and hole-doped (0.1 holes/u.c.)  $\text{Li}_3\text{N}$ . The incident angle and the permittivity of exit medium are set as  $\theta_i = 10^\circ$  and  $\varepsilon_3 = 1.0$ , respectively.



**Fig. S10** Spin Hall shifts of light after artificially shifting the whole spectrum of  $\text{Im}(\varepsilon_{xx})$  of -6% strained  $\text{Li}_3\text{N}$  toward high-energy side by 0.2 eV, and all the other factors are the same with those for -6% strained  $\text{Li}_3\text{N}$ . For comparison, the spin Hall shifts of light for  $H$ - and  $V$ -polarized light incident on thin films of  $\text{Li}_3\text{N}$  and -6% strained  $\text{Li}_3\text{N}$  are also shown. The incident angle and the permittivity of exit medium are set as  $\theta_i = 10^\circ$  and  $\varepsilon_3 = 1.0$ , respectively.

## References

- [1] C. Ambrosch-Draxl, S. Sagmeister, C. Meisenbichler and J. Spitaler, exciting-code, in press, <http://exciting-code.org>.
- [2] J. P. Perdew, K. Burke and M. Ernzerhof, *Phys. Rev Lett*, 1996, 77, 3865.
- [3] J. Heyd, G. E. Scuseria and M. Ernzerhof, *J. Chem. Phys.* 2003, 118, 8207-8215.
- [4] K. Hummer, J. Harl and G. Kresse, *Phys. Rev B* 2009, 80, 115205.
- [5] W. Hanke and L. J. Sham, *Phys. Rev B* 1988, 38, 13361.
- [6] G. Strinati, *La Rivista dell'Uovo Cimento* (1978-1999), 1988, 11, 1-86.
- [7] G. Onida, L. Reining, and A. Rubio, *Rev. Mod. Phys.* 2002, 74, 601.
- [8] P. Puschnig and C. Ambrosch-Draxl, *Phys. Rev B* 2002, 66, 165105.
- [9] M. Rohlfing and S. G. Louie, *Phys. Rev B* 2000, 62, 4927.
- [10] E. K. U. Gross and W. Kohn, *Phys. Rev Lett*, 1985, 55, 2850.